[CS M51A Fall 18] Midterm exam

Date: 10/30/18

- The midterm is closed book, and up to 4 sheets (= 8 pages) of summary notes are allowed. You can use a calculator but not smart phones.
- For multiple choice questions, wrong answers may have a negative score value so choose carefully. It is possible that questions can have multiple answers.
- Please show all your work and write legibly, otherwise no partial credit will be given.
- **This should strictly be your own work; any form of collaboration will be penalized.**

Name : Elaine Shyu

Student ID : ______________________

Problem	Points	Score
1	10	10
2	15	15
3	15	15
4	15	15
5	20	17
6	10	10
7	~~15~~ 9	9
Total	~~100~~ 94	91

13 $\lceil \log_{10}(13) \rceil = 2$

0001 0111 $2(4) = 8$

Problem 1 (10 points)

1. **(4 points)** How many bits are required to encode a color spectrum capable of supporting 16 million colors using:

   a. Decimal digits in BCD

4 bits for each digit in BCD

$10^x = 16000000$

$x = 8$

$8(4) = \boxed{32 \text{ bits}}$

$\lceil \log_{10}(16 \text{ mil}) \rceil$ → 8 BCD digits

   b. Hexadecimal representation

4 bits for each digit in hex

$16^x = 16000000$

$x = 6$

$6(4) = \boxed{24 \text{ bits}}$

$\lceil \log_{16}(16 \text{ million}) \rceil$ → 6 hex digits

Which representation is more efficient? Why?

hexadecimal is more efficient because you can represent the range 0-15 numbers in a 4-bit value, but for BCD, you can only represent 0-9.

2. **(6 points)** Fill in the missing entries in the table.

$5(16^2) + 16 + 7$

$8^2(5) + 8 + 7$

Radix	Digit vector x	Value x in decimal
16	$(5, 1, 7)$	1303
8	$(5, 1, 7)$	335
7	(4,5,3,2)	1640

	remainder
1640/7	2
234/7	3
33/7	5
4/7	4

check

$4(7^3) + 5(7^2) + 3(7) + 2 = 1640$

15

Problem 2 (15 points)

$a+b=b+a$	$ab=ba$	Commutativity
$a+(bc)=(a+b)(a+c)$	$a(b+c)=(ab)+(ac)$	Distributivity
$a+(b+c)=(a+b)+c=a+b+c$	$a(bc)=(ab)c=abc$	Associativity
$a+a=a$	$aa=a$	Idempotency
$a+a'=1$	$aa'=0$	Complement
$1+a=1$	$0a=0$	
$0+a=a$	$1a=a$	Identity
$(a')'=a$		Involution
$a+ab=a$	$a(a+b)=a$	Absorption
$a+a'b=a+b$	$a(a'+b)=ab$	Simplification
$(a+b)'=a'b'$	$(ab)'=a'+b'$	DeMorgan's law

Given $E(a,b,c,d)=(ab+c)'(ac+(b'+c'+a'cd)')+a((b+c)(b+d)+c)'$, which of the following represents the same function as $E(a,b,c,d)$? Show all your work.

1. $a+b+c+d'$
2. $a'+b+c$
3. $b+c'+d$
4. $a'b'c'd$
5. $ab'c'$
6. $b'cd'$

$(ab+c)'(ac+(b'+c'+a'cd)')+a((b+c)(b+d)+c)'$

DeMorgans, Distributivity: $((ab)'c')(ac+(b'+(c'+a'cd))')+a((b+cd)+c)'$

DeMorgans, absorption: $((a'+b')c')(ac+(b'+(c'+a'd))')+a(b+(cd+c))'$ ← absorption

absorption: $((a'+b')c')(ac+(b'+c'+a'd)')+a((b+c))'$

DeMorgans: $c'(a'+b')(ac+(bc(a'd)'))+a(b'c')$

DeMorgans: $c'(a'+b')(ac+(bc(a+d')))+a(b'c')$

distributive: $c'(a'+b')(ac+bca+bcd')+a(b'c')$

distributive: $c'(a'+b')(c(a+ab+cd'))+a(b'c')$

commutativity: $c'c(a'+b')(a+ab+cd'))+a(b'c')$

Complement: $0+ab'c'$

ab \ cd		d	d	
	0	0	0	0
b	0	0	0	0
a, b	0	0	0	0
a	1	1	0	0
			c	c

← no way to represent this other ways ∴ only one answer

Problem 3 (15 points)

Show if the gate G, described by $G(x,y,z) = one-set\{3,4,6,7\}$, can implement NOT and AND gates. Assume that 0 and 1 are available. If it can, then use G gates to implement the following expression and show the corresponding network of G gates

$2^2\ 2^1\ 2^0$
100

$$E(a,b,c) = (a+b')(b+c')$$

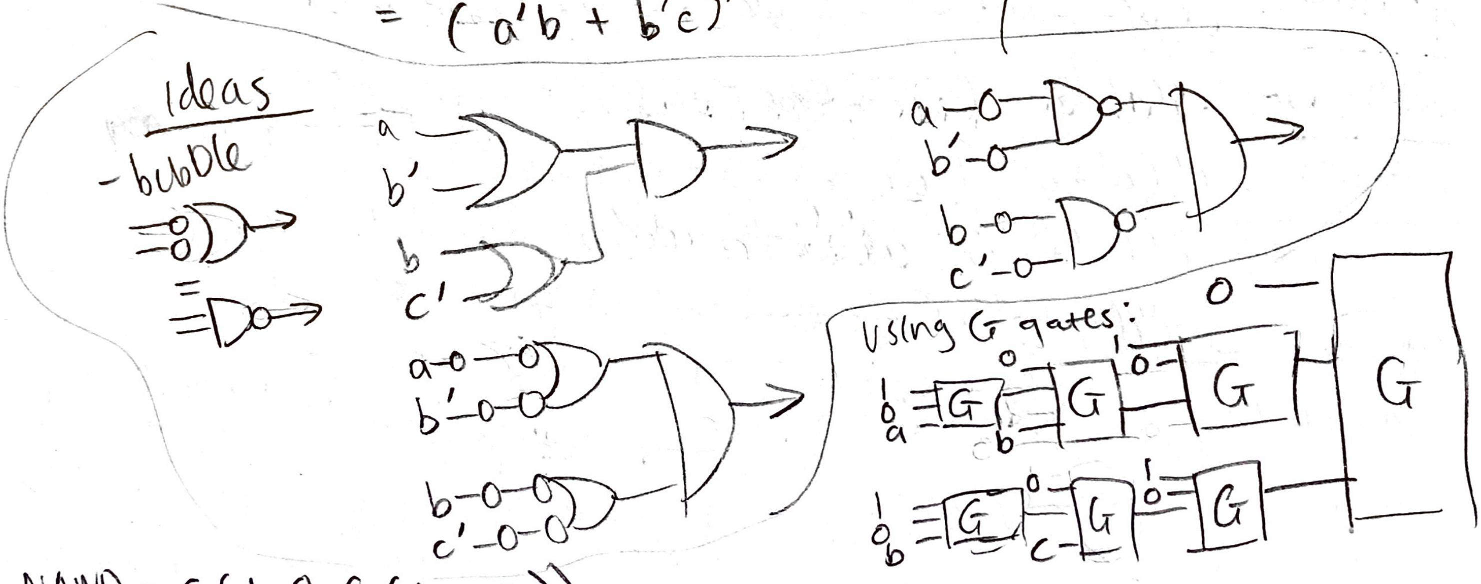
x \ yz	00	01	11	10
0			1	
1	1		1	1

$G(x,y,z) = zy + z'x$

$G(0,y,z) = zy + z'(0)$
$= zy \rightarrow \boxed{\text{AND gate}}$

$G(1,0,z) = z(0) + z'(1)$
$= z' \rightarrow \boxed{\text{NOT gate}}$

$E(a,b,c) = (a+b')(b+c')$
$= ((a+b')(b+c'))''$
$= ((a+b')' + (b+c')')'$
$= (a'b + b'c)'$



NAND = G(1, 0, G(0, y, z))

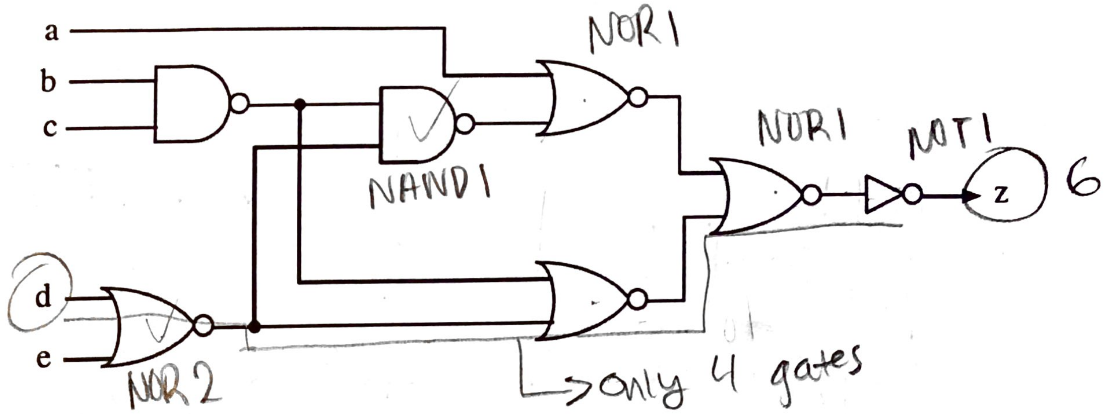
$\boxed{G(\underset{\text{AND}}{0}, \underset{\text{NOT}}{G}(1,0, \underset{\text{AND}}{G}(0, \underset{\text{not}}{G}(1,0,a), b)), \underset{\text{NOT}}{G}(1,0, \underset{\text{AND}}{G}(0, \underset{\text{not}}{G}(1,0,b), c))}$

AND(NOT(AND(NOT(a), b)), NOT(AND(NOT(b), c))

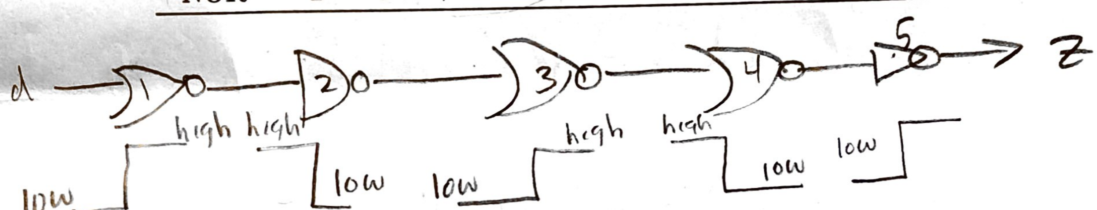
15

Problem 4 (15 points)

With the help of the table below, determine the low to high propagation delay $t_{pLH}(d, z)$ of the output z of the network shown below. Assume the network output has a load of 6.



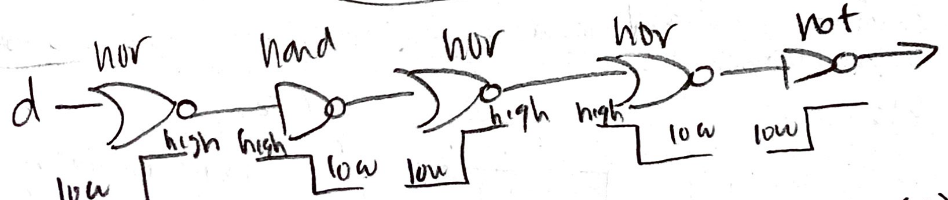
Gate Type	Fan-in	Propagation Delays (ns) t_{pLH}	t_{pHL}	Load Factor
NOT	1	$0.02 + 0.038L$	$0.05 + 0.017L$	1.0
NAND	2	$0.05 + 0.038L$	$0.08 + 0.027L$	1.0
NOR	2	$0.06 + 0.075L$	$0.07 + 0.016L$	1.0



1 2 3 4

$(0.06+0.075(2)) + (0.08+0.027(1)) + (0.06+0.075(1)) + (0.07+0.016(1)) + (0.02+0.038(6))$

$\boxed{0.786 \text{ ns}}$



$0.06+0.075(2) + (0.08+0.027(1)) + 0.06+0.075(1) + 0.07+0.016(1) + (0.02+0.038(6))$

$= 0.786$ ✓

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Problem 5 (20 points)

Obtain a two-level gate network of the following system.

Inputs:	$x, y \in \{0, 1, 2, 3\}$
Outputs:	$z \in \{0, 1, 2, 3\}$
Function:	$z = \{3xy + 1\} \bmod 4$

1. **(2 points)** Complete the switching table using binary encoding for all values.

x	y		x_1	x_0	y_1	y_0	z_1	z_0	
0	0	1	0	0	0	0	0	1	0
0	1	1	0	0	0	1	0	1	1
0	2	1	0	0	1	0	0	1	2
0	3	1	0	0	1	1	0	1	3
1	0	1	0	1	0	0	0	1	4
1	1	0	0	1	0	1	0	0	5
1	2	3	0	1	1	0	1	1	6
1	3	2	0	1	1	1	1	0	7
2	0	1	1	0	0	0	0	1	8
2	1	3	1	0	0	1	1	1	9
2	2	1	1	0	1	0	0	1	
2	3	3	1	0	1	1	1	1	11
3	0	1	1	1	0	0	0	1	12
3	1	2	1	1	0	1	1	0	13
3	2	3	1	1	1	0	1	1	14
3	3	0	1	1	1	1	0	0	

z_1

$x_1x_0 \backslash y_1y_0$	00	01	11	10
00	0	0	0	0
01	0	0	1	1
11	0	1	0	1
10	0	1	1	0

y_0, x_0, x_1, y_1

2. **(5 points)** Show the switching expressions of z_1 and z_0 in sum of minterms form.

$z_1 = \sum m(6, 7, 9, 11, 13, 14)$

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$z_0 = \sum m(0, 1, 2, 3, 4, \cancel{6}, 8, 9, 10, 11, 12, 14)$

NAND-NAND:

z_1: NAND (NAND(x_1, y_0, y_1'), NAND(y_0, x_0', x_1), NAND(x_0, y_0', y_1), NAND(y_1, x_0, x_1'))

z_0: NAND(y_0, x_0)

3. **(8 points)** Show the minimal sum of products expressions of z_1 and z_0. In each case, show a K-map, indicate all prime implicants, and all essential prime implicants. Show NAND-NAND networks.

z_1

$x_1x_0 \backslash y_1y_0$	00	01	11	10
00	0	0	0	0
01	0	0	1	1
11	0	1	0	1
10	0	1	1	0

y_0, x_0, x_1, y_1

PI: $x_1 y_0 y_1'$
$y_0 x_0' x_1$
$x_0 y_0' y_1$
$y_1 x_0 x_1'$

EPI: same as PI / all essential

SOP: $x_1 y_0 y_1' + y_0 x_0' x_1 + x_0 y_0' y_1 + y_1 x_0 x_1'$

z_0

$x_1x_0 \backslash y_1y_0$	00	01	11	10
00	1	1	1	1
01	1			1
11	1			1
10	1	1	1	1

y_0, x_0, x_1, y_1

prime implicants:
y_0', x_0'

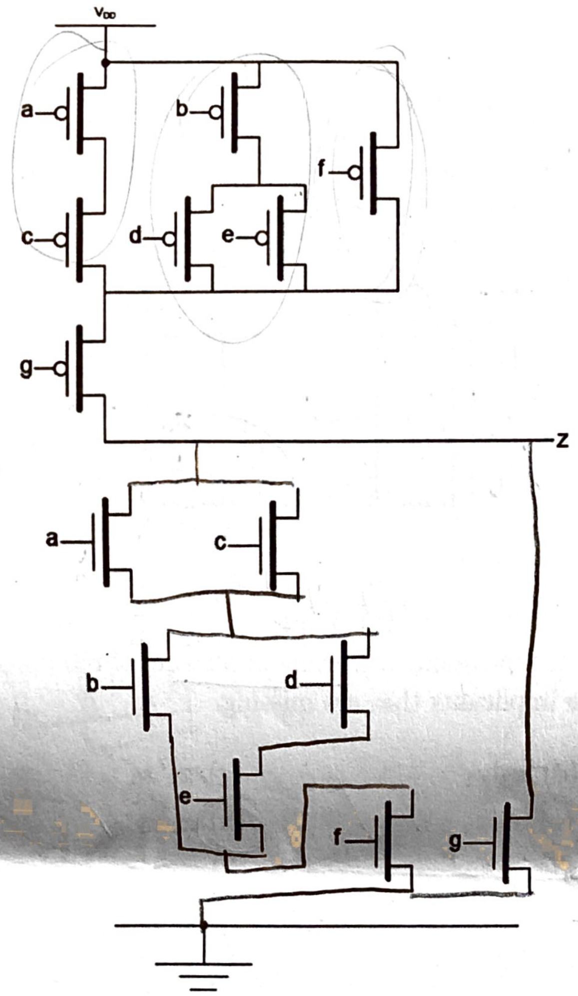
essential prime implicants:
y_0', x_0'

SOP: $y_0' + x_0'$

$(y_0' + x_0')''$

Problem 6 (10 points)

We are given the following partial CMOS network.



1. **(5 points)**

   Write the expression for the pull-up network. From this, derive the expression for the pull-down network using switching algebra.

2. **(5 points)** Connect NMOS transistors to complete the circuit according to the pull-down expression. Please only add missing wires.

1) $\boxed{z = (a'c' + b'(d'+e') + f')\,g'}$ ← pull up

$z' = (\,(a'c' + b'(d'+e') + f')\,g'\,)'$

$= (\,(a'c')'\,(b'(d'+e'))'\,f\,) + g\,)$

$= (\,(a+c)(\,b + (d'+e')')f + g\,)$

$\boxed{z' = (\,(a+c)(\,b + ed)f + g\,)}$ ← pull down

Problem 7 (15 points)

4 6 9 1001

For $f(x_3, x_2, x_1, x_0) = (x_3 + x_2' + x_1 + x_0)(x_3 + x_2' + x_1' + x_0)(x_3' + x_2 + x_1 + x_0')(x_3' + x_2' + x_1 + x_0)(x_3' + x_2' + x_1 + x_0')(x_3' + x_2' + x_1' + x_0)$

1110 → 14 1100 (12) 1101 → 13

1. **(2 points)** Fill out the following K-map.

$x_3x_2 \backslash x_1x_0$	00	01	11	10
00	1	1	1	1
01	0	1	1	0
11	0	0	1	0
10	1	0	1	1

$X_2'X_3'$ X_0X_3' $X_0'X_2'$ X_1X_2'

2. **(4 points)** Which of the given expressions are prime implicants of the function given above? Circle **all** that apply. Write down any prime implicants that are missing.

(a) x_1
(b) x_3x_1
(c) $x_3'x_2'$
(d) $x_3'x_1$
(e) $x_3'x_0$
(f) x_2x_1
(g) $x_2'x_0$
(h) x_1x_0
(i) x_1x_0'
(j) $x_3'x_2'x_1$
(k) $x_2x_1x_0$
(l) $x_3x_2x_1x_0$

$\boxed{X_0'X_2'}$ $\boxed{X_1X_2'}$

$X_3X_2 \backslash X_1X_0$				
	1	1	1	1
		1	1	
			1	
	1		1	1

X_0X_3' $X_0'X_2'$ $X_2'X_3'$ X_1X_0 X_1X_2'

3. **(2 points)** Write down the complete set of **essential** prime implicants.

X_0X_3', $X_0'X_2$, X_1X_0

4. **(1 point)** Write the minimal sum of products expression for f. Is it unique?

$\boxed{X_0'X_2' + X_1X_0 + X_0X_3'}$

Yes, it is unique.