[CS M51A FALL 18] MIDTERM EXAM

Date: 10/30/18

- The midterm is closed book, and up to 4 sheets (= 8 pages) of summary notes are allowed. You ca use a calculator but not smart phones.
- For multiple choice questions, wrong answers may have a negative score value so choose carefully. is possible that questions can have multiple answers.
- Please show all your work and write legibly, otherwise no partial credit will be given.
- **This should strictly be your own work; any form of collaboration will be penalized.**

Name : __________

Student ID : __________

Problem 1 (10 points)

1. **(4 points)** How many bits are required to encode a color spectrum capable of supporting 16 million using:

   a. Decimal digits in BCD

   BCD encodes each digit (0-9) as 4 bits

   -> (4 of 16) million bits

   64 million bits

   ✗

   b. Hexadecimal representation

   ~~7 digits~~ $16^6 \approx 16.7$ million

   ∴ we need ~~7~~ 6 digits in Hex

   ✗

   => Each Hex = 4 bits

   $= 7 \times 4 = 28$ bits.

   Which representation is more efficient? Why?

   ✓ Hex is a more compact representation w/ 28 bits ~~[illegible]~~ at BCD's 64m bits. Also it has 7 Hex digits which is enough to encode key color propertities like R, G, B, Y, alpha, gamma and within its 7 ~~bits~~ hex digits.

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2. **(6 points)** Fill in the missing entries in the table.

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Radix	Digit vector $\underline{x}$	Value x in decimal
16	(5, 1, 7)	1303
8	(5, 1, 7)	335
7	4 5 3 2	1640

$a+b=b+a$	$ab=ba$	Commutativity
$a+(bc)=(a+b)(a+c)$	$a(b+c)=(ab)+(ac)$	Distributivity
$a+(b+c)=(a+b)+c=a+b+c$	$a(bc)=(ab)c=abc$	Associativity
$a+a=a$	$aa=a$	Idempotency
$a+a'=1$	$aa'=0$	Complement
$1+a=1$	$0a=0$	
$0+a=a$	$1a=a$	Identity
$(a')'=a$		Involution
$a+ab=a$	$a(a+b)=a$	Absorption
$a+a'b=a+b$	$a(a'+b)=ab$	Simplification
$(a+b)'=a'b'$	$(ab)'=a'+b'$	DeMorgan's law

Given $E(a,b,c,d)=(ab+c)'(ac+(b'+c'+a'cd)')+a((b+c)(b+d)+c)'$, which of the following represents the same function as $E(a,b,c,d)$? Show all your work.

$E(a,b,c,d) = (ab+c)'(ac+(b'$ ... $a'c$

1. $a+b+c+d'$
2. $a'+b+c$
3. $b+c'+d$
4. $a'b'c'd$
5. $ab'c'$
6. $b'cd'$

①

$E = (ab+c)'(ac+(b'+c'+a'cd)')$ → De morgans

$= (ab+c)'(ac+bcac'd')$ → 0, $abd' \to 0$, Complement ($cc'=0$)

$= (ab+c)'$ De Morgan's

$= (ab)'c'(ac) \to 0$

$= (a'+b')c'(ac)$ → Associativity

$= (a'c'+b'c')(ac)$

$= acc'(a'+b')$ → 0

$= 0\cdot(a'+b') = 0$ ①

②

$a((b+c)(b+d)+c)'$

$= a(b+cd+c)'$ → Distributivity

$= a(b+c)'$ → Absorption

$= a(b'c')$ → De morgan's

$= ab'c'$ ②

Final $=$ ① $+$ ② $= 0 + ab'c' =$ $ab'c'$

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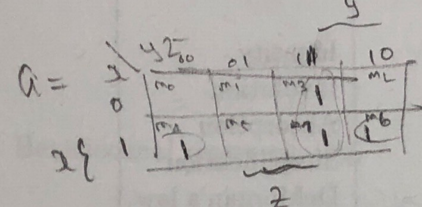
Problem 3 (15 points)

Global

Show if the gate G, described by $G(x, y, z) = one - set\{3, 4, 6, 7\}$, can implement NOT and AND gates. Assume that 0 and 1 are available. If it can, then use G gates to implement the following expression and show the corresponding network of G gates

$$E(a, b, c) = (a + b')(b + c')$$

G		F
	000	0
	001	0
	010	0
	011	1
	100	1
	101	0
	110	1
	111	1



∴ Suitable Expression for G : $xz' + yz$

$G(x, y, z) = xz' + yz$

Not : ~~Not(g)~~ $Not(x) = x'$

$G(x,y,z) = xz' + yz$

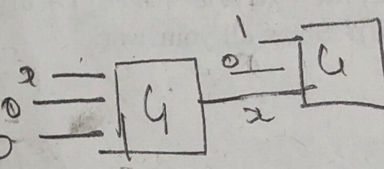
$G(x, 0, 0) = x$

$G(x, 0, 1) = 0 + y$

$G(x, 1, 0) = x$

$G(1, 0, x) = x'$ ✓

For NOT :

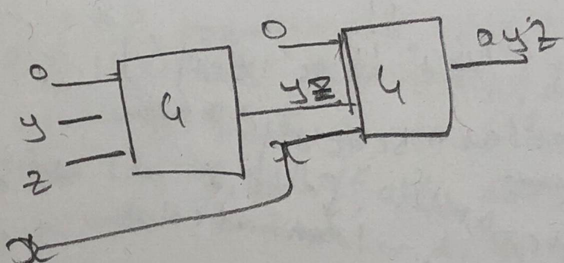


⇒ NOT Gate

AND : $G(0, y, z) = yz$ ✓

$G(0, yz, x) = xyz$

AND :



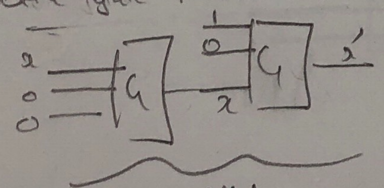
AND =

$E(a,b,c) = (a+b')(b+c') = ab + ac' + b'c'$

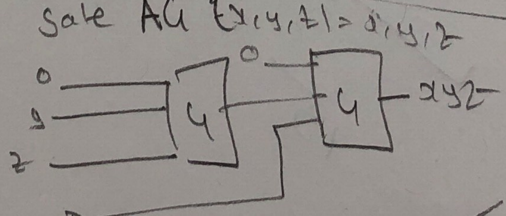
$= ((ab)'(ac')'(b'c')')'$ ⇒ can be implemented using AND and NOT gates.

⇒ Can be implemented with G.

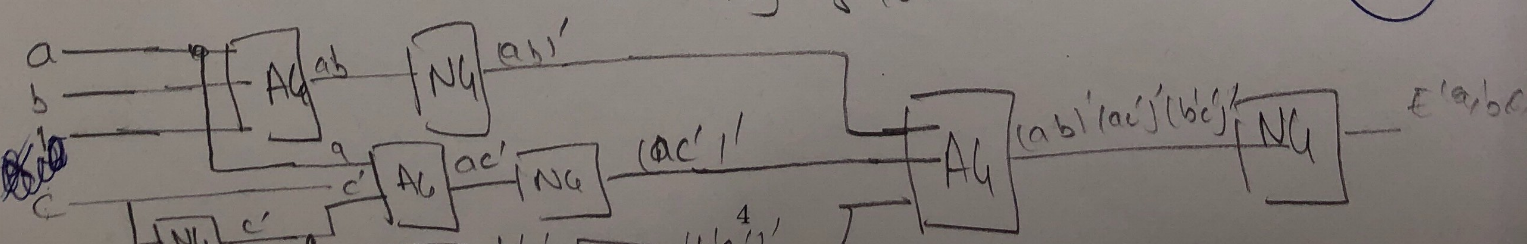
Define gate $NG(x) = x'$:



Define gate $AG(x, y, z) = x, y, z$



Then we can implement $E(a, b, c)$ as follows :



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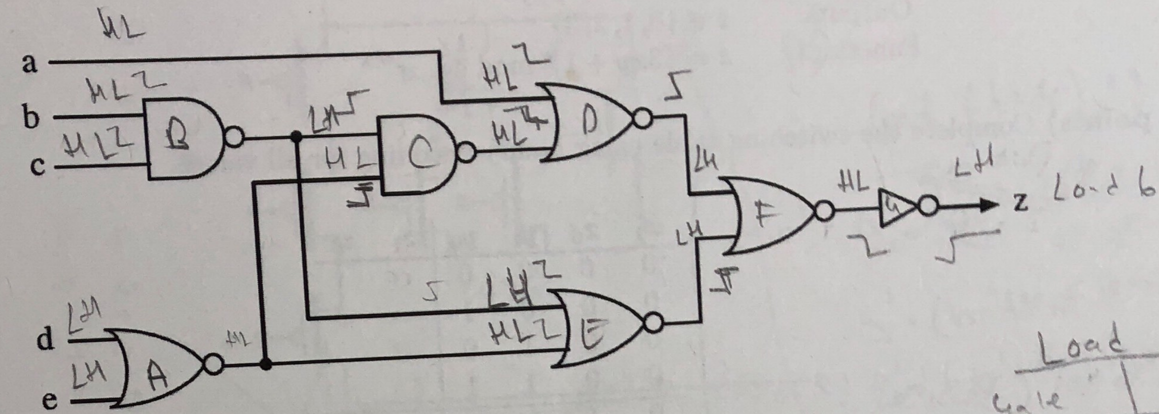
Problem 4 (15 points)

With the help of the table below, determine the low to high propagation delay $t_{pLH}(d, z)$ of the output z of the network shown below. Assume the network output has a load of 6.



Gate	Load
A	2
B	2
C	1
D	1
E	1
F	1
G	1

Gate Type	Fan-in	Propagation Delays (ns) t_{pLH}	t_{pHL}	Load Factor
NOT	1	$0.02 + 0.038L$	$0.05 + 0.017L$	1.0
NAND	2	$0.05 + 0.038L$	$0.08 + 0.027L$	1.0
NOR	2	$0.06 + 0.075L$	$0.07 + 0.016L$	1.0

$t_A(HL)$ NOR $+ t_B(LH)$ NAND $+ t_C(HL)$ NAND $+ t_D(LH)$ NOR $+ t_F(HL)$ NOR $+ t_G(LH)$ NOT $+ t_E(LH)$ NOR

$= (0.07 + 0.016*2) + (0.05 + 0.038*2) + (0.08 + 0.027*1) + (0.06 + 0.075) + (0.06 + 0.075) + (0.07 + 0.016) + (0.02 + 0.038*6)$

$= 0.939\,ns$

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Problem 5 (20 points)

Obtain a two-level gate network of the following system.

Inputs:	$x, y \in \{0, 1, 2, 3\}$
Outputs:	$z \in \{0, 1, 2, 3\}$
Function:	$z = \{3xy + 1\}$ mod 4

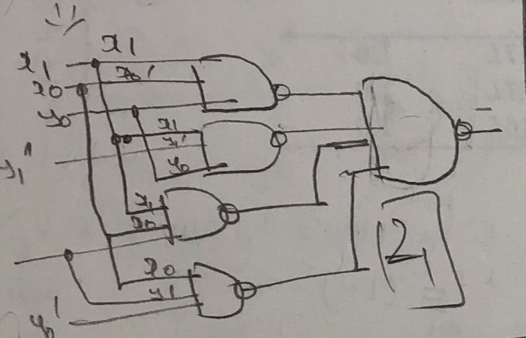
1. **(2 points)** Complete the switching table using binary encoding for all values.

For Part 3 -

$Z_1 : x_1 x_0' y_0 + x_1 y_1' y_0 + x_1' x_0 y_1 + x_0 y_1 y_0'$

$= \left((x_1 x_0' y_0)' (x_1 y_1' y_0)' (x_1 x_0 y_1)' (x_0 y_1 y_0')' \right)'$

Assume complemented inputs exist.



x_1	x_0	y_1	y_0	z_1	z_0
0	0	0	0	0	1
0	0	0	1	0	1
0	0	1	0	0	1
0	0	1	1	0	1
0	1	0	0	0	1
0	1	0	1	0	0
0	1	1	0	1	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	0	1	1	1
1	0	1	0	0	0
1	0	1	1	1	1
1	1	0	0	0	1
1	1	0	1	1	0
1	1	1	0	1	1
1	1	1	1	0	0

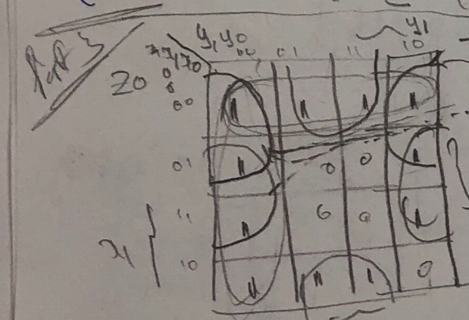
x	y	(3xy+1)
0	0	0 + 1
0	1	0 + 1
0	2	0 + 1
0	3	0 + 1
1	0	0 + 1
1	1	3 + 1
1·2		6 + 1
1·3		9 + 1
2·0		
2·1		6 + 1
2·2		
2·3		19
3·0		
3·1		9 + 1
3·2		19
3·3·3 ·3		28

2. **(5 points)** Show the switching expressions of z_1 and z_0 in sum of minterms form.

$Z_0 = \sum m(0, 1, 2, 3, 4, 6, 8, 9, 11, 12, 14)$

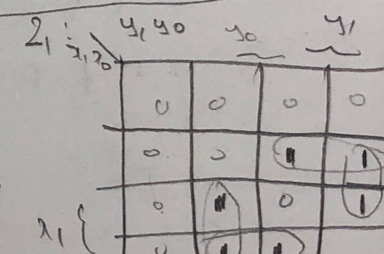
$Z_1 = \sum m(6, 7, 9, 11, 13, 14)$

Part 3



Not essential $\{m_0, m_2, m_4, m_6\}$

Essential :- $\{m_0, m_1, m_2, m_3\}, \{m_0, m_4, m_8, m_{12}\}$
$\{m_1, m_3, m_9, m_{11}\}, \{m_4, m_{12}, m_{14}, m_6\}$



For Z_1 all prime impl are essen

3. **(8 points)** Show the minimal sum of products expressions of z_1 and z_0. In each case, show a K-map, indicate all prime implicants, and all essential prime implicants. Show NAND-NAND networks.

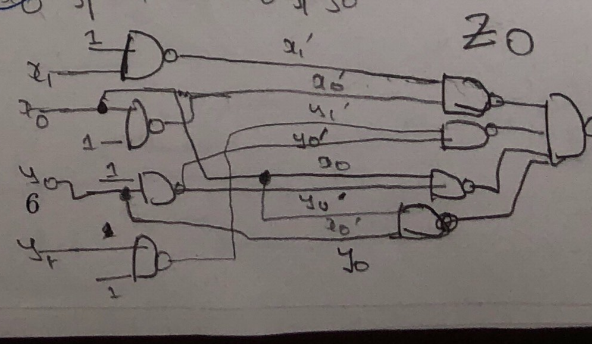
Z_1 above near part 1

Minimal SOP:

$Z_0 : x_1' x_0' + y_1' y_0' + x_0 y_0' + x_0' y_0$

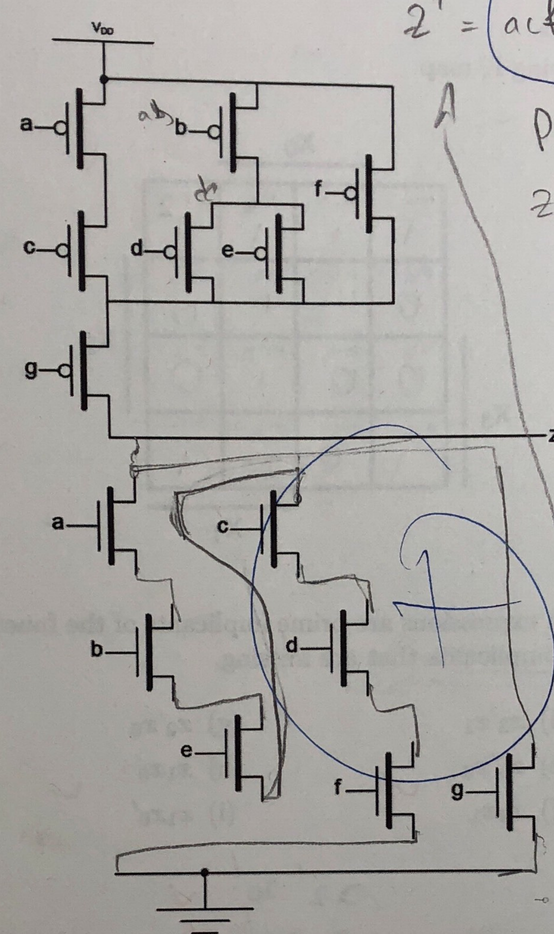
$Z_1 : x_1 x_0' y_0 + x_1 y_1' y_0 + x_1' x_0 y_1 + x_0 y_1 y_0'$

$Z_0 : \left((x_1' x_0')' (y_1' y_0')' (x_0 y_0')' (x_0' y_0)' \right)'$ ⇒



...blem 6 (10 points)

...ve are given the following partial CMOS network.



$z' = (acf(b+de) + g)$

PD:

$z = (acf(b+de) + g)'$

$g' \cdot (acf(b+de))'$

$g' \cdot ((acf)' + (b+de)')$

$g' \cdot ((a'+c'+f') + b'(de)')$

$g' \cdot ((a'+c'+f') + b'(d'+e'))$

$g' \cdot (a'b'+c'+d'e'+f')$

1. **(5 points)** Write the expression for the pull-up network. From this, derive the expression for the pull-down network using switching algebra.

2. **(5 points)** Connect NMOS transistors to complete the circuit according to the pull-down expression. Please only add missing wires.

Problem 7 (15 points)

For $f(x_3, x_2, x_1, x_0) = (x_3 + x_2' + x_1 + x_0)(x_3 + x_2' + x_1' + x_0)(x_3' + x_2 + x_1 + x_0')(x_3' + x_2' + x_1 + x_0)(x_3' + x_2' + x_1 + x_0')(x_3' + x_2' + x_1' + x_0)$

0100 M_4 0110 M_6 1001 M_9 1100 M_{12} 1101 M_{13} 1110 M_{14}

1. **(2 points)** Fill out the following K-map.

X0 (columns 2–3), X2 (rows 2–3), X3 (rows 3–4), X1 (columns 3–4)

m_0: 1	m_1: 1	m_3: 1	m_2: 1
m_4: 0	m_5: 1	m_7: 1	m_6: 0
m_{12}: 0	m_{13}: 0	m_{15}: 1	m_{14}: 0
m_8: 1	m_9: 0	m_{11}: 1	m_{10}: 1

Prime implicants

$x_2'x_3' + x_0x_1$
$+ x_0x_3'$
$+ x_2'x_0'$
$+ x_1x_2'$
$+ x_3x_2'$

2. **(4 points)** Which of the given expressions are prime implicants of the function given above? Circle **all** that apply. Write down any prime implicants that are missing.

SOP

(a) x_1
(b) x_3x_1
(c) $x_3'x_2'$ ✓
(d) $x_3'x_1$
(e) $x_3'x_0$ ✓
(f) x_2x_1
(g) $x_2'x_0$
(h) x_1x_0 ✓
(i) x_1x_0'
(j) $x_3'x_2'x_1$
(k) $x_2x_1x_0$
(l) $x_3x_2x_1x_0$

$x_2'x_0'$ ✓
x_1x_2' ✗
x_3x_2' ✗

−1

3. **(2 points)** Write down the complete set of **essential** prime implicants.

$x_2'x_3'$, x_0x_1, x_0x_3', $x_2'x_0'$

−1

4. **(1 point)** Write the minimal sum of products expression for f. Is it unique?

Minimum SOP $f = x_2'x_3' + x_0x_1 + x_0x_3' + x_2'x_0'$

→ Unique