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Last First

UCLA COMPUTER SCIENCE DEPARTMENT

MIDTERM EXAMINATION

CS M51A/EE M16 Summer 2015 Section 1
Logic Design of Digital Systems

July 21, 2015

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Rules:

This is a closed-textbook, closed-note, and independent exam (110 minutes). You may use two-page 8.5"x11" single-sided note. No scratch paper or calculator is allowed. Points are assigned to the problems based on estimates of how long they should take. PACE YOURSELF ACCORDINGLY. The order may not reflect the degree of difficulty. BROWSE THROUGH THE ENTIRE SET first to decide the order you want to follow. READ THE PROBLEM DESCRIPTION CAREFULLY. Be sure to include all final answers at indicated locations. Write down your Student ID at the top of each page. Use provided space for all work. Have fun and good luck!

Honor Code:

I attest that I have not given or received any aid or discussion in relationship to this exam.

Panse

Your Signature

Your Score:

No.	Your Score	Maximal Score
#1	10	10
#2	15	15
#3	13	15
#4	10	10
#5	15	15
#6	14	20
#7	15	15
Total	92	100

Problem No. 1 (10 points) 10

Part (a) A car odometer with five decimal digits wraps around at 100,000 miles. If it uses five radix-8 digits instead, at how many miles does it wrap around?

Your Answer: 32,768 ✓ miles. $(A,B,C,D,E)_{10}$ $10^5 = 100,000$

Show all your work below for full credit: $(A,B,C,D,E)_8$ $8^5 = 8\cdot8\cdot8\cdot8\cdot8$

$64 \cdot 8 = 512 = 8^3 \cdot 8 = 4096 \cdot 8 = 32,768$

Part (b) NASA's first manned mission to Mars in 2030 finds only the ruins of a civilization. From the artifacts, astronauts find the following expression: $\sqrt{41} = 5$. Extensive research discovers that the Martian people used a position number system and the expression is correct. What is the radix of the Martian number system?

Your Answer: 6 ✓

Show all your work below for full credit:

$\sqrt{4R+1}^2 = 5^2$

$4R + 1 = 25$

$R = 6$

Part (c) A electronic ruler *iRuler* measures length by one 3-digit mixed-radix number system (*yard*, *foot*, *inch*). Recall their relationships are: 1 *yard* = 3 *feet*, 1 *foot* = 12 *inches*. Assume that radix for digit *yard* is twelve.

(c.1) Using binary code, at least how many bits are needed to encode the largest number of *inches* that *iRuler* can measure?

Your Answer: 9 ✓ bits.

(c.2) How many *inches* are represented by a reading of $X = (10, 2, 9)$?

Your Answer: 393 ✓ *inches*.

Show all your work below for full credit:

Yard	Foot	Inches
12	3	12
3	12	1

c.1. $12 \cdot 3 \cdot 12 - 1 = 144 \cdot 3 - 1 = 432 - 1 = 433$ is the largest number of inches $2^5 = 32$ $2^6 = 64$ $2^7 = 128$ $2^8 = 256$ $2^9 = 512$ $2^{10} = 1024$

$2^8 < 433 < 2^9 \rightarrow 9$ bits $11 \cdot 11 \cdot 2$

c.2. $10 \cdot 3 \cdot 12 \cdot 1 + 2 \cdot 12 \cdot 1 + 9 \cdot 1$

$30 \cdot 12 = 360 + 24 + 9 = 393$

Problem No. 2 (15 points) 15

A high-tech startup *HotCom, Inc.* is operated by the CEO A, the CFO B, and two members of the Board of Directors, C and D.

To make a decision, A needs support from at least another member of the board, while B needs support from at least two other members of the board.

Part (a)

Fill in the function table below for F. Function $F = 1$ if a decision has been approved.

A	B	C	D	F	
0	0	0	0	0	0
0	0	0	1	0	1
0	0	1	0	0	2
0	0	1	1	0	3
0	1	0	0	0	4
0	1	0	1	0	5
0	1	1	0	0	6
0	1	1	1	1	7
1	0	0	0	0	8
1	0	0	1	1	9
1	0	1	0	1	10
1	0	1	1	1	11
1	1	0	0	1	12
1	1	0	1	1	13
1	1	1	0	1	14
1	1	1	1	1	15

Part (b)

(b.1) Write the *minterm* expression for $F(A, B, C, D)$ in compact form:

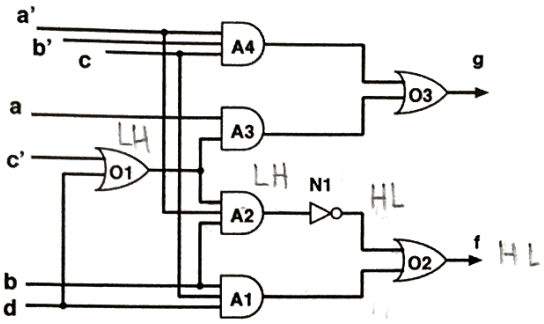
$$F(A, B, C, D) = \sum m \{7, 9, 10, 11, 12, 13, 14, 15\}$$

(b.2) The switching expression for maxterm M_8 is:

$$M_8 = (A' + B + C + D)$$ ✓

Problem No. 3 (15 points) 13

Given the gate network below, answer the following questions:



Part(a) Assume that negated variables are available. Find the *critical path* of the above network by listing its gates along the path, starting at the inputs:

O1 → A2 → N1 → O2

Part(b) Assume that load factors of all gates equal to 1 and that both outputs f and g have the output load value L. List the output load value of every gate in the *critical path* (e.g. A1: 1):

O1 : 2 A2 : 1 N1 : 1 O2 : L

Part(c) Write the expression of the longest network propagation delay T_{pHL} in terms of delays of each gate (You **do not** have to compute the final result but transition direction on each gate has to be indicated):

Think about it as the gate's output

$T_{pHL} = T_{pLH}(O1) + T_{pLH}(A2) + T_{pHL}(N1) + T_{pHL}(O2)$

Part(d) Assume that negated variables are available. Write the switching expression of the output f in two-level AND-OR (sum-of-product) form. It **does not** have to be minimal. Show all your work below for full credit.

$f(a,b,c,d) = bcd + (d + c')a'b \Rightarrow bcd + a'bd + a'bc'$.

-2

Show all your work below for full credit:

$f = T_1 + T_2$

$f = bcd + T_3'$

$f = bcd + T_4 a'b$

$f = bcd + (d + c')a'b$

$T_2 = T_3'$ $T_3' = T_4 a'b$

$T_4 = c' + d$ $T_1 = bcd$

$bcd + T_3'$

$bcd + T_4 a'b$

$bcd + (c' + d)a'b$

$bcd + (a'bc') + a'bd$

Problem No. 4 (10 points) 10

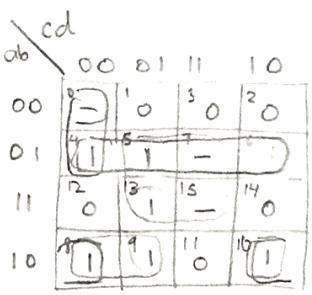
A former CS M51/EE M16 student Goofy is interviewed for an internship in a Silicon Valley company, Pokerface, Inc.. He is asked to minimize the following 4-input switching function using the Quine-McCluskey algorithm.

$$f(a,b,c,d) = \sum m(4,5,6,8,9,10,13) + \sum d.c.(0,7,15)$$

Goofy has completed the first step and generated the *prime implicant chart* as shown below. You have to help him identify the essential prime implicants and then give the minimal AND-OR (sum-of-product) expression of $f(a,b,c,d)$.

Prime Implicant Chart

Prime Implicants	Minterms 4	5	6	8	9	10	13
✓ 0 – 00	X						
✓ –000				X			
✓ 100–				X	X		
✓ 10 – 0				X		(X)	
- 1 – 01					X		X
✓ 01 – –	X	X	(X)				
–1 – 1		X					X



Part (a) The essential prime-implicants (in switching expressions) are:

a'b, ab'd'

Part (b) The minimal switching expression in two-level AND-OR form is:

$f(a,b,c,d) =$ a'b + ab'd' + ac'd ✓

Show your work on the Prime Implicant Chart above for full credit.

Problem No. 5 (15 points) 15

A newly-hired summer intern Bruinie at *Googlic, Inc.* is given the following instructions for her first assignment by her mentor, Dr. D. Logic, on the first day at work:

1. The electric power should be turned on unless nobody is in the office and the automatic monitoring system is not in operation.
2. The automatic monitoring system will be in operation if and only if nobody is in the office or a large payroll is left in the office.

With a solid A in her CSM51A class, Bruinie figures out that she should just always leave the electric power on. Please help check whether her interpretation on Mr. Logic's instruction is correct. If not, give the correct one.

Your answer: Correct

Show all your work below for full credit.

E = Electric power

P = Office has people inside

A = AMS is in operation

L = Payroll is in office

unless means $(\quad)'$

$$E = (P' \cdot A')'$$

$$A = (P' + L)$$

$$E = (\underbrace{P'}_{X} \cdot (\underbrace{P'}_{Y} + L)')' \text{ by De Morgan's law}$$

$$(XY)' = X' + Y' \longrightarrow P + (P' + L)$$

$$E = P + P' + L = 1$$

This entity will always be 1 because either P or P' will be 1 ✓

people not in office

E = El

N = Nobody

A = in operation

L = left in office

$$E = (N \cdot A')' = N' + A = N' + N + L$$

$$A = N + L$$

people are in office

Problem No. 6 (20 points) 14

A former CSM51A class student Minnie has easily landed a job as a digital system designer in *YourTubix, Inc.* after graduation. She is motivated to revolutionarize the logic design and has invented a new type of logic gate, called *BILL* gates. Its symbol and truth table are given below.

x	y	BILL(x,y)
0	0	1
0	1	1
1	0	0
1	1	1

x' y'
x' y
x y'
x y

x' + y = 1
x' + y = 1
x' + y = 0

Part(a) Is the *BILL* gate a universal set?

Your answer: Yes

Show all your work below for full credit.

Univ set: NOT NOR OR
* NOT OR AND
NOT AND

F = x' + y

NOT gate: Bill(x, 0) = x' + 0 = x'

OR gate: we need x+y → Bill(x', y) = x'' = x + y
· Bill(Bill(x, 0), y)

NOR = (x+y)'

AND = xy

AND gate: we need xy = (x'+y')'
x' + y' = Bill(x, y') (x'+y')' = (Bill(x, y'))'

Bill(Bill(x, y'), 0)
still need to eliminate this

Bill(Bill(x, Bill(y, 0)), 0)
x' + y = y' + 0 = y'

✓

Part(b) Implement a three-input XOR gate using one of the following approaches:
(1) use *BILL* gates only if the answer in Part (a) is yes, otherwise
(2) use at least one *BILL* gate and any other types of gates as needed.

Show all your work below for full credit.

XOR: XY' + X'Y

3 inputs? −6

(Extra space available on the next page)

(Extra space for Problem No. 6)

$XOR = XY' + X'Y$ $\quad f = x' + y$

$Bill((XY')', X'Y)$

$Bill(Bill(XY', 0), X'Y)$

AND: $Bill(Bill(x, Bill(y, 0)), 0) = xy$

OR: $Bill(Bill(x, 0), y)$

$XY + YZ + X'Z = XY + X'Z$

$Y = Y' \quad Z = Y$

$XY' + YY' + X'Y$

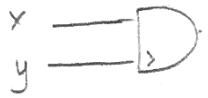
$X' \qquad Y$

$Y = X'Y$

$X = (XY')'$

$X = X' + Y$

$x' + y$



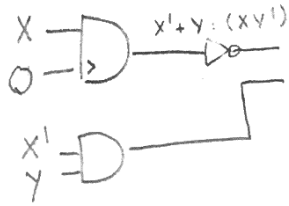
$Bill(x, y)$ 1st input

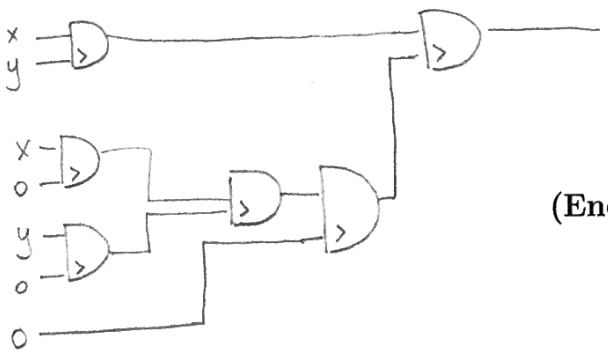
$X'Y = (X + Y')' \quad X + Y' = (Bill(X', Y'))' = (Bill(Bill(X, 0), Y'))'$

$(XY')' = X' + Y \quad 0 \quad X' + Y = X$

$Bill(Bill(Bill(x, 0), y'), 0)$

$Bill(Bill(Bill(x, 0), Bill(y, 0)), 0)$





(End of Problem No. 6)

Problem No. 7 (15 points) 15

Fill in blanks in the following table by performing conversions and arithmetic operations in specified **binary number systems**. For one's complement and two's complement in parts (c) and (d), use **only** complementation and addition, and **indicate overflow if it occurs**. For part (e), use **only** shifting, complementation, and addition as needed, and **no** subtraction, multiplication, or division is allowed. Use provided space for all your work. Please **clearly LABEL** your steps and the final answer.

Operations	Sign/Magnitude		1's Complement		2's Complement	
	Bit Vector	Signed Integer	Bit Vector	Signed Integer	Bit Vector	Signed Integer
Part (a): x	11000	-8	11000	-7	11000	**N/A**
Part (b): y	01011	+11	01011	+11	01011	+11
Part (c): $s = x + y$	**N/A**	**N/A**	00100	+4	**N/A**	**N/A**
Part (d): $d = x - y$	overflow		**N/A**	**N/A**	overflow	
Part (e): $z = \frac{1}{2}x + 2y$	**N/A**	**N/A**	**N/A**	**N/A**	010010	+18

✓

Show all your work below for full credit.

Part (a): x SM 11000 = -8

sign

1's) 11000 - complement form

00111 - -7

Part (b): y SM 01011 + 11

sign: +

1's 01011 - true form → +11

2's 01011 - true form → +11

(Continue on the next page)

(Continue from Problem No. 7)

Part (c): $s = x + y$ $\quad x = 11000 \quad y = 01011$

1's C.S.

$$\begin{array}{r} {\scriptstyle 1\ \ 1\ \ \ \ \ \ } \\ 11000 \\ +\ 01011 \\ \hline 100011 \end{array}$$

↳ carry $+1$

$00100 \rightarrow +4$

Part (d): $d = x - y$ $\quad x = 11000 \quad y = 01011 \quad x - y = x + (-y)$

S. M.

$$\begin{array}{r} {\scriptstyle 1\ 1\ 0\ 0\ 0\ \ } \\ 11000 \\ +\ 11011 \\ \hline 1.10011 \end{array}$$

carry in ≠ carry out

Overflow

2's C.S $\quad x = 11000 \quad y = 01011$

$d = x - y =$

$x + (-y) =$

$$\begin{array}{r} {\scriptstyle 1\ 1\ 0\ 0\ 0\ \ } \\ 11000 \\ +\ 10101 \\ \hline 1.01101 \end{array}$$

$y = 01011$

$-y = 10100 + 1 = 10101$

carry in ≠ carry out

Overflow

(Continue on the next page)

(Continue from Problem No. 7)

Part (e): $z = \frac{1}{2}x + 2y$

x divided by 2
y multiplied by 2

x = 11000
y = 01011

x = 111000 } because mult/div
y = 001011 } If you don't extend you lose the sign of y

shift
$\frac{x}{2}$ = 111100

shift
2y = 010110

$z = \frac{x}{2} + 2y$

```
  111100
+ 010110
 (1)010010
```

throw away carry

since x is negative and y is positive, the result needs to be positive

z = 010010 → signed int.

$0 \cdot 2^0 + 1 \cdot 2 + 0 \cdot 4 - 0 \cdot 8 + 1 \cdot 16 + 0\ 32$

2 + 16 = +18

(End of Midterm)