[CS M51A Fall 18] Midterm exam

Date: 10/30/18

- The midterm is closed book, and up to 4 sheets (= 8 pages) of summary notes are allowed. You can use a calculator but not smart phones.
- For multiple choice questions, wrong answers may have a negative score value so choose carefully. It is possible that questions can have multiple answers.
- Please show all your work and write legibly, otherwise no partial credit will be given.
- **This should strictly be your own work; any form of collaboration will be penalized.**

Name : ______________________________

Student ID : ______________________________

Problem	Points	Score
1	10	10
2	15	15
3	15	11
4	15	15
5	20	20.
6	10	10.
7	~~15~~ 9	8
Total	~~100~~ 94	89

16,000,000

Problem 1 (10 points)

1. **(4 points)** How many bits are required to encode a color spectrum capable of supporting 16 million colors using:

   a. Decimal digits in BCD

   $4 \times 8 = 32$ (although 3 can be eliminated since the tens millions place has a max value of 1)

   b. Hexadecimal representation

   F _ _ _ _ _

   16^5 16^4 16^3 16^2 16^1 16^0

   $6 \Rightarrow 6 \times 4 = 24$

   Which representation is more efficient? Why?

   Hexadecimal is more efficient because it takes up considerably fewer bits than BCD does.

2. **(6 points)** Fill in the missing entries in the table.

Radix	Digit vector $\underline{x}$	Value x in decimal
16	(5, 1, 7)	1303
8	(5, 1, 7)	335
7	(4, 5, 3, 2)	1640

5 1 7

16^2 16^1 16^0

4 5 3 2

7^3 7^2 7^1 7^0

5 1 7

8^2 8^1 8^0

Problem 2 (15 points)

$a+b=b+a$	$ab=ba$	Commutativity
$a+(bc)=(a+b)(a+c)$	$a(b+c)=(ab)+(ac)$	Distributivity
$a+(b+c)=(a+b)+c=a+b+c$	$a(bc)=(ab)c=abc$	Associativity
$a+a=a$	$aa=a$	Idempotency
$a+a'=1$	$aa'=0$	Complement
$1+a=1$	$0a=0$	
$0+a=a$	$1a=a$	Identity
$(a')'=a$		Involution
$a+ab=a$	$a(a+b)=a$	Absorption
$a+a'b=a+b$	$a(a'+b)=ab$	Simplification
$(a+b)'=a'b'$	$(ab)'=a'+b'$	DeMorgan's law

0 + 0 ← simplification

Given $E(a,b,c,d)=(ab+c)'(ac+(b'+c'+a'cd)')+a((b+c)(b+d)+c)'$, which of the following represents the same function as $E(a,b,c,d)$? Show all your work.

b + cd ← distributivity

1. $a+b+c+d'$
2. $a'+b+c$
3. $b+c'+d$
4. $a'b'c'd$
5. $ab'c'$
6. $b'cd'$

← absorption

$E(a,b,c,d) = (ab+c)'(ac+(b'+c'+a'd)')+a(b+c\cancel{d}+c)'$

demorgan's → $E(a,b,c,d) = (a'+b')c'(ac+bc(a+d'))+a(b'c')$

distributivity → $= c'((a'+b')(ac+bc(a+d'))+ab')$

distributivity → $= c'((a'+b')(ac+abc+bcd')+ab')$

distributivity → $= c'(a'\cancel{a}c + a'\cancel{a}bc + a'bcd' + b'ac + b'\cancel{abc}$
$+ \cancel{b'b}cd' + ab')$

complements → $= c'(a'bcd' + b'ac + ab')$

distributivity → $= \cancel{a'bcc'd'} + \cancel{b'acc'} + ab'c'$

complements → $= ab'c'$

Problem 3 (15 points)

Show if the gate G, described by $G(x, y, z) = one - set\{3, 4, 6, 7\}$, can implement NOT and AND gates. Assume that 0 and 1 are available. If it can, then use G gates to implement the following expression and show the corresponding network of G gates

$$E(a, b, c) = (a + b')(b + c')$$

	x	y	z	G
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

$G = x'yz + xy'z' + xyz' + xyz$

		z		
	0	0	1	0
x	1	0	1	1
			y	

$G = yz + xz'$

$G(1, 0, z) = z'$ NOT ✓

$G(0, y, z) = yz$ AND ✓

$E(a,b,c) = ab + \overset{\text{complement}}{\cancel{bb'}} + ac' + b'c'$ ✓ distributivity

N/W ~~circuit~~ missing

$= ab + ac' + b'c'$

$= \left((ab)' \cdot (ac')' \cdot (b'c')'\right)'$ NOT(AND(NOT(b), NOT(c)))

NOT(AND(a, b)) NOT(AND(a, NOT(c)))

11

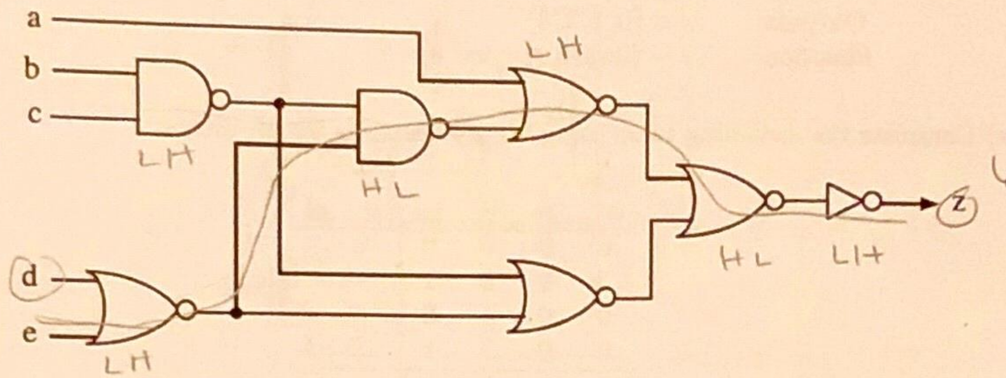
12 G gates

$= \text{NOT}(\text{AND}(\quad , \text{AND}(\quad , \quad)))$

$= G(1,0, G(0, G(1,0, G(0,a,b)), G(0, G(1,0, G(0,a,G(1,0,c))), G(1,0,G(0,G(1,0,b),G(1,0,c)))))))$

Problem 4 (15 points)

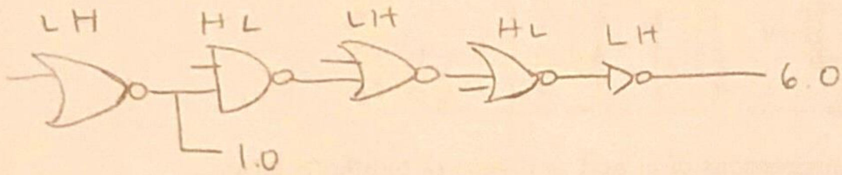
With the help of the table below, determine the low to high propagation delay $t_{pLH}(d, z)$ of the output z of the network shown below. Assume the network output has a load of 6.



critical path

NOR worse than NAND for LH

Gate Type	Fan-in	Propagation Delays (ns) t_{pLH}	Propagation Delays (ns) t_{pHL}	Load Factor
NOT	1	$0.02 + 0.038L$	$0.05 + 0.017L$	1.0
NAND	2	$0.05 + 0.038L$	$0.08 + 0.027L$	1.0
NOR	2	$0.06 + 0.075L$	$0.07 + 0.016L$	1.0



$0.06 + 0.075(2) = 0.21$

$0.08 + 0.027(1) = 0.107$

$0.06 + 0.075(1) = 0.135$

$0.07 + 0.016(1) = 0.086$

$0.02 + 0.038(6) = 0.248$

$\boxed{t_{pLH}(d,z) = 0.786}$

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Problem 5 (20 points)

Obtain a two-level gate network of the following system.

Inputs:	$x, y \in \{0,1,2,3\}$
Outputs:	$z \in \{0,1,2,3\}$
Function:	$z = \{3xy+1\} \bmod 4$

1. **(2 points)** Complete the switching table using binary encoding for all values.

3xy+1	x	y	x_1	x_0	y_1	y_0	z_1	z_0	z	
1	0	0	0	0	0	0	0	1	1	0
1	0	1	0	0	0	1	0	1	1	1
1	0	2	0	0	1	0	0	1	1	2
1	0	3	0	0	1	1	0	1	1	3
1	1	0	0	1	0	0	0	1	1	4
4	1	1	0	1	0	1	0	0	0	5
7	1	2	0	1	1	0	1	1	3	6
10	1	3	0	1	1	1	1	0	2	7
1	2	0	1	0	0	0	0	1	1	8
7	2	1	1	0	0	1	1	1	3	9
13	2	2	1	0	1	0	0	1	1	10
19	2	3	1	0	1	1	1	1	3	11
1	3	0	1	1	0	0	0	1	1	12
10	3	1	1	1	0	1	1	0	2	13
19	3	2	1	1	1	0	1	1	3	14
28	3	3	1	1	1	1	0	0	0	15

2. **(5 points)** Show the switching expressions of z_1 and z_0 in sum of minterms form.

$Z_1 = \sum m(6,7,9,11,13,14)$

$Z_0 = \sum m(0,1,2,3,4,6,8,9,10,11,12,14)$

20

assume inputs are complemented.

3. **(8 points)** Show the minimal sum of products expressions of z_1 and z_0. In each case, show a K-map, indicate all prime implicants, and all essential prime implicants. Show NAND-NAND networks.

Z_1:

			y_0	y_0	
	0	0	0	0	
x_0	0	0	1	1	
x_0	0	1	0	1	x_1
	0	1	1	0	x_1
			y_1	y_1	

PI: $y_0 y_1' x_1$, $y_0 x_0' x_1$, $y_1 x_0 x_1'$, $y_0' y_1 x_0$

All are essential.

$$\boxed{Z_1 = y_0 y_1' x_1 + y_0 x_0' x_1 + y_1 x_0 x_1' + y_0' y_1 x_0}$$

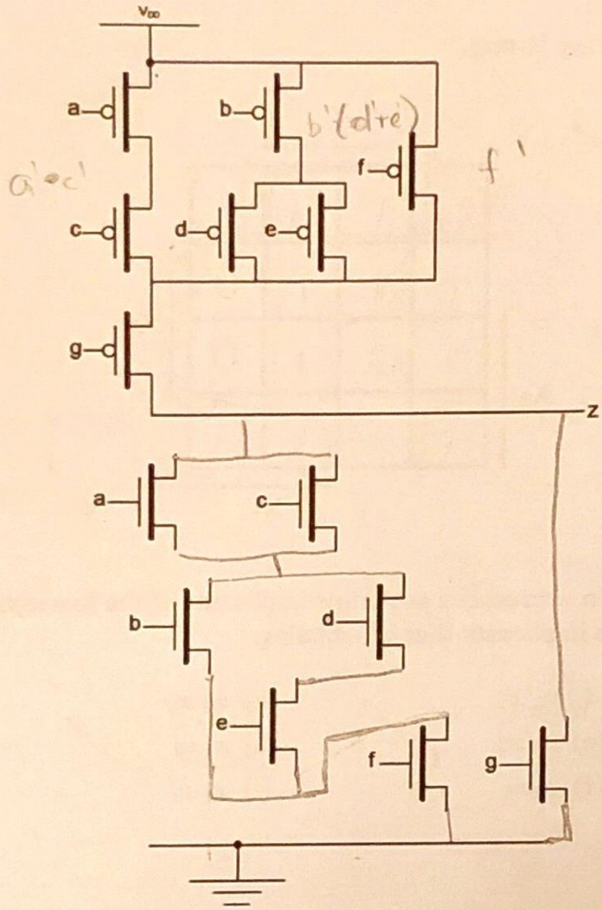
⇓ NAND-NAND

$$Z_1 = ((y_0 y_1' x_1)' \cdot (y_0 x_0' x_1)' \cdot (y_1 x_0 x_1')' \cdot (y_0' y_1 x_0)')'$$

(Z_0 is at back of exam)

Problem 6 (10 points)

We are given the following partial CMOS network.



1. **(5 points)**
   Write the expression for the pull-up network. From this, derive the expression for the pull-down network using switching algebra.
2. **(5 points)** Connect NMOS transistors to complete the circuit according to the pull-down expression. Please only add missing wires.

1.

$$z = g'(a'c' + b'(d'+e') + f')$$

$$z' = (g'(a'c' + b'(d'+e') + f'))' \leftarrow \text{de morgan's}$$

$$= g + (a+c)(b+de)(f)$$

Problem 7 (15 points) M_4 M_6 M_9 M_{12}

For $f(x_3, x_2, x_1, x_0) = (x_3 + x_2' + x_1 + x_0)(x_3 + x_2' + x_1' + x_0)(x_3' + x_2 + x_1 + x_0')(x_3' + x_2' + x_1 + x_0)(x_3' + x_2' + x_1 + x_0')(x_3' + x_2' + x_1' + x_0)$ M_{13}

M_{14}

1. **(2 points)** Fill out the following K-map.

X_0

1	1	1	1
0	1	1	0
0	0	1	0
1	0	1	1

X_2

X_3

X_1

2. **(4 points)** Which of the given expressions are prime implicants of the function given above? Circle all that apply. Write down any prime implicants that are missing.

(a) x_1	(d) $x_3'x_1$	(g) $x_2'x_0$	(j) $x_3'x_2'x_1$
(b) x_3x_1	(e) $x_3'x_0$	(h) x_1x_0	(k) $x_2x_1x_0$
(c) $x_3'x_2'$	(f) x_2x_1	(i) x_1x_0'	(l) $x_3x_2x_1x_0$

$x_0'x_2'$, $x_3'x_0$, x_0x_1 $x_2'x_1$

missing

$x_3'x_2'$

-1

3. **(2 points)** Write down the complete set of **essential** prime implicants.

x_0x_1, $x_3'x_0$ $x_2'x_0'$

4. **(1 point)** Write the minimal sum of products expression for f. Is it unique?

$f = x_0x_1 + x_3'x_0 + x_0'x_2'$ yes. it is unique

5.3)

$$\Sigma m(0,1,2,3,4,6,8,9,10,11,12,14)$$

z_0:

		y_0			
	1	1	1	1	
x_0	1	0	0	1	
x_0	1	0	0	1	x_1
	1	1	1	1	x_1
			y_1		

PI: x_0', y_0'

both are essential:

$$z_0 = x_0' + y_0'$$

$\Downarrow$ NAND-NAND

$$z_0 = (x_0 y_0)'$$