[CS M51A W17] Solution to Quiz 4

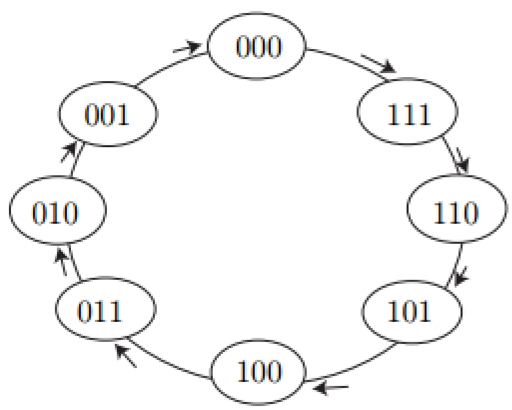
Date: 03/10/2017

Quiz Problems (50 points total)

Problem 1 (20 Points)

We would like to design a 3-bit binary **synchronous down-counter**.

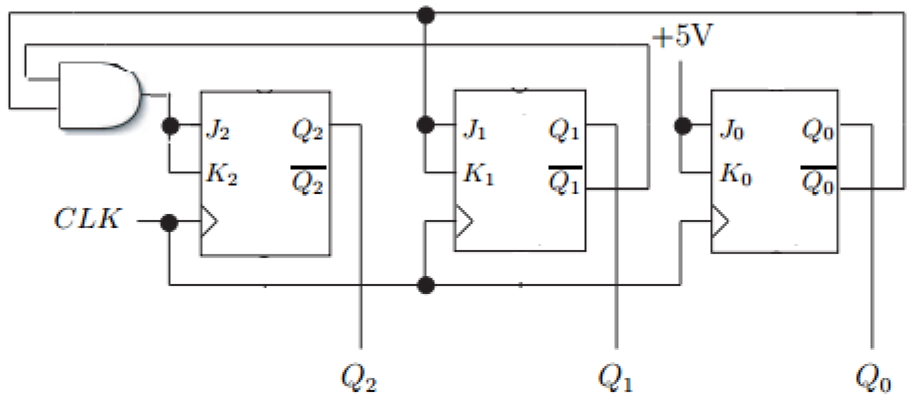
1. (4 points) Draw the state-transition diagram, with the three bit flip flop outputs as the State. Begin with the state 111.
   Solution:



2. (16 points) We will be using JK Flip-Flops to implement this down-counter. Complete the truth table given below.
Solution:

Present State			Next State			Inputs					
$Q_2(t)$	$Q_1(t)$	$Q_0(t)$	$Q_2(t+1)$	$Q_1(t+1)$	$Q_0(t+1)$	J_2	K_2	J_1	K_1	J_0	K_0
0	0	0	1	1	1	1	X	1	X	1	X
0	0	1	0	0	0	0	X	0	X	X	1
0	1	0	0	0	1	0	X	X	1	1	X
0	1	1	0	1	0	0	X	X	0	X	1
1	0	0	0	1	1	X	1	1	X	1	X
1	0	1	1	0	0	X	0	0	X	X	1
1	1	0	1	0	1	X	0	X	1	1	X
1	1	1	1	1	0	X	0	X	0	X	1

3. (10 points) Complete the circuit based on the truth table above. Use Kmaps if required.
   $J_0 = K_0 = 1$
   $J_1 = K_1 = Q_0'$
   $J_2 = K_2 = Q_0'.Q_1'$



Problem 2 (10 points)

Implement the given function using only one **4 input MUX**. You can use additional gates if needed. Do not use gates if they are not required.
$Z = f(A, B, C) = A'B + B'C + BC + AB'C'$
Solution:
Expanding to Sum of Products form:

$$\begin{aligned} Z &= A'B(C + C') + B'C(A + A') + BC(A + A') + AB'C' \\ &= A'BC + A'BC' + AB'C + A'B'C + ABC + A'BC + AB'C' \\ &= A'BC + A'BC' + AB'C + A'B'C + ABC + AB'C' \\ &= A'B'(C) + AB'(C + C') + A'B(C + C') + ABC \\ &= A'B'(C) + AB'(1) + A'B(1) + ABC \end{aligned}$$

Thus we can use A and B as select lines: