[CS M51A Fall 18] Solutions for Midterm exam

Date: 10/30/18

Problem 1 (10 points)

1. **(4 points)** How many bits are required to encode a color spectrum capable of supporting 16 million colors using:

   a. Decimal digits in BCD

   b. Hexadecimal representation

   Which representation is more efficient? Why?

   Solution 16 million colors are represented with 8 decimal digits. Each decimal digit needs 4 bits in BCD, therefore $8 \times 4 = 32$ bits.

   $2^{23} = 8,388,608 < 16,000,000 < 16,777,216 = 2^{24}$.

   Therefore we need 24 bits or 6 hex digits. The hex representation is more efficient than BCD representation. With four bits per digit, and ten digit values, BCD uses only 10 out of 16 possible bit-vectors per digit while in hex representation has no wasted bits.

2. **(6 points)** Fill in the missing entries in the table.

Radix	Digit vector $\underline{x}$	Value **x** in decimal
16	(5, 1, 7)	
8	(5, 1, 7)	

Solution

$$(1) \quad = 7 + 1 \times 16 + 5 \times 16^2 = 1303$$
$$(2) \quad = 7 + 1 \times 8 + 5 \times 8^2 = 335$$

Problem 2 (15 points)

$a+b=b+a$	$ab=ba$	Commutativity
$a+(bc)=(a+b)(a+c)$	$a(b+c)=(ab)+(ac)$	Distributivity
$a+(b+c)=(a+b)+c=a+b+c$	$a(bc)=(ab)c=abc$	Associativity
$a+a=a$	$aa=a$	Idempotency
$a+a'=1$	$aa'=0$	Complement
$1+a=1$	$0a=0$	
$0+a=a$	$1a=a$	Identity
$(a')'=a$		Involution
$a+ab=a$	$a(a+b)=a$	Absorption
$a+a'b=a+b$	$a(a'+b)=ab$	Simplification
$(a+b)'=a'b'$	$(ab)'=a'+b'$	DeMorgan's law

Given $E(a,b,c,d)=(ab+c)'(ac+(b'+c'+a'cd)')+a((b+c)(b+d)+c)'$, which of the following represents the same function as $E(a,b,c,d)$? Show all your work.

1. $a+b+c+d'$
2. $a'+b+c$
3. $b+c'+d$
4. $a'b'c'd$
5. $ab'c'$
6. $b'cd'$

Solution The correct simplification is shown below.

$$\begin{aligned}
E(a,b,c,d) &= (ab+c)'(ac+(b'+c'+a'cd)')+a((b+c)(b+d)+c)' \\
&= (ab+c)'(ac+(b'+c'+a'cd)')+a((b+c)(b+d))'c' \\
&= (ab+c)'(ac+(b'+c'+a'cd)')+a(b+cd)'c' \\
&= (ab+c)'(ac+(b'+c'+a'cd)')+ab'(cd)'c' \\
&= (ab+c)'(ac+(b'+c'+a'cd)')+ab'(c'+d')c' \\
&= (ab+c)'(ac+(b'+c'+a'cd)')+ab'c' \\
&= (ab)'c'(ac+(b'+c'+a'cd)')+ab'c' \\
&= (ab)'c'(ac+bc(a'cd)')+ab'c' \\
&= (ab)'c'c(a+b(a'cd)')+ab'c' \\
&= ab'c' \\
\overline{E} &= (ab'c')' = a'+b+c
\end{aligned}$$

Expressions corresponding to E(a,b,c) are: (e) sum of products (single PT), (b) product of sums (single ST) describing E'(a,b,c).

Problem 3 (15 points)

Show if the gate G, described by $G(x, y, z) = one - set\{3, 4, 6, 7\}$, can implement NOT and AND gates. Assume that 0 and 1 are available. If it can, then use G gates to implement the following expression and show the corresponding network of G gates

$$E(a, b, c) = (a + b')(b + c')$$

Solution

$G(x, y, z) = x'yz + xy'z' + xyz' + xyz = xz' + yz$

NOT: $G(1, 0, z) = z'$ or $G(1, 1, z) = z'$

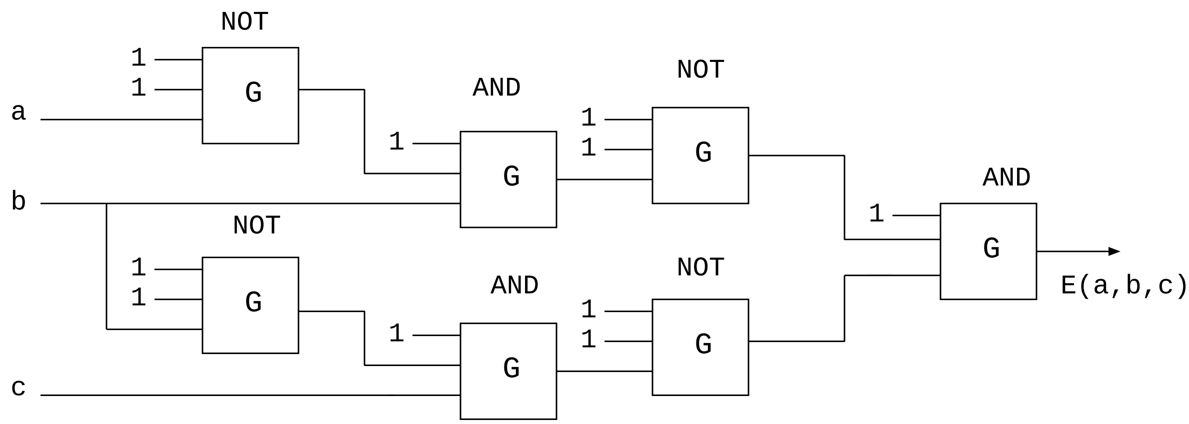
AND: $G(0, y, z) = yz$ or $G(1, y, z) = yz$ or $G(x, y, 0) = xy$

Since the set {AND, NOT} is a universal gate set, so is the set {G}.

Given expression $E(a, b, c) = (a + b')(b + c') = (a'b)'(b'c)'$ can be implemented with G gates as follows

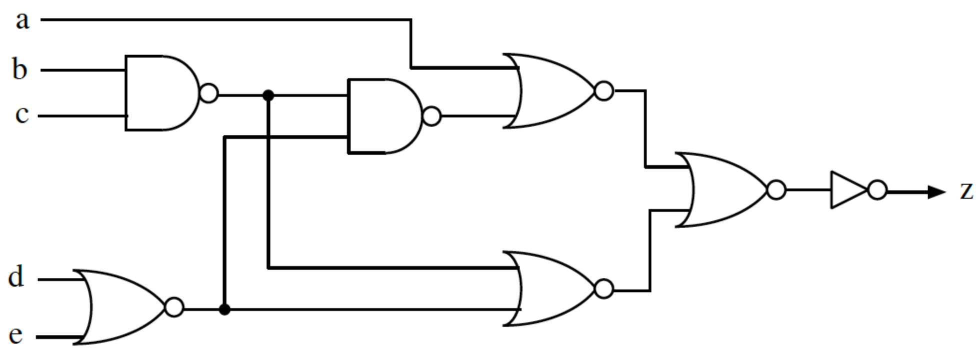
$$\begin{aligned} E(a, b, c) &= AND(NOT(AND(NOT(a), b)), NOT(AND(NOT(b), c))) \\ &= G(0, G(1, 0, G(0, G(1, 0, a), b), G(1, 0, G(0, G(1, 0, b), c)) \end{aligned}$$

The G network is shown in the figure



Problem 4 (15 points)

With the help of the table below, determine the low to high propagation delay $t_{pLH}(d,z)$ of the output z of the network shown below. Assume the network output has a load of 6.



Gate Type	Fan-in	Propagation Delays (ns) t_{pLH}	t_{pHL}	Load Factor
NOT	1	$0.02 + 0.038L$	$0.05 + 0.017L$	1.0
NAND	2	$0.05 + 0.038L$	$0.08 + 0.027L$	1.0
NOR	2	$0.06 + 0.075L$	$0.07 + 0.016L$	1.0

Solution

$t_{pLH}(d,z) = t_{pLH}(NOT) + t_{pHL}(NOR) + t_{pLH}(NOR) + t_{pHL}(NAND) + t_{pLH}(NOR)$

$t_{pLH}(d,z) = (0.02+0.038*6)+(0.07+0.016*1)+(0.06+0.075*1)+(0.08+0.027*1)+(0.06+0.075*2) = 0.786ns$

Problem 5 (20 points)

Obtain a two-level gate network of the following system.

Inputs:	$x, y \in \{0,1,2,3\}$
Outputs:	$z \in \{0,1,2,3\}$
Function:	$z = \{3xy+1\} \mod 4$

1. **(2 points)** Complete the switching table using binary encoding for all values.

   Solution

x_1	x_0	y_1	y_0	$3xy+1$	z	z_1	z_0
0	0	0	0	1	1	0	1
0	0	0	1	1	1	0	1
0	0	1	0	1	1	0	1
0	0	1	1	1	1	0	1
0	1	0	0	1	1	0	1
0	1	0	1	4	0	0	0
0	1	1	0	7	3	1	1
0	1	1	1	10	2	1	0
1	0	0	0	1	1	0	1
1	0	0	1	7	3	1	1
1	0	1	0	13	1	0	1
1	0	1	1	19	3	1	1
1	1	0	0	1	1	0	1
1	1	0	1	10	2	1	0
1	1	1	0	19	3	1	1
1	1	1	1	28	0	0	0

2. **(5 points)** Show the switching expressions of z_1 and z_0 in sum of minterms form.

   Solution Looking at the table, we can get:

$$z_1 = \sum m(6,7,9,11,13,14)$$
$$z_0 = \sum m(0,1,2,3,4,6,8,9,10,11,12,14)$$

3. **(8 points)** Show the minimal sum of products expressions of z_1 and z_0. In each case, show a K-map, indicate all prime implicants, and all essential prime implicants. Show NAND-NAND networks.

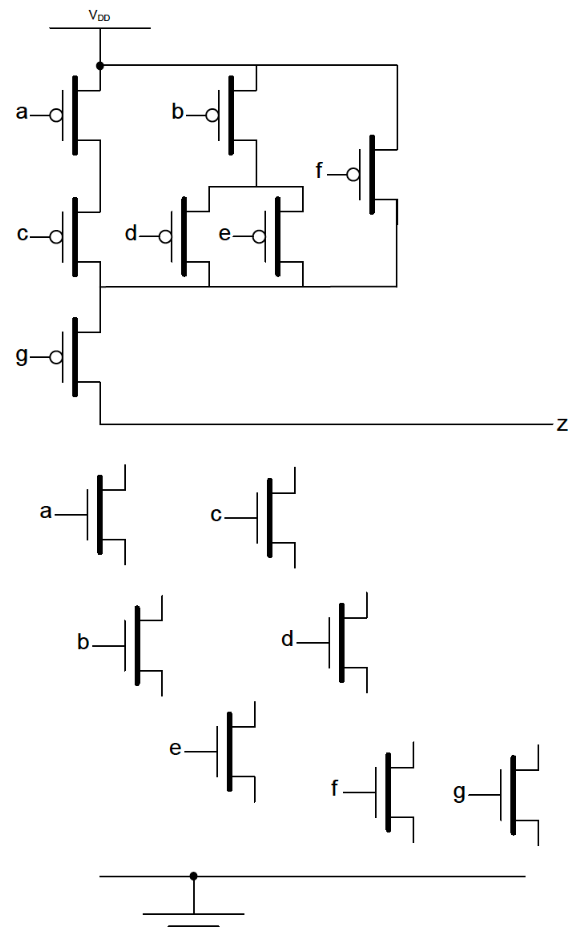
Solution All rectangles are prime implicants; all are essential. The minimal sum of products are:

$$\begin{aligned} z_1 &= x_1x_0'y_0 + x_1'x_0y_1 + x_1y_1'y_0 + x_0y_1y_0' \\ z_0 &= x_0' + y_0' \end{aligned}$$

Figure to be done

Problem 6 (10 points)

We are given the following partial CMOS network.



1. **(5 points)**

   Write the expression for the pull-up network. From this, derive the expression for the pull-down network using switching algebra.

   Solution

   From the given circuit we can directly write expression for the pull-up network:

   $z = (a'c' + b'(d' + e') + f')g'$

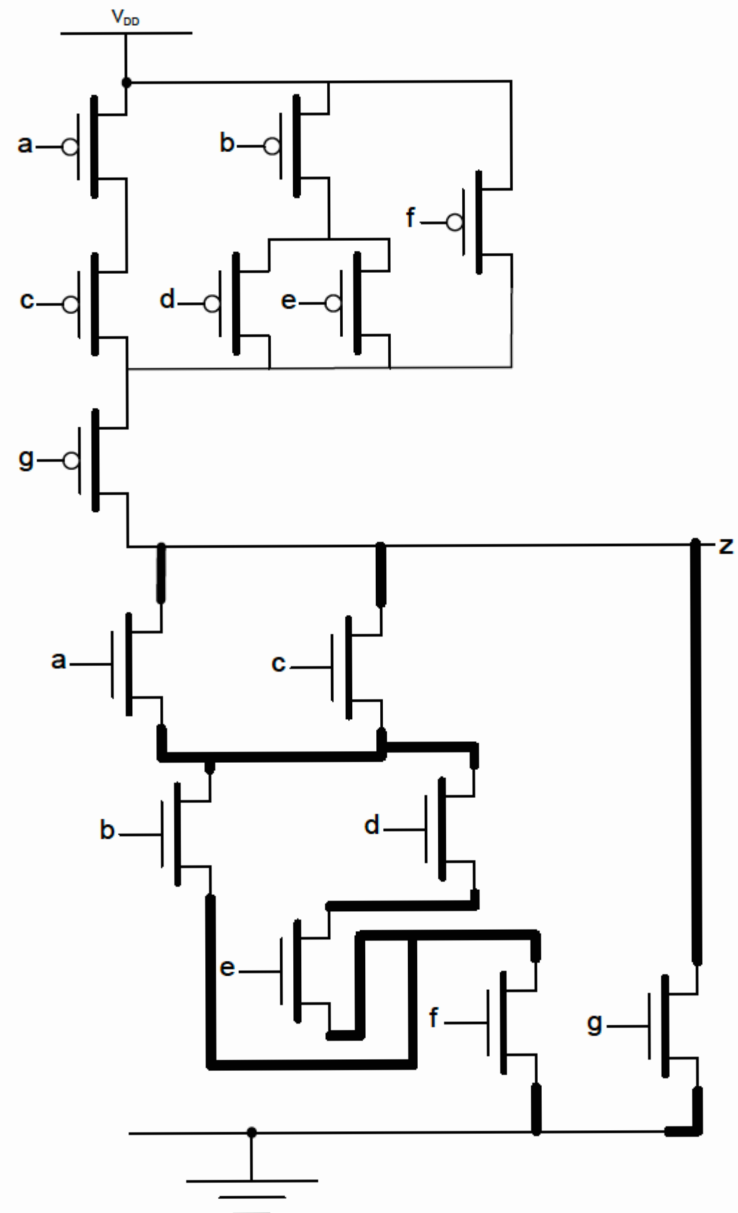
   The expression for the pull-down network is

   $z' = [(a'c' + b'(d' + e') + f')g']' = (a'c')'(b'(d' + e'))'f + g = (a + c)(b + de)f + g$

2. **(5 points)** Connect NMOS transistors to complete the circuit according to the pull-down expression. Please only add missing wires.

Solution

The completed circuit is shown bellow



V_DD
a
b
f
c
d
e
g
z
a
c
b
d
e
f
g

Problem 7 (15 points)

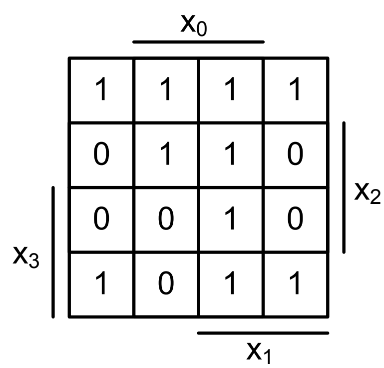
For $f(x_3, x_2, x_1, x_0) = (x_3 + x_2{}' + x_1 + x_0)(x_3 + x_2{}' + x_1{}' + x_0)(x_3{}' + x_2 + x_1 + x_0{}')(x_3{}' + x_2{}' + x_1 + x_0)(x_3{}' + x_2{}' + x_1 + x_0{}')(x_3{}' + x_2{}' + x_1{}' + x_0)$

1. **(2 points)** Fill out the following K-map.

   Solution From the given equation, we can get

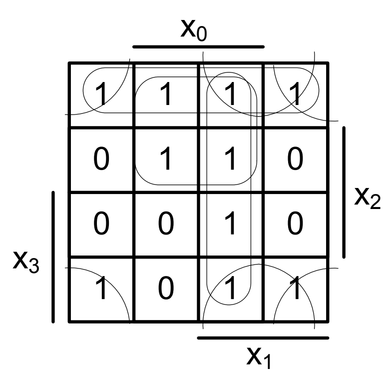
$$f(x_3, x_2, x_1, x_0) = \prod M(4, 6, 9, 12, 13, 14)$$

   The completed K-map is shown:



2. **(4 points)** Which of the given expressions are prime implicants of the function given above? Circle **all** that apply. Write down any prime implicants that are missing.

   Solution The prime implicants are shown in the K-map.



   The equivalent product terms are $x_3{}'x_2{}'$ (c), $x_3{}'x_0$ (e), $x_2{}'x_1$ (missing), $x_2{}'x_0{}'$ (missing) and x_1x_0 (h).

3. **(2 points)** Write down the complete set of **essential** prime implicants.

   Solution The 1 squares with single coverage are 5, 8 and 15. So we have three essential implicants, $x_3{}'x_0$, $x_2{}'x_0{}'$ and x_1x_0.

4. **(1 point)** Write the minimal sum of products expression for f. Is it unique?

Solution Since all 1 squares are covered by the essential prime implicants, the minimal SOP is $x_3{}'x_0 + x_2{}'x_0{}' + x_1x_0$. Also, it is unique.