# [CS M51A F19] Solution to Quiz 3

Date: 11/15/19

TAs

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## Quiz Problems (30 points total)

### Problem 1 (15 points)

Design a pattern recognizer which takes a stream of binary bits as input, one bit at each clock. The output is 1 when it detects a string of 001, 011 or 100. The input signal is $x(t)$, and the output signal is $z(t)$.

1. **(7 points)** Fill in the empty slots in the state transition and output table.

   ***Solution***

| $PS$ | $x = 0$ | $x = 1$ |
|---|---|---|
| $S_{init}$ | $S_0$, 0 | $S_1$, 0 |
| $S_0$ | $S_{00}$, 0 | $S_{01}$, 0 |
| $S_1$ | $S_{10}$, 0 | $S_{11}$, 0 |
| $S_{00}$ | $S_{00}$, 0 | $S_{01}$, 1 |
| $S_{01}$ | $S_{10}$, 0 | $S_{11}$, 1 |
| $S_{10}$ | $S_{00}$, 1 | $S_{01}$, 0 |
| $S_{11}$ | $S_{10}$, 0 | $S_{11}$, 0 |
| | $NS$, $z$ | |

2. **(8 points)** Minimize the number of states in the transition table, and show the final minimized table. Use $G_1$, $G_2$, ... for the states in the minimal state table.

   ***Solution*** Looking at the output of the previous table, we first get $P_1$ as shown:

   $P_1 = (S_{init}, S_0, S_1, S_{11})$ (output 0/0), $(S_{00}, S_{01})$ (output 0/1), and $(S_{10})$ (output is 1/0).

| | group 1 | | | | group 2 | | g3 |
|---|---|---|---|---|---|---|---|
| | $S_{init}$ | $S_0$ | $S_1$ | $S_{11}$ | $S_{00}$ | $S_{01}$ | $S_{10}$ |
| 0 | 1 | 2 | 3 | 3 | 2 | 3 | |
| 1 | 1 | 2 | 1 | 1 | 2 | 1 | |

$P_2 = (S_{init}), (S_0), (S_1, S_{11}), (S_{00}), (S_{01}), (S_{10})$

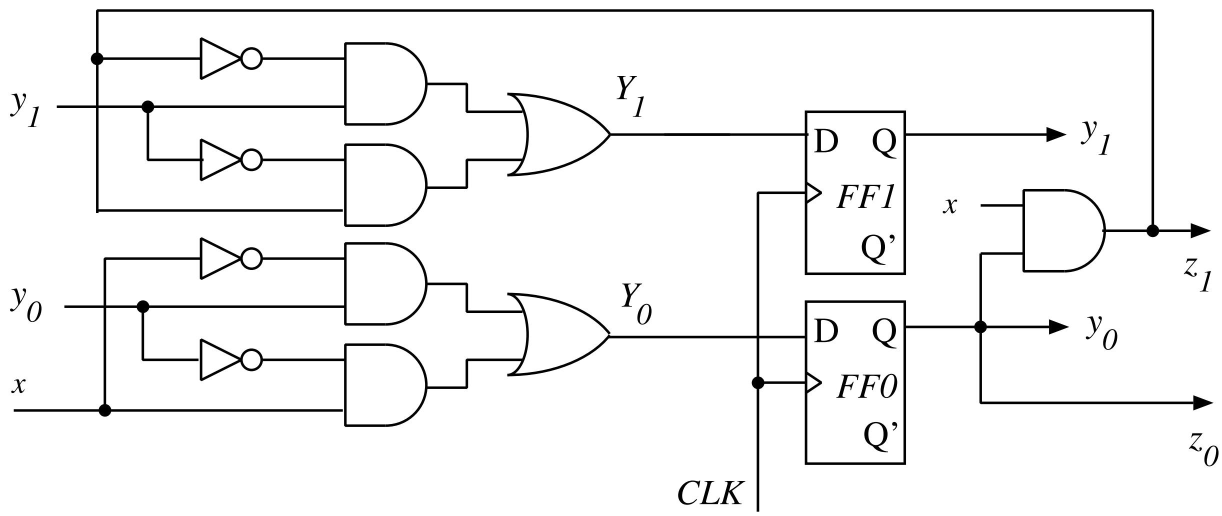
| | g1 | g2 | g3 | | g4 | g5 | g6 |
|---|---|---|---|---|---|---|---|
| | $S_{init}$ | $S_0$ | $S_1$ | $S_{11}$ | $S_{00}$ | $S_{01}$ | $S_{10}$ |
| 0 | | | 6 | 6 | | | |
| 1 | | | 3 | 3 | | | |

Since $P_3 = P_2$, we stop . By naming each group, we can write the following table.

| $PS$ | $x = 0$ | $x = 1$ |
|---|---|---|
| $G1$ | $G2$, 0 | $G3$, 0 |
| $G2$ | $G4$, 0 | $G5$, 0 |
| $G3$ | $G6$, 0 | $G3$, 0 |
| $G4$ | $G4$, 0 | $G5$, 1 |
| $G5$ | $G6$, 0 | $G3$, 1 |
| $G6$ | $G4$, 1 | $G5$, 0 |
| | $NS$, $z$ | |

## Problem 2 (15 points)

For the canonical sequential network shown bellow, determine:




**a. (3 points)** The switching expressions for the next-state variables and the outputs:

$Y_1 =$

$Y_0 =$

$z_1 =$

$z_0 =$

***Solution***

$$Y_1 = xy_0 \oplus y_1$$

$$Y_0 = x \oplus y_0$$

$$z_1 = xy_0$$
$$z_0 = y_0$$

**b. (4 points)** The next-state and the output table (binary-level):

| $PS$ | $y_1y_0$ | $x=0$ | $x=1$ |
|---|---|---|---|
| $A$ | 00 | $00, 00$ | $01, 00$ |
| $B$ | 01 | | |
| $C$ | 10 | | |
| $D$ | 11 | | |
| | | $Y_1Y_0,\ z_1z_0$ | |

***Solution***

| $PS$ | $y_1y_0$ | $x=0$ | $x=1$ |
|---|---|---|---|
| $A$ | 00 | $00,00$ | $01,00$ |
| $B$ | 01 | $01,01$ | $10,11$ |
| $C$ | 10 | $10,00$ | $11,00$ |
| $D$ | 11 | $11,01$ | $00,11$ |
| | | $Y_1Y_0,\ z_1z_0$ | |

**c. (4 points)** The next-state and the output table (high-level). Let $z = 2z_1 + z_0$:

| $PS$ | $x=0$ | $x=1$ |
|---|---|---|
| A | $A$, 0 | $B$, 0 |
| B | | |
| C | | |
| D | | |
| | $NS,\ z$ | |

***Solution***

| $PS$ | $x=0$ | $x=1$ |
|---|---|---|
| A | $A$, 0 | $B$, 0 |
| B | $B$,1 | $C$,3 |
| C | $C$,0 | $D$,0 |
| D | $D$,1 | $A$,3 |
| | $NS,\ z$ | |

**d. (2 points)** Is this a minimal FSM? If not, find the minimal FSM.

***Solution***

Not minimal.

$P_1 = (\{A, C\}, \{B, D\}) = P_2$. The minimal FSM is

| $PS$ | $x=0$ | $x=1$ |
|---|---|---|
| A | $A$, 0 | $B$, 0 |
| B | $B$,1 | $A$,3 |
| | $NS,\ z$ | |

If the initial state is $A$ and the input is $x(0,5) = 011010$, show $z(0,5)$

***Solution***

| $t$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $x(t)$ | 0 | 1 | 1 | 0 | 1 | 0 |
| $s(t)$ | A | A | B | C | C | D |
| $z(t)$ | 0 | 0 | 3 | 0 | 0 | 1 |