[CS M51A Fall 18] Midterm exam

Date: 10/30/18

- The midterm is closed book, and up to 4 sheets (= 8 pages) of summary notes are allowed. You can use a calculator but not smart phones.
- For multiple choice questions, wrong answers may have a negative score value so choose carefully. It is possible that questions can have multiple answers.
- Please show all your work and write legibly, otherwise no partial credit will be given.
- **This should strictly be your own work; any form of collaboration will be penalized.**

Name : ______________________

Student ID : ______________________

Problem	Points	Score
1	10	10
2	15	15
3	15	9
4	15	15
5	20	20
6	10	4
7	~~15~~	2
Total	~~100~~ 94	75

Problem 1 (10 points)

1. **(4 points)** How many bits are required to encode a color spectrum capable of supporting 16 million co[lors] using:

   a. Decimal digits in BCD

   16,000,000

   4 bits per digit

   8 digit places

   32 bits

   b. Hexadecimal representation

   $16^6 = 16777216$ possible #s

   So only 6 bits needed

   Which representation is more efficient? Why?

   Hexadecimal is more efficient since it can represent more numbers with less bits needed.

2. **(6 points)** Fill in the missing entries in the table.

Radix	Digit vector $\underline{x}$	Value x in decimal
16	(5, 1, 7)	1303
8	(5, 1, 7)	335
7	4532	1640

$517_{16} \rightarrow X_{10}$

$5 \times 16^2 + 1 \times 16^1 + 7 \times 16^0 = 1303_{10}$

$517_8 \rightarrow X_{10}$

$5 \times 8^2 + 1 \times 8^1 + 7 \times 8^0 = 335$

$1640 \div 7 = 234$ R2 ($1640 - 1638 = 2$)

$234 \div 7 = 33$ R3 ($234 - 231 = 3$)

$33 \div 7 = 4$ R5 ($33 - 28 = 5$)

$4 \div 7 = 0$ R4

em 2 (15 points)

$a+b=b+a$	$ab=ba$	Commutativity
$a+(bc)=(a+b)(a+c)$	$a(b+c)=(ab)+(ac)$	Distributivity
$a+(b+c)=(a+b)+c=a+b+c$	$a(bc)=(ab)c=abc$	Associativity
$a+a=a$	$aa=a$	Idempotency
$a+a'=1$	$aa'=0$	Complement
$1+a=1$	$0a=0$	
$0+a=a$	$1a=a$	Identity
$(a')'=a$		Involution
$a+ab=a$	$a(a+b)=a$	Absorption
$a+a'b=a+b$	$a(a'+b)=ab$	Simplification
$(a+b)'=a'b'$	$(ab)'=a'+b'$	DeMorgan's law

Given $E(a,b,c,d)=(ab+c)'(ac+(b'+c'+a'cd)')+a((b+c)(b+d)+c)'$, which of the following represents the same function as $E(a,b,c,d)$? Show all your work.

1. $a+b+c+d'$
2. $a'+b+c$
3. $b+c'+d$
4. $a'b'c'd$
5. $ab'c'$
6. $b'cd'$

$(ab+c)'(ac+(b'+c'+a'cd)')+a((b+c)(b+d)+c)'$

$((ab)'\cdot c')(ac+(bc(a+c'+d)))+a((b+c)'+(b+d)'\cdot c')$ — DeMorgans

$[(a'+b')c'][ac+abc+bcd]+[a(b'c'+b'd'c')]$ — Distributive, Complement

$[a'c'+b'c'][ac+abc+bcd]+ab'c'+ab'd'c'$ — Distributive

$0+0+0+0+0+0+0+ab'c'+ab'd'c'$

$ab'c'+ab'd'c'$ — Distributive, Complement

$ab'c'(1+d')$ — Distributive

$ab'c'$ — Complement

$\boxed{ab'c'}$ #5

Problem 3 (15 points)

Show if the gate G, described by $G(x, y, z) = one - set\{3, 4, 6, 7\}$, can implement NOT and AND gates. Ass that 0 and 1 are available. If it can, then use G gates to implement the following expression and show t corresponding network of G gates

$$E(a, b, c) = (a + b')(b + c')$$

$G(x,y,z) = x'yz + xy'z' + xyz' + xyz = x'yz + xy'z' + xy$

x \ yz	00	01	11	10
0	(0)	(1)	(3) 1	(2)
1	(4) 1	(5)	(7) 1	(6) 1

$G(x,y,z) = xz' + yz$

$G(1,0,z) = z' + 0 = z'$ NOT Gate ✓

$G(0,y,z) = 0 + yz = yz$ AND Gate ✓

Yes, it can implement {AND, NOT}

$E(a,b,c) = (a+b')(b+c')$

$c' \Rightarrow G(1,0,c) = c'$ OR $\Rightarrow$ NAND − NAND

$b' \Rightarrow G(1,0,b) = b'$

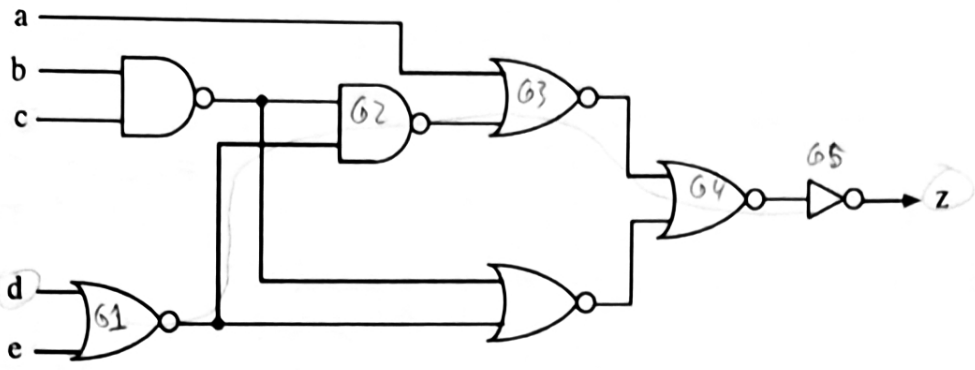
$G(1,0,G(0,a,G(1,0,b))) \Rightarrow [ab']'$

$G(1,0,G(0,$

9

...em 4 (15 points)

...the help of the table below, determine the low to high propagation delay $t_{pLH}(d, z)$ of the output z of the ...work shown below. Assume the network output has a load of 6.



Gate Type	Fan-in	Propagation Delays (ns) t_{pLH}	t_{pHL}	Load Factor
NOT	1	$0.02 + 0.038L$	$0.05 + 0.017L$	1.0
NAND	2	$0.05 + 0.038L$	$0.08 + 0.027L$	1.0
NOR	2	$0.06 + 0.075L$	$0.07 + 0.016L$	1.0

G1-2NOR → G2-2NAND → G3-2 NOR → G4-2 NOR → G5-NOT

L=2 L=1 L=1 L=1 L=6

0.06+0.075L 0.08+0.027L 0.06+0.075L 0.07+0.016L 0.02+0.038L

= 0.21 0.107 0.135 0.086 0.248

= $\boxed{0.786 \text{ ns}}$

15

Problem 5 (20 points)

Obtain a two-level gate network of the following system.

Inputs:	$x, y \in \{0, 1, 2, 3\}$
Outputs:	$z \in \{0, 1, 2, 3\}$
Function:	$z = \{3xy + 1\} \bmod 4$

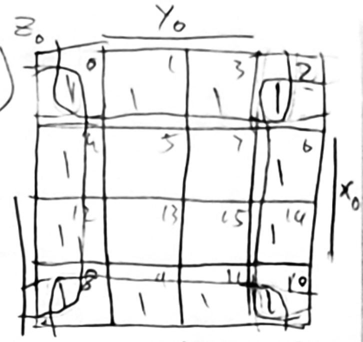
1. **(2 points)** Complete the switching table using binary encoding for all values.

$x \backslash y$	0	1	2	3
0	1	1	1	1
1	1	0	3	2
2	1	3	1	3
3	1	2	3	0

	x_1	x_0	y_1	y_0	z_1	z_0
0	0	0	0	0	0	1
1	0	0	0	1	0	1
2	0	0	1	0	0	1
3	0	0	1	1	0	1
4	0	1	0	0	0	1
5	0	1	0	1	0	0
6	0	1	1	0	1	1
7	0	1	1	1	1	0
8	1	0	0	0	0	1
9	1	0	0	1	1	1
10	1	0	1	0	0	1
11	1	0	1	1	1	1
12	1	1	0	0	0	1
13	1	1	0	1	1	0
14	1	1	1	0	1	1
15	1	1	1	1	0	0

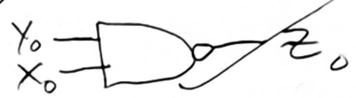
20

#3 (cont)



$z_0 = [(y_0' + x_0')']'$

$= (y_0 x_0)'$

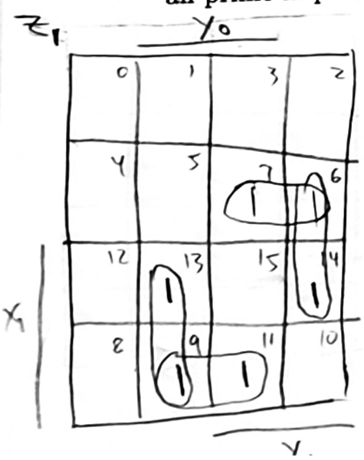


2. **(5 points)** Show the switching expressions of z_1 and z_0 in sum of minterms form.

$z_1 = x_1'x_0y_1y_0' + x_1'x_0y_1y_0 + x_1x_0'y_1'y_0 + x_1x_0'y_1y_0 + x_1x_0y_1'y_0 + x_1x_0y_1y_0'$

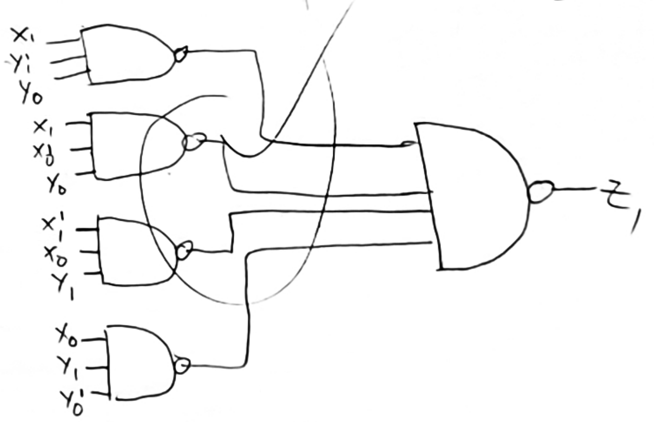
$z_0 = x_1'x_0'y_1'y_0' + x_1'x_0'y_1'y_0 + x_1'x_0'y_1y_0' + x_1'x_0'y_1y_0 + x_1'x_0y_1'y_0' + x_1'x_0y_1y_0' + x_1'x_0'y_1'y_0' + x_1x_0'y_1'y_0 + x_1x_0'y_1y_0' + x_1x_0'y_1y_0 + x_1x_0y_1'y_0' + x_1x_0y_1y_0'$

3. **(8 points)** Show the minimal sum of products expressions of z_1 and z_0. In each case, show a K-map, indicate all prime implicants, and all essential prime implicants. Show NAND-NAND networks.



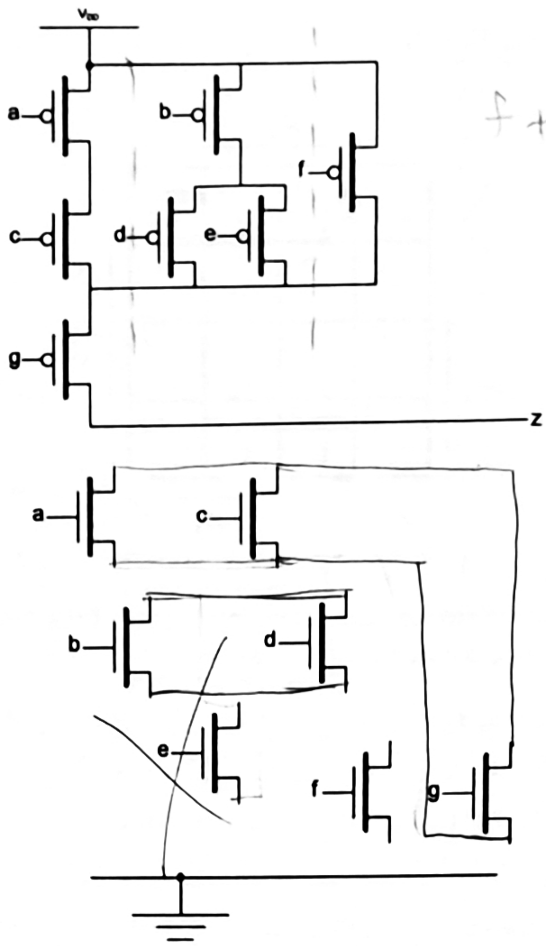
$z_1 = x_1y_1'y_0 + x_1x_0'y_0 + x_1'x_0y_1 + x_0y_1y_0'$

$z_1 = [(x_1y_1'y_0)'(x_1x_0'y_0)'(x_1'x_0y_1)'(x_0y_1y_0')']'$



,em 6 (10 points)

are given the following partial CMOS network.



$f + [b(d+e)] + acg$

1. **(5 points)**

   Write the expression for the pull-up network. From this, derive the expression for the pull-down network using switching algebra.

2. **(5 points)** Connect NMOS transistors to complete the circuit according to the pull-down expression. Please only add missing wires.

1) $a'c'g$, $b'(d'+e')$, f' $\boxed{z = a'c'g' + b'(d'+e') + f'}$

$z' = [a'c'g' + b'd' + b'e' + f']' = (a'c'g')'(b'd')'(b'e')' f = (a+c+g)(b+d)(b+e)f$

$z' = (a+c+g)(b+d)(b+e)f$

11 9 6 3

Problem 7 (15 points)

For $f(x_3, x_2, x_1, x_0) = (x_3 + x_2' + x_1 + x_0)(x_3 + x_2' + x_1' + x_0)(x_3' + x_2 + x_1 + x_0')(x_3' + x_2' + x_1 + x_0)(x_3' +$
$x_1 + x_0')(x_3' + x_2' + x_1' + x_0)$

2

1. **(2 points)** Fill out the following K-map.

		x_0	x_0	
	0: 0	1: 1	3: 1	2: 1
x_2	4: 0	5: 0	7: 0	6: 1
x_3, x_2	12: 0	13: 0	15: 0	14: 0
x_3	8: 0	9: 1	11: 1	10: 0
			x_1	x_1

, 2

$x_2'x_0$

$x_3'x_1x_0'$

$x_3'x_2'x_1$

2. **(4 points)** Which of the given expressions are prime implicants of the function given above? Circle **all that** apply. Write down any prime implicants that are missing.

(a) x_1	(d) $x_3'x_1$	(g) $x_2'x_0$	(j) $x_3'x_2'x_1$
(b) x_3x_1	(e) $x_3'x_0$	(h) x_1x_0	(k) $x_2x_1x_0$
(c) $x_3'x_2'$	(f) x_2x_1	(i) x_1x_0'	(l) $x_3x_2x_1x_0$

$x_3'x_1x_0'$

, 3

3. **(2 points)** Write down the complete set of **essential** prime implicants.

$x_2'x_0$, $x_3'x_1x_0'$

, 2

4. **(1 point)** Write the minimal sum of products expression for f. Is it unique?

$x_2'x_0 + x_3'x_1x_0'$, yes it is unique