[CS M51A Fall 15] Solution to Quiz 1

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Quiz Problems (50 points total)

Problem 1 (10 points)

Find x and y such that the following conditions are satisfied and show all the steps of your work.

1. **(5 points)** $(818)_9 = (x)_3$

   Solution

   $(818)_9 = (220122)_3$

2. **(5 points)** What is the largest number y that can be represented with 4 digit-vector in radix 5. Show y in radix 5 and decimal.

   Solution

   $(4444)_5 = (5^4 - 1)_{10} = (624)_{10}$

Problem 2 (16 points)

Solve the following problems using the postulates and theorems of Boolean algebra. **Do not use a truth table**.

1. **(8 points)** The Boolean function f is defined as $f(a, b, c) = ac' + a'b$ and the Boolean function g is defined as $g(a, b, c) = ac + b'c + a'b'$. Show that $g(a, b, c)' = f(a, b, c)$.

   Solution

$$
\begin{aligned}
g(a,b,c)' &= (ac + b'c + a'b')' \\
&= (ac + ab'c + a'b'c + a'b')' \\
&= (ac + a'b')' \\
&= (ac)'(a + b) \\
&= (a' + c')(a + b) \\
&= a'b + ac' + bc' \\
&= a'b + ac' + abc' + a'bc' \\
&= a'b + ac' \\
&= f(a,b,c)
\end{aligned}
$$

2. **(8 points)** Simplify the following expression.

$$xyzw' + xyz' + xy' + x'$$

Solution

$$\begin{aligned} xyzw' + xyz' + xy' + x' &= xy(zw' + z') + xy' + x' \\ &= xy(w' + z') + xy' + x' \\ &= x(y(w' + z') + y') + x' \\ &= x((w' + z') + y') + x' \\ &= x(w' + z' + y') + x' \\ &= (w' + z' + y') + x' \\ &= w' + z' + y' + x' \end{aligned}$$

Problem 3 (24 points)

F is a function that accepts inputs $x \in \{0,1,2\}$, $y \in \{1,2,3\}$, and outputs $z = max(x^2, y)$. Suppose you use binary code to encode x, y, and z. x is encoded as x_1x_0, y is encoded as y_1y_0, z is encoded as $z_2z_1z_0$.

1. **(16 points)** Fill in the table below.

   Solution

x_1	x_0	y_1	y_0	z_2	z_1	z_0
0	0	0	0	-	-	-
0	0	0	1	0	0	1
0	0	1	0	0	1	0
0	0	1	1	0	1	1
0	1	0	0	-	-	-
0	1	0	1	0	0	1
0	1	1	0	0	1	0
0	1	1	1	0	1	1
1	0	0	0	-	-	-
1	0	0	1	1	0	0
1	0	1	0	1	0	0
1	0	1	1	1	0	0
1	1	0	0	-	-	-
1	1	0	1	-	-	-
1	1	1	0	-	-	-
1	1	1	1	-	-	-

2. **(8 points)** Fill in the sets in the forms specified below.

   Solution From the table, we can write

$$\begin{aligned} z_2 &= \sum m(9, 10, 11) \\ z_1 &= \sum m(2, 3, 6, 7) \\ z_0 &= \prod M(2, 6, 9, 10, 11) \\ dc - set\ of\ z_1 &= dc(0, 4, 8, 12, 13, 14, 15) \end{aligned}$$