

Name

First

Last

Student ID #

University of California
Los Angeles
Computer Science Department

CSM51A/EEM16 Midterm Exam
Winter Quarter 2016
February 8th 2016

This is a closed book exam. Absolutely nothing is permitted except pen, pencil and eraser to write your solutions. Any academic dishonesty will be prosecuted to the full extent permissible by university regulations.

Time allowed 100 minutes.

Problem (possible points)	Points
1 (20)	20
2 (20)	20
3 (20)	20
4 (20)	12
5 (20)	20
Total (100)	92

20

Problem 1 (20 points)

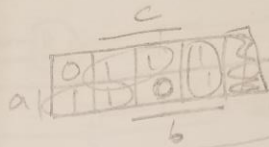
Use only the "E" gate defined below to implement Boolean function:

$$F = w'xy' + wxz + w'x'z + wx'y'z'$$

You may also use constants 0 and 1 as inputs.

a	b	c	E(a,b,c)
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

$a'b'c$
 $a'b'c'$
 $a'bc$
 $a'bc'$
 $ab'c$
 $ab'c'$
 abc

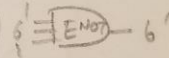


$$E(a,b,c) = ab' + a'c + bc'$$

$$F = w'xy' + wxz + w'x'z + wx'y'z'$$

NOT gate

$$E(1,b,1) = b'$$



$$E(0,0,c) = c$$

$$E(a,1,c) = a'c + c' = a' + c'$$

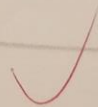
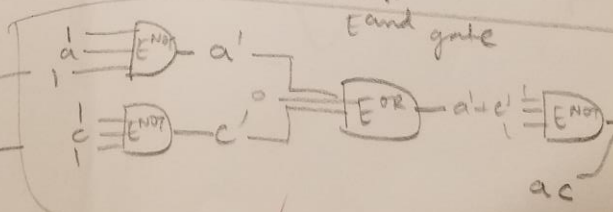
$$(a \cdot b)' = a' + b'$$

$$E(a,0,c) = a + a'c$$

$$E(a,0,c) = a + c$$

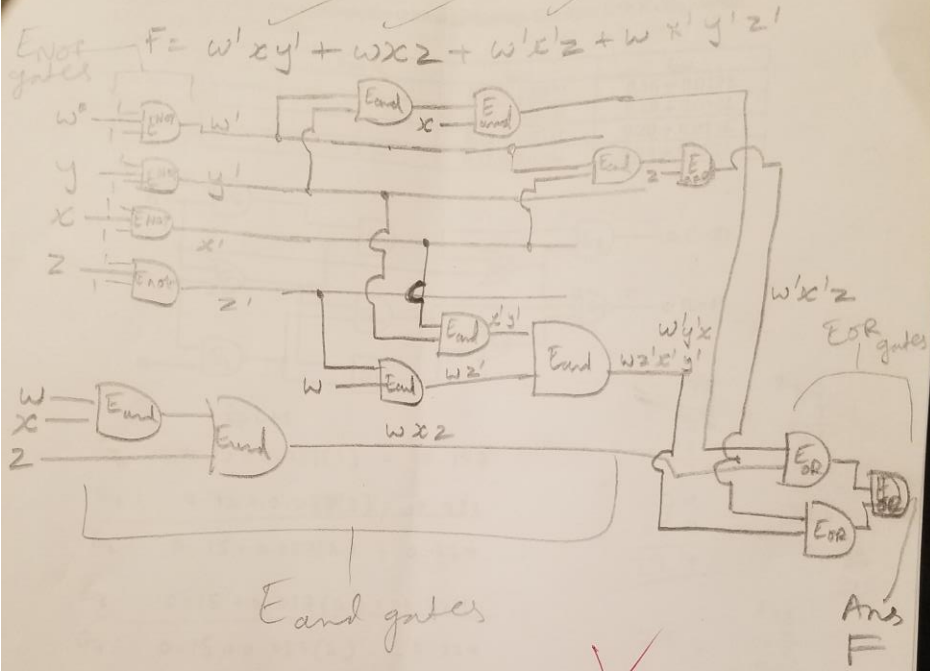


~~$(a+c)' = a'c'$~~
 $(a' + c')' = ac$



Problem 1) Extra Page

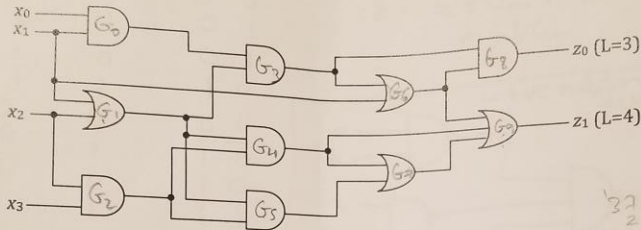
All-2-input type



Problem 2 (20 points)

Given the network below, calculate the critical path delay. Consider L → H delay when calculating the critical path.

Gate	Fan-in	t_{pLH}	t_{pHL}
AND	2	$0.15 + 0.037L$	$0.16 + 0.017L$
AND	3	$0.20 + 0.038L$	$0.18 + 0.018L$
OR	2	$0.12 + 0.037L$	$0.20 + 0.019L$
OR	3	$0.12 + 0.038L$	$0.34 + 0.022L$



t_{pLH}

$G_0 : 0.15 + 0.037(1) = 0.187$

$G_1 : 0.12 + 0.037(3) = 0.231$

$G_2 : 0.15 + 0.037(2) = 0.224$

$G_3 : 0.15 + 0.037(2) = 0.224$

$G_4 : 0.15 + 0.037(2) = 0.224$

$G_5 : 0.15 + 0.037(1) = 0.187$

$G_6 : 0.12 + 0.037(2) = 0.194$

$G_7 : 0.12 + 0.037(1) = 0.157$

$G_8 : 0.15 + 0.037(3) = 0.261$

$G_9 : 0.12 + 0.038(4) = 0.272$

Handwritten calculations for propagation delays:

$$\begin{array}{r} 37 \\ 2 \\ \hline 74 \\ 120 \\ \hline 194 \end{array}$$

$$\begin{array}{r} 277 \\ 3 \\ \hline 111 \\ 150 \\ 74 \\ \hline 277 \end{array}$$

$$\begin{array}{r} 74 \\ 150 \\ \hline 224 \end{array}$$

$$\begin{array}{r} 328 \\ 4 \\ \hline 152 \\ 120 \\ \hline 272 \end{array}$$

$$\begin{array}{r} 272 \\ 2 \\ \hline 194 \\ 224 \\ 231 \\ \hline 0.921 \end{array}$$

∴ Critical path delay = Delay for network (LH)

$G_1 G_3 G_6 G_9$

$= 0.272 + 0.194 + 0.224 + 0.272$

$= 0.921 \text{ ns}$

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Problem 3 (20 points)

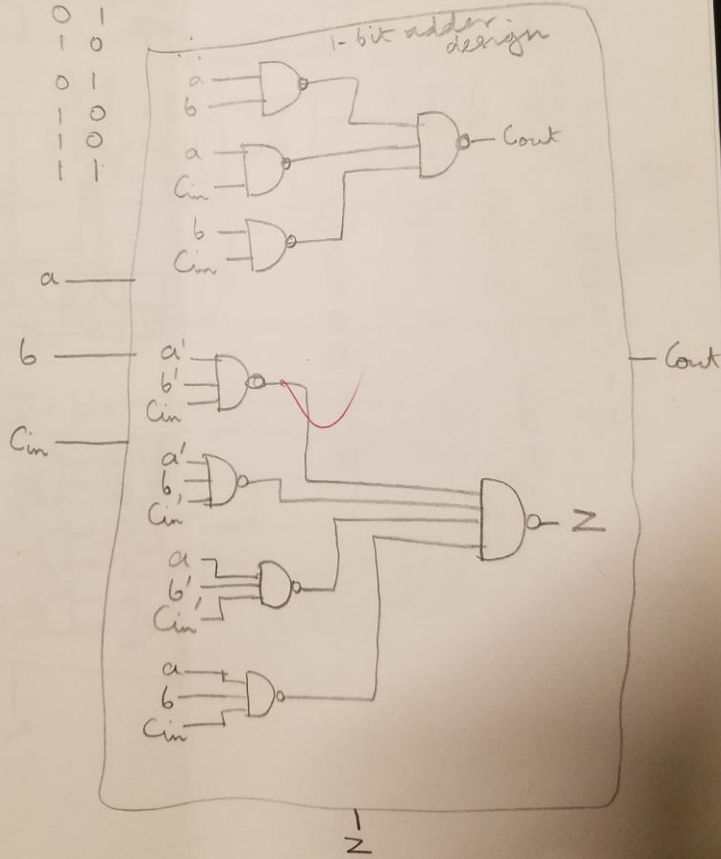
Four 4-bit numbers A, B, C, and D are given as inputs, $E=A+B$, $F=C+D$. Design a system that outputs the larger number between E and F. If $E=F$, output either E or F. You can use any type of gates to implement your design.

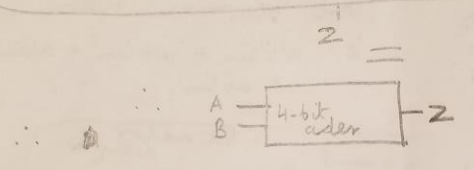
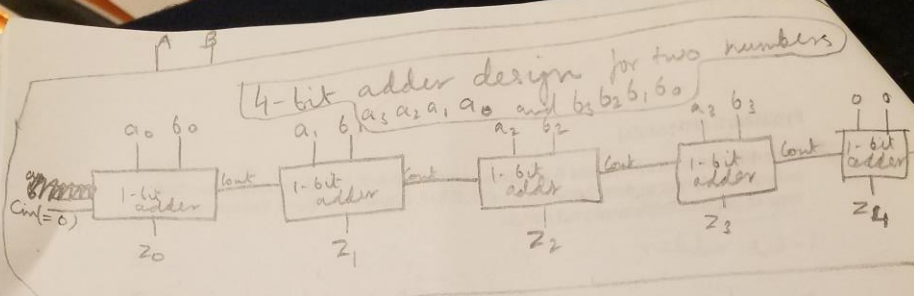
1-bit adder

a	b	Cin	Count	Z
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

$$\text{Count} = bC_{in} + aC_{in} + ab$$

$$Z = a'b'C_{in} + a'bC_{in}' + abC_{in}' + abC_{in}$$

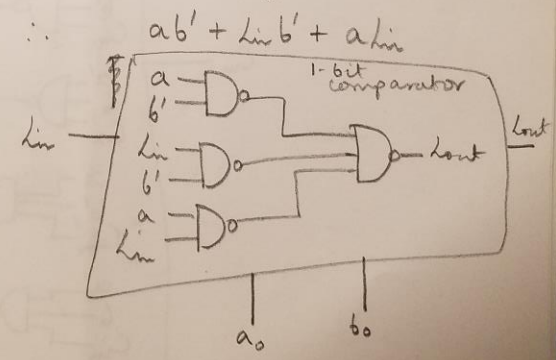
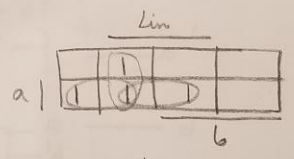




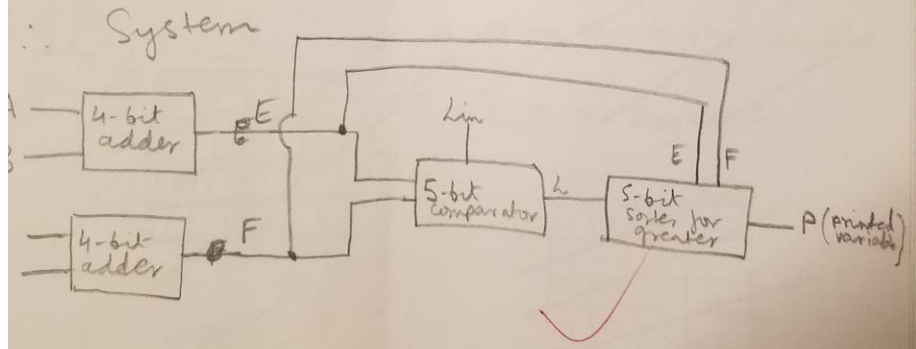
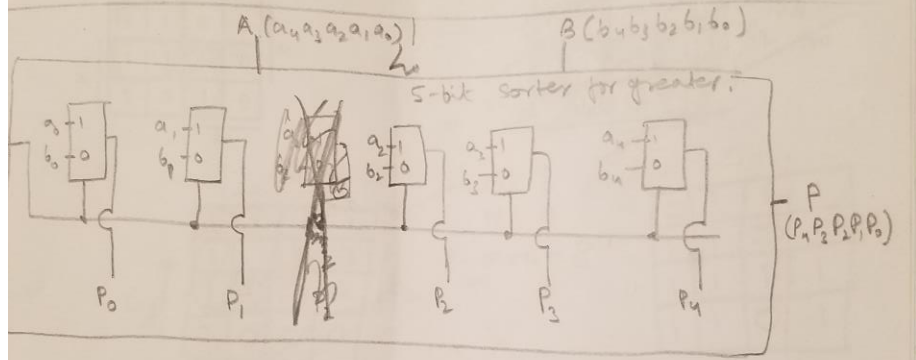
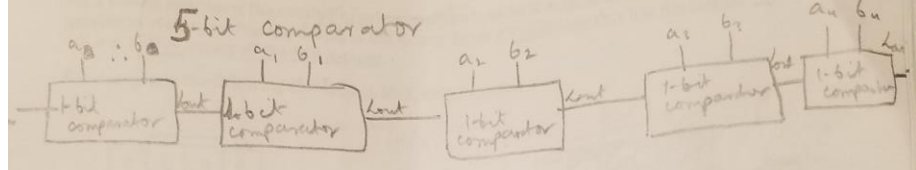
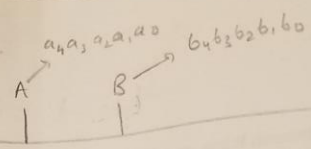
1-bit comparator

$l_{in} = 1$ if $a > b$
 0 if $a \leq b$

a	b	l_{in}	l_{out}
0	0	0	0
0	1	0	0
1	0	1	0
1	1	0	0



Problem 3) Extra Page



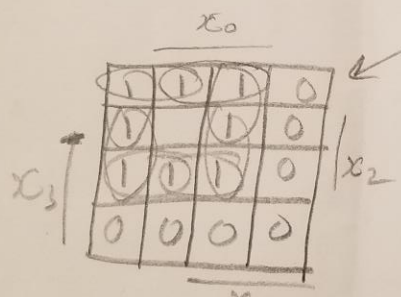
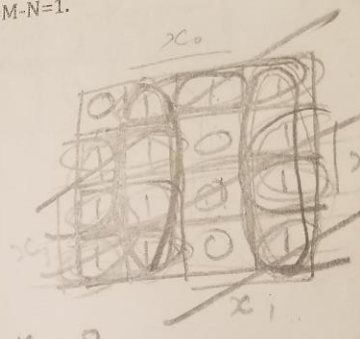
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Problem 4 (20 points)

For a K-map, M denotes the number of prime implicants of the K-map, and N denotes the number of essential prime implicants of the K-map. Draw a 4×4 K-map that has the largest value of $P=M-N$ among all the 4×4 K-maps.

For example, in the following 4×4 K-map, $M=3$, $N=2$, $P=M-N=1$.

	x_0				
	0	0	0	0	
	1	1	0	0	x_2
x_3	1	1	1	0	
	0	0	1	0	
	x_1				

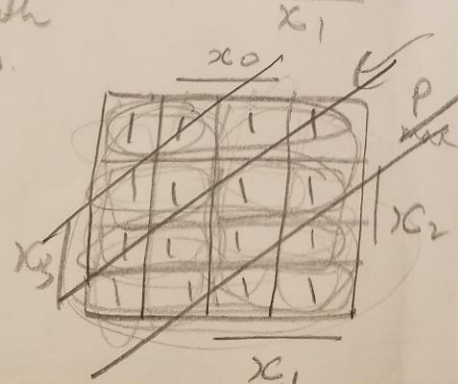


$M=8$
 $N=0$
 $P=M-N=8$
max

~~X~~

0	1
1	1
1	1
1	1

with
1s.



~~$M = \dots$ very high~~
 ~~$N = 0$~~
 ~~$P = M - N = \dots$ highest~~

0	0	1
0	0	1
0	0	1
0	0	1

Problem 5 (20 points)

Use only multiplexers to design a system with input $x \in \{0, 1, 2, \dots, 8\}$, outputs y and z that implements the following equation

In the system, x is encoded as $x_3x_2x_1x_0$ in binary, y is encoded as y_1y_0 in binary, and z is encoded as z_1z_0 in binary.

Note that the outputs y and z represent the two digits of a base-3 number. For example, if $x=7$ ($x_3x_2x_1x_0=0111$), then the system will solve: $(7)_{10} = (21)_3$. Thus $y = 2$ ($y_1y_0=10$) and $z = 1$ ($z_1z_0=01$).

20

x_3	x_2	x_1	x_0	y_1	y_0	z_1	z_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	1	0	0
0	1	0	0	0	1	0	0
0	1	0	1	0	1	0	1
0	1	1	0	1	0	0	0
0	1	1	1	1	0	0	1
1	0	0	0	1	0	1	0
1	0	0	1	1	0	1	0
1	0	1	0	-	-	-	-
1	0	1	1	-	-	-	-
1	1	0	0	-	-	-	-
1	1	0	1	-	-	-	-
1	1	1	0	-	-	-	-
1	1	1	1	-	-	-	-

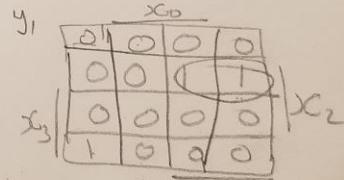
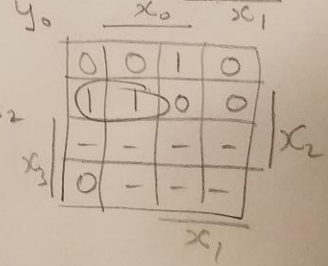
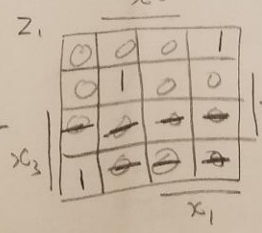
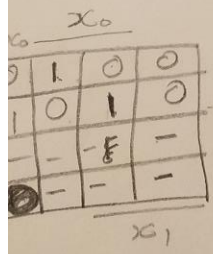
22
 $6 + 2 = 8$

$z_0 = x_3'x_2'x_1x_0 + x_3'x_2x_1x_0' + x_3'x_2x_1x_0$

$z_1 = x_3'x_2'x_1x_0' + x_3'x_2x_1x_0' + x_3x_2'x_1x_0'$

$y_0 = x_3'x_2'x_1x_0 + x_3'x_2x_1x_0' + x_3'x_2x_1x_0$

$y_1 = x_3'x_2x_1x_0' + x_3'x_2x_1x_0 + x_3x_2'x_1x_0'$



Problem 5) Extra Page

$$y_1 = x_2 x_1 x_3' + x_3 x_2' x_1' x_0'$$

$$y_1 x_3 x_2' = x_1' x_0'$$

$$y_1 x_3 x_2' x_1' = x_0'$$

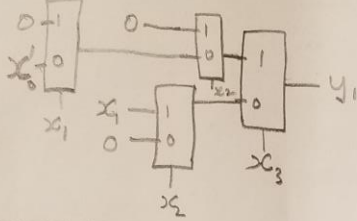
$$y_{x_3} = x_2' x_1' x_0'$$

$$y_{x_2} = x_2 x_1$$

$$y_{x_3 x_2} = 0$$

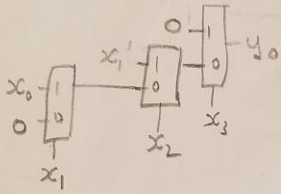
$$y_{x_2 x_1} = x_1$$

$$y_{x_3 x_1} = 0$$



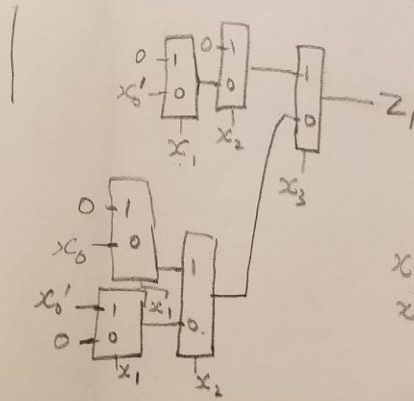
$$y_0 = x_3' x_2 x_1' + x_3' x_2' x_1 x_0$$

$$x_2 x_1' + x_2' x_1 x_0$$



$$z_1 = x_3' x_2' x_1' x_0' + x_3' x_2 x_1' x_0 + x_3 x_2' x_1 x_0'$$

$$x_2' x_1' x_0'$$



$$x_2' x_1' x_0'$$

$$x_2' x_1' x_0$$