Problem 1 (10 points)

1. **(4 points)** How many bits are required to encode a color spectrum capable of supporting 16 million colors using:

   a. Decimal digits in BCD

   28 bits

   16000000

   $8 \times 4 = 32$

   b. Hexadecimal representation

   6 bits

   Which representation is more efficient? Why?

   Hexadecimal representation is more effcient since it ~~only needs~~ 6 bits while BCD requires 28 bits.

2. **(6 points)** Fill in the missing entries in the table.

Radix	Digit vector $\underline{x}$	Value x in decimal
16	(5, 1, 7)	1303
8	(5, 1, 7)	335
7	(4,5,3,2)	1640

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lem 2 (15 points)

$a+b=b+a$	$ab=ba$	Commutativity
$a+(bc)=(a+b)(a+c)$	$a(b+c)=(ab)+(ac)$	Distributivity
$a+(b+c)=(a+b)+c=a+b+c$	$a(bc)=(ab)c=abc$	Associativity
$a+a=a$	$aa=a$	Idempotency
$a+a'=1$	$aa'=0$	Complement
$1+a=1$	$0a=0$	
$0+a=a$	$1a=a$	Identity
$(a')'=a$		Involution
$a+ab=a$	$a(a+b)=a$	Absorption
$a+a'b=a+b$	$a(a'+b)=ab$	Simplification
$(a+b)'=a'b'$	$(ab)'=a'+b'$	DeMorgan's law

Given $E(a,b,c,d) = (ab+c)'(ac+(b'+c'+a'cd)') + a((b+c)(b+d)+c)'$, which of the following represents the same function as $E(a,b,c,d)$? Show all your work.

1. $a+b+c+d'$
2. $a'+b+c$
3. $b+c'+d$
4. $a'b'c'd$
5. $ab'c'$
6. $b'cd'$

$E(a,b,c,d) = (ab+c)'(ac+(b'+c'+a'cd)') + a((b+c)(b+d)+c)'$

$= (ab)' \cdot c' \cdot (ac + b \cdot c \cdot (a'cd)') + a(\underbrace{b \cdot b}_{b} + cb + cd + bd + c)'$

$= (ab)' \cdot \underbrace{c' \cdot c}_{0}(a + b \cdot (a'cd)') + a(b + c(1+b+d) + bd)'$

Demorgan's law

distributivity

Idempotency

distributivity

$= 0 + a(b+c+bd)'$ complement identity

$= 0 + a(b(1+d)+c)'$ → distributivity

$= ab'c'$ Identity

Demorgan's Law

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Only are
are given

Problem 3 (15 points)

Show if the gate G, described by $G(x, y, z) = one - set\{3, 4, 6, 7\}$, can implement NOT and AND gates. Assume that 0 and 1 are available. If it can, then use G gates to implement the following expression and show the corresponding network of G gates

$$E(a, b, c) = (a + b')(b + c')$$

$G(x,y,z)$

$= x'yz + xy'z' + xyz' + xyz$

$= (x'+x)yz + xz'(y'+y)$

$= yz + xz'$

$NOT(z) = z' = 0 \cdot z + 1 \cdot z'$

$= G(1, 0, z)$

$AND(y, z) = y \cdot z = G(0, y, z)$

$\Rightarrow G(x,y,z)$ can implement AND and NOT gates.

$b' = G(1, 0, b)$

$c' = G(1, 0, c)$

$a' = G(1, 0, a)$

~~$E(a,b,c) = ab + \underbrace{b'b}_{0} + b'c' + ac'$~~

~~$= G(0, a, b) + G(0, G(1,0,b), G(1,0,c$~~

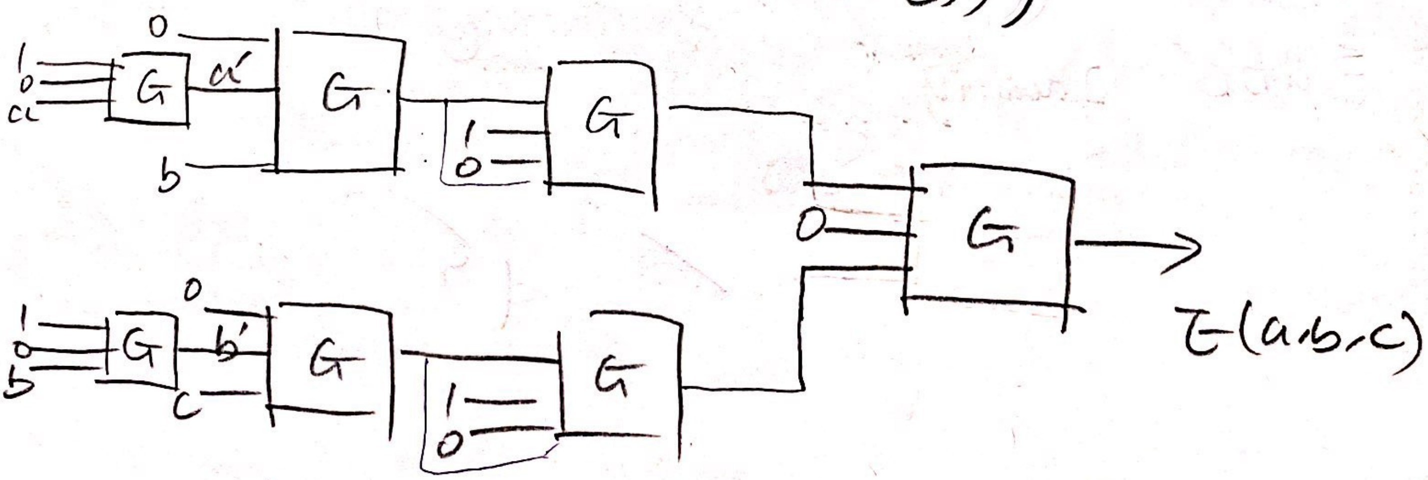
$E(a,b,c)$

$= (a'b)' \cdot (b'c)'$

$= G(0, G(1, 0, G(0, G(1, 0, a), b)),$
$G(1, 0, G(0, G(1, 0, b), c)))$

$a'b = G(0, G(1, 0, a), b)$

$b'c = G(0, G(1, 0, b), c)$



Ordering in N/W wrong.

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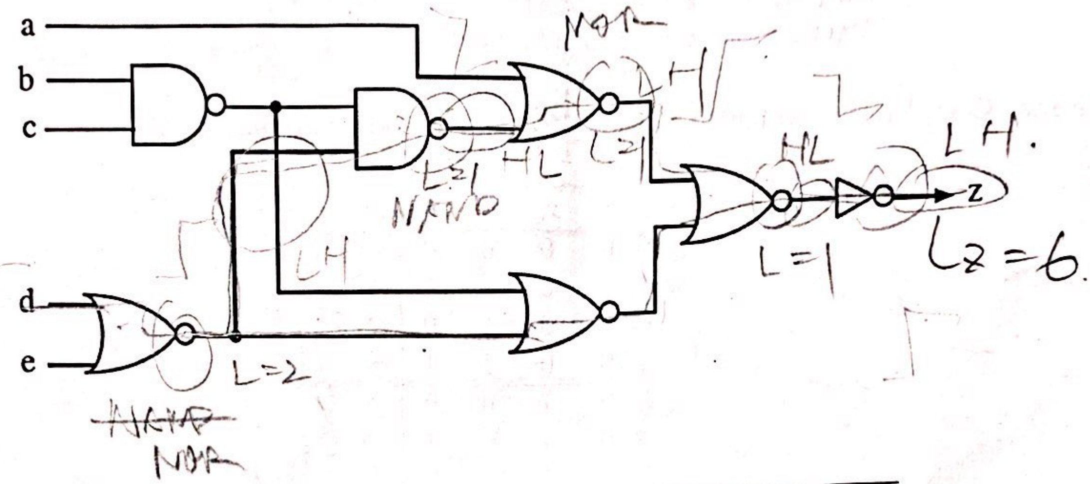
...blem 4 (15 points)

...th the help of the table below, determine the low to high propagation delay $t_{pLH}(d, z)$ of the output z of the network shown below. Assume the network output has a load of 6.



Gate Type	Fan-in	Propagation Delays (ns) t_{pLH}	t_{pHL}	Load Factor
NOT	1	$0.02 + 0.038L$	$0.05 + 0.017L$	1.0
NAND	2	$0.05 + 0.038L$	$0.08 + 0.027L$	1.0
NOR	2	$0.06 + 0.075L$	$0.07 + 0.016L$	1.0

The longest path from d to z is NOR → NAND → NOR → NOR → NOT

$$t_{pLH}(d,z) = t_{pLH}(NOR^2) + t_{pHL}(NAND^1) + t_{pLH}(NOR^1) + t_{pHL}(NOR^1) + t_{pLH}(NOT^6)$$

$$= \underbrace{0.06 + 0.075 \times 2}_{0.21} + \underbrace{0.08 + 0.027 \times 1}_{0.107} + \underbrace{0.06 + 0.075 \times 1}_{0.135} + \underbrace{0.07 + 0.016 \times 1}_{0.086} + \underbrace{0.02 + 0.038 \times 6}_{0.248}$$

$= 0.786$ ns

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Problem 5 (20 points)

Obtain a two-level gate network of the following system.

Inputs:	$x, y \in \{0, 1, 2, 3\}$
Outputs:	$z \in \{0, 1, 2, 3\}$
Function:	$z = \{3xy + 1\}$ mod 4

1. **(2 points)** Complete the switching table using binary encoding for all values.

	x_1	x_0	y_1	y_0	z_1	z_0
0	0	0	0	0	0	1
1	0	0	0	1	0	1
2	0	0	1	0	0	1
3	0	0	1	1	0	1
4	0	1	0	0	0	1
5	0	1	0	1	0	0
6	0	1	1	0	1	1
7	0	1	1	1	1	0
8	1	0	0	0	0	1
9	1	0	0	1	1	1
10	1	0	1	0	0	1
11	1	0	1	1	1	1
12	1	1	0	0	0	1
13	1	1	0	1	1	0
14	1	1	1	0	1	1
15	1	1	1	1	0	0

2. **(5 points)** Show the switching expressions of z_1 and z_0 in sum of minterms form.

$z_1 = \sum m(6, 7, 9, 11, 13, 14)$

$z_0 = \sum m(0, 1, 2, 3, 4, 6, 8, 9, 10, 11, 12, 14)$

3. **(8 points)** Show the minimal sum of products expressions of z_1 and z_0. In each case, show a K-map, indicate all prime implicants, ~~and all~~ essential prime implicants. Show NAND-NAND networks.

The answer is on the last page.

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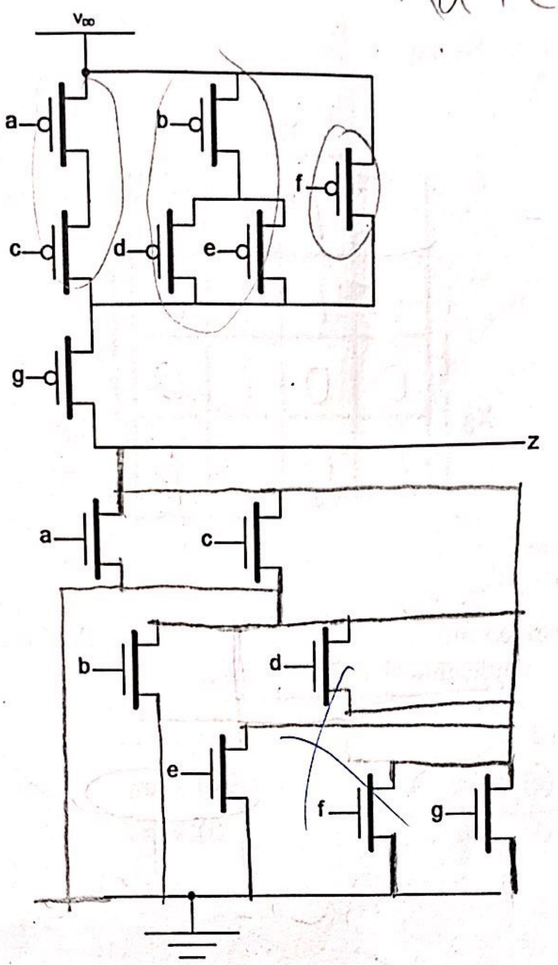
blem 6 (10 points)

e are given the following partial CMOS network.

$(a' \oplus c') + (d'+e') \cdot b' + f'$



1. **(5 points)**

   Write the expression for the pull-up network. From this, derive the expression for the pull-down network using switching algebra.

2. **(5 points)** Connect NMOS transistors to complete the circuit according to the pull-down expression. Please only add missing wires.

1. Pull up: $Z = g' \cdot [a'c' + (d'+e') \cdot b' + f']$

   Pull down: $Z = g'a'c' + g'd'b' + g'e'b' + g'f'$

   $Z' = (g'a'c')' \cdot (g'd'b')' \cdot (g'e'b')' \cdot (g'f')'$

   $= (g+a\cancel{+c}) \cdot (g\cancel{+a}+b) \cdot (g+e+b) \cdot (g+f)$ d

implicates

Problem 7 (~~15~~ 9 points)

For $f(x_3, x_2, x_1, x_0) = (x_3 + x_2' + x_1 + x_0)(x_3 + x_2' + x_1' + x_0)(x_3' + x_2 + x_1 + x_0')(x_3' + x_2' + x_1 + x_0)(x_3' + x_2$ $x_1 + x_0')(x_3' + x_2' + x_1' + x_0)$

1. **(2 points)** Fill out the following K-map.

		x_0	x_0		
	1	1	1	1	
	0	1	1	0	x_2
x_3	0	0	1	0	x_2
x_3	1	0	1	1	
			x_1	x_1	

2. **(4 points)** Which of the given expressions are prime implicants of the function given above? Circle **all** that apply. Write down any prime implicants that are missing.

(a) x_1 (d) $x_3'x_1$ (g) $x_2'x_0$ (j) $x_3'x_2'x_1$

(b) x_3x_1 (e) $x_3'x_0$ (h) x_1x_0 (k) $x_2x_1x_0$

(c) $x_3'x_2'$ (f) x_2x_1 (i) x_1x_0' (l) $x_3x_2x_1x_0$

$X_2'X_0'$
$X_3X_2'X_1$
} missing prime implicants

3. **(2 points)** Write down the complete set of **essential** prime implicants.

$X_2'X_0'$
X_1X_0 $X_3'X_0$

4. **(1 point)** Write the minimal sum of products expression for f. Is it unique?

minimal sp $= X_2'X_0' + X_1X_0 + X_3'X_0$ it is unique

P5.3

0	1	3	2
4	5	7	6
12	13	15	14
8	9	11	10

Z_1

y_0

0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0

x_0, x_1, y_1

Prime implicants:

$x_1 y_1' y_0$,

$x_1 x_0' y_0$,

$x_1' x_0 y_1$,

$x_0 y_1 y_0'$

All of them are essential.

Z_0

y_0

1	1	1	1
1	0	0	1
1	0	0	1
1	1	1	1

x_0, x_1, y_1

Prime implicants:

y_0',

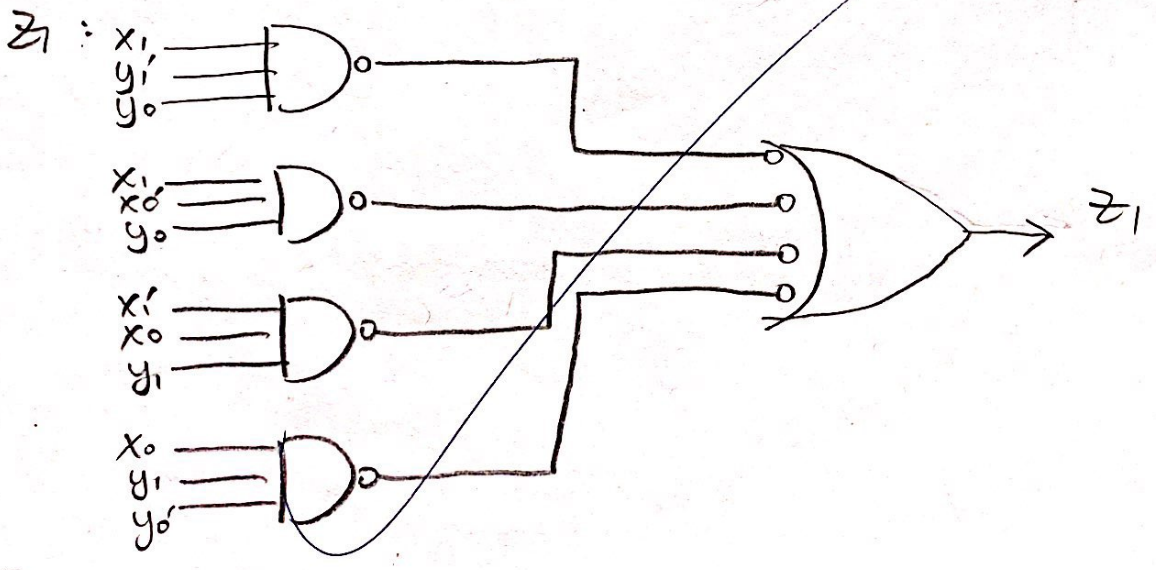
x_0'

All of them are essential.

$$Z_1 = x_1 y_1' y_0 + x_1 x_0' y_0 + x_1' x_0 y_1 + x_0 y_1 y_0'$$

$$Z_0 = y_0' + x_0'$$

Z_1:



Z_0