[CS M51A Fall 18] Midterm exam

Date: 10/30/18

- The midterm is closed book, and up to 4 sheets (= 8 pages) of summary notes are allowed. You can use a calculator but not smart phones.
- For multiple choice questions, wrong answers may have a negative score value so choose carefully. It is possible that questions can have multiple answers.
- Please show all your work and write legibly, otherwise no partial credit will be given.
- **This should strictly be your own work; any form of collaboration will be penalized.**

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Problem	Points	Score
1	10	10
2	15	15
3	15	10
4	15	7
5	20	20
6	10	8
7	~~15~~ 9	7
Total	~~100~~ 94	77

Problem 1 (10 points)

1. **(4 points)** How many bits are required to encode a color spectrum capable of supporting 16 million colors using:

16 million colors

a. Decimal digits in BCD

4 bits for each digit

16,000,000

8 digits

$\boxed{\text{32 bits}}$

b. Hexadecimal representation

1 bit → 16 values
2 bits → 256 values

$16^x \geq 16{,}000{,}000$ $x = 6$

$\boxed{\text{6 bits}}$

Which representation is more efficient? Why?

The $\boxed{\text{hexadecimal}}$ since it uses much fewer bits.

2. **(6 points)** Fill in the missing entries in the table.

Radix	Digit vector $\underline{x}$	Value x in decimal
16	(5, 1, 7)	1303
8	(5, 1, 7)	335
7	(4, 5, 3, 2)	1640

1640

234   23

spectrum capable of supporting 16 million

lem 2 (15 points)

$a+b=b+a$	$ab=ba$	Commutativity
$a+(bc)=(a+b)(a+c)$	$a(b+c)=(ab)+(ac)$	Distributivity
$a+(b+c)=(a+b)+c=a+b+c$	$a(bc)=(ab)c=abc$	Associativity
$a+a=a$	$aa=a$	Idempotency
$a+a'=1$	$aa'=0$	Complement
$1+a=1$	$0a=0$	
$0+a=a$	$1a=a$	Identity
$(a')'=a$		Involution
$a+ab=a$	$a(a+b)=a$	Absorption
$a+a'b=a+b$	$a(a'+b)=ab$	Simplification
$(a+b)'=a'b'$	$(ab)'=a'+b'$	DeMorgan's law

Given $E(a,b,c,d)=(ab+c)'(ac+(b'+c'+a'cd)')+a((b+c)(b+d)+c)'$, which of the following represents the same function as $E(a,b,c,d)$? Show all your work.

1. $a+b+c+d'$
2. $a'+b+c$
3. $b+c'+d$
4. $a'b'c'd$
5. $\boxed{ab'c'}$
6. $b'cd'$

$(ab+c)'(ac+(b'+c'+a'cd)')+a((b+c)(b+d)+c)'$

$(ab)'(c)'(ac+(b)(c)(a'cd)')+a((b+c)(b+d)+c)'$ DeMorgan's

$((ab)'(c)'(ac+(b)(c)(a'cd)')+a(bb+bd+bc+cd+c)'$ Distributive

$(ab)'(c)'(ac+bc(a+c'+d'))+a(bb+bd+bc+cd+c)'$ DeMorgan's

$(ab)'(c)'(ac+abc+bcc'+bcd')+a(bb+bd+bc+cd+c)'$ Distributive

$(ab)'(acc'+abcc'+bcc'c'+bcc'd')+a(bb+bd+bc+cd+c)'$ Distributive

$(ab)'(a(0)+ab(0)+b(0)c'+b(0)d')+a(bb+bd+bc+cd+c)'$ Complement

$(ab)'(0)+a(bb+bd+bc+cd+c)'$ complement

$a(b+bd+bc+cd+c)'$ Idempotency

$a((b+c)')$ Absorption

$=ab'c'$ DeMorgan's.

Problem 3 (15 points)

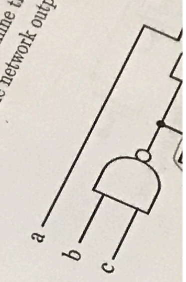
Show if the gate G, described by $G(x, y, z) = one - set\{3, 4, 6, 7\}$, can implement NOT and AND gates. ... help of the table below, determine the low t... Assume that 0 and 1 are available. If it can, then use G gates to implement the following expression and sho... corresponding network of G gates

$$E(a, b, c) = (a + b')(b + c')$$

4 (15 points)

help of the table below, determine the low t...

shown below. Assume the network output has a...



$G(x,y,z) = x'yz + xy'z' + xyz' + xyz$

$= x'yz + xz'(y+y') + xyz$

$= x'yz + xz' + xyz$

$= x'yz + x(z' + yz)$

$= x'yz + x(z' + y)$

$= x'yz + xz' + xy$

$= (x'z + x)y + xz' = (x+z)y + xz'$

$= xy + yz + xz'$

NOT: $G(1,0,z) = z'$

AND: $G(0,y,z) = yz$, so $G(x,y,z)$ is universal.

N/W missing ~~circuit~~

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$E(a,b,c) = (a+b')(b+c')$

$= ab + ac' + bb' + b'c'$

$= ab + ac' + b'c'$

$= [(ab)'(ac')'(b'c')']'$

= NOT(AND(AND(NOT(AND(b, NOT(c))), NOT(AND(a, NOT(c)))), NOT(AND(b,a))))

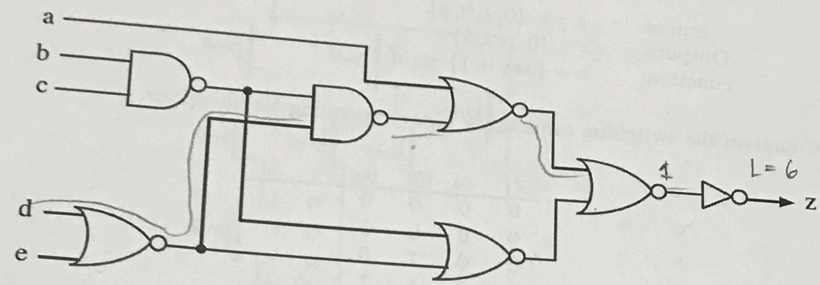
Hence,

$E(a,b,c) = G(1, 0, G(0, G(1,0,G(0,a,b)), G(0, G(1,0,G(0,b,G(1,0,c))),$ $G(1,0,G(0,a,G(1,0,c)))$ $))$

NOT and AND gates.
owing expression and sho

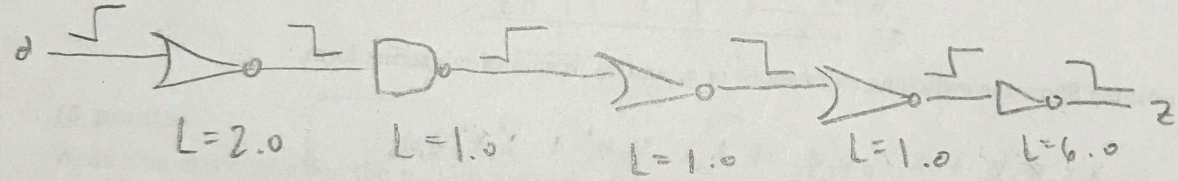
…lem 4 (15 points)

…the help of the table below, determine the low to high propagation delay $t_{pLH}(d, z)$ of the output z of the …work shown below. Assume the network output has a load of 6.



Gate Type	Fan-in	Propagation Delays (ns) t_{pLH}	t_{pHL}	Load Factor
NOT	1	$0.02 + 0.038L$	$0.05 + 0.017L$	1.0
NAND	2	$0.05 + 0.038L$	$0.08 + 0.027L$	1.0
NOR	2	$0.06 + 0.075L$	$0.07 + 0.016L$	1.0

Path with greatest delay:



$$t_{pLH}(d,z) = \underset{LH}{t_{pHL}(NOR_1)} + \underset{HL}{t_{pLH}(NAND_1)} + \underset{LH}{t_{pHL}(NOR_2)} + \underset{HL}{t_{pLH}(NOR_3)} + \underset{LH}{t_{pHL}(NOT_1)}$$

$$= [0.07 + 0.016(2)] + [0.05 + 0.038(1)] + [0.07 + 0.016(1)] + [0.06 + 0.075(1)] + [0.05 + 0.017(6)]$$

$$t_{pLH}(d,z) = .102 + .088 + .086_5 + .135 + .152 = \boxed{0.563 \text{ ns.}}$$

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Problem 5 (20 points)

Obtain a two-level gate network of the following system.

Inputs:	$x, y \in \{0, 1, 2, 3\}$
Outputs:	$z \in \{0, 1, 2, 3\}$
Function:	$z = \{3xy + 1\} \bmod 4$

1. **(2 points)** Complete the switching table using binary encoding for all values.

	x_1	x_0	y_1	y_0	z_1	z_0
0,0 = 1	0	0	0	0	0	1
0,1 =	0	0	0	1	0	1
0,2	0	0	1	0	0	1
0,3	0	0	1	1	0	1
1,0	0	1	0	0	0	1
1,1	0	1	0	1	0	0
1,2 = 3	0	1	1	0	1	1
1,3 = 2	0	1	1	1	1	0
2,0	1	0	0	0	0	1
2,1 = 3	1	0	0	1	1	1
2,2 = 1	1	0	1	0	0	1
2,3 = 3	1	0	1	1	1	1
3,0	1	1	0	0	0	1
3,1 = 2	1	1	0	1	1	0
3,2 = 3	1	1	1	0	1	1
3,3	1	1	1	1	0	0

2. **(5 points)** Show the switching expressions of z_1 and z_0 in sum of minterms form.

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$$z_0 = x_1'x_0'y_1'y_0' + x_1'x_0'y_1'y_0 + x_1'x_0'y_1y_0' + x_1'x_0'y_1y_0 + x_1'x_0y_1'y_0' + x_1'x_0y_1y_0' + x_1x_0y_1'y_0' + x_1x_0y_1y_0' + x_1x_0'y_1'y_0' + x_1x_0'y_1'y_0 + x_1x_0'y_1y_0' + x_1x_0'y_1y_0$$

$$z_1 = x_1'x_0y_1y_0' + x_1'x_0y_1y_0 + x_1x_0'y_1'y_0 + x_1x_0'y_1y_0 + x_1x_0y_1'y_0 + x_1x_0y_1y_0'$$

3. **(8 points)** Show the minimal sum of products expressions of z_1 and z_0. In each case, show a K-map, indicate all prime implicants, and all essential prime implicants. Show NAND-NAND networks.

z_0

$x_1x_0 \backslash y_1y_0$	00	01	11	10
00	1	1	1	1
01	1	0	0	1
11	1	0	0	1
10	1	1	1	1

EPI: x_0', y_0'

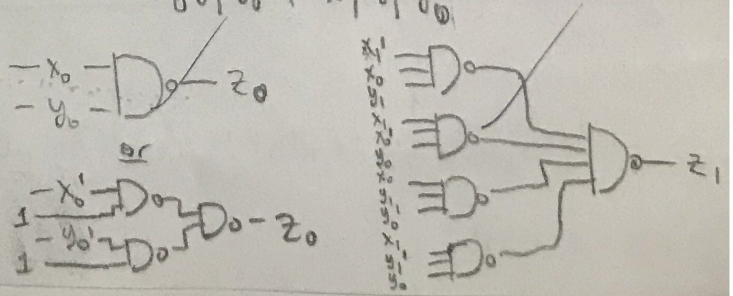
z_1

$x_1x_0 \backslash y_1y_0$	00	01	11	10
00	0	0	0	0
01	0	0	1	1
11	0	1	0	1
10	0	1	1	0

EPI: $x_1'x_0y_1$, $x_1x_0'y_0$, $x_0y_1y_0'$, $x_1y_1'y_0$

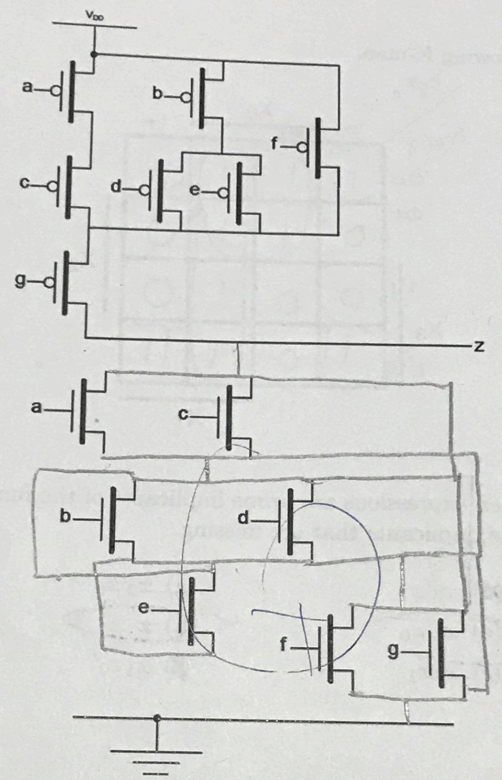
$$z_0 = x_0' + y_0'$$

$$z_1 = x_1'x_0y_1 + x_1x_0'y_0 + x_0y_1y_0' + x_1y_1'y_0$$



blem 6 (10 points)

are given the following partial CMOS network.



1. **(5 points)**

   Write the expression for the pull-up network. From this, derive the expression for the pull-down network using switching algebra.

2. **(5 points)** Connect NMOS transistors to complete the circuit according to the pull-down expression. Please only add missing wires.

1) $z = (a'c' + b'(e' + d') + f')g'$

$= (a'c' + b'e' + b'd' + f')g'$

$= a'c'g' + b'e'g' + b'd'g' + f'g'$

$= (a+c+g)' + (b+e+g)' + (b+d+g)' + (f+g)'$

Take complement,

Pull down $= (a+c+g)(b+e+g)(b+d+g)(f+g)$

Problem 7 (15 points)

For $f(x_3, x_2, x_1, x_0) = (x_3 + x_2' + x_1 + x_0)(x_3 + x_2' + x_1' + x_0)(x_3' + x_2 + x_1 + x_0')(x_3' + x_2' + x_1 + x_0)(x_3' + x_2' +$ $x_1 + x_0')(x_3' + x_2' + x_1' + x_0)$

1. **(2 points)** Fill out the following K-map.

$x_3x_2 \backslash x_1x_0$	00	01	11	10
00	1	1	1	1
01	0	1	1	0
11	0	0	1	0
10	1	0	1	1

X0, X1, X2, X3

2. **(4 points)** Which of the given expressions are prime implicants of the function given above? Circle **all** that apply. Write down any prime implicants that are missing.

(a) x_1 (d) $x_3'x_1$ (g) $x_2'x_0$ (j) $x_3'x_2'x_1$

(b) x_3x_1 (e) $x_3'x_0$ (h) x_1x_0 (k) $x_2x_1x_0$

(c) $x_3'x_2'$ (f) x_2x_1 (i) x_1x_0' (l) $x_3x_2x_1x_0$

$X_2'X_1,\ X_2'X_0'$

-2

$X_3X_2'X_0',\ X_3X_2'X_1\ X_3X_1X_0,\ X_3'X_1X_0,\ X_3'X_1'X_0,\ X_3X_2X_0$
$X_3'X_2'X_0,\ X_3'X_2'X_1',\ X_3'X_2'X_0'$
$X_3X_2'X_1X_0,\ X_3X_2'X_1X_0',\ X_3X_2'X_1'X_0',\ X_3'X_2'X_1'X_0$
$X_3'X_2X_1'X_0,\ X_3'X_2X_1X_0,\ X_3'X_2'X_1'X_0',\ X_3'X_2'X_1X_0,\ X_3'X_2X_1X_0'$

3. **(2 points)** Write down the complete set of **essential** prime implicants.

Essential P.I.: $X_3'X_0,\ X_2'X_0',\ X_1X_0$

4. **(1 point)** Write the minimal sum of products expression for f. Is it unique?

$f = X_3'X_0 + X_2'X_0' + X_1X_0$

unique

$\boxed{f = (X_1 + X_3)X_0 + X_2'X_0'}$