[CS M51A Fall 15] Quiz 1

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- The quiz is closed book, and closed notes (30mins).
- Please show all your work and write legibly, otherwise no partial credit will be given.
- **This should strictly be your own work; any form of collaboration will be penalized.**

Name :

Student ID :

Quiz Problems (50 points total)

Problem 1 (10 points)

Find x and y such that the following conditions are satisfied and show all the steps of your work.

1. **(5 points)** $(818)_9 = (x)_3$

$8\cdot9^2 + 1\cdot9^1 + 8\cdot9^0 = 8\cdot81 + 9 + 8 = 648 + 9 + 8 = 665$

$(x)_3 = 2\cdot3^5 + 2\cdot3^4 + 0\cdot3^3 + 1\cdot3^2 + 2\cdot3^1 + 2\cdot3^0$

$\boxed{(x)_3 = 220122}$

$81 \times 8 = 648$ $\quad 648 + 17 = 665$

$81 \times 3 = 243$ $\quad 243 \times 3 = 729$

$81 \times 2 = 162$ $\quad 243 \times 2 = 486$

$665 - 486 = 179$ $\quad 179 - 162 = 17$ $\quad 17 - 9 = 8$ $\quad 8 - 6 = 2$

2. **(5 points)** What is the largest number y that can be represented with 4 digit-vector in radix 5. Show y in radix 5 and decimal.

decimal

$(Y)_5 = 4\ 4\ 4\ 4$ ← 4digit vector

$(Y)_{10} = 4\cdot 5^3 + 4\cdot 5^2 + 4\cdot 5^1 + 4\cdot 5^0$

$= 4\cdot 125 + 4\cdot 25 + 20 + 4$

$= 500 + 100 + 20 + 4$

$= 624$

$\boxed{(Y)_5 = 4444 \quad (Y)_{10} = 624}$

Problem 2 (16 points)

Solve the following problems using the postulates and theorems of Boolean algebra. **Do not use a truth table.**

1. **(8 points)** The Boolean function f is defined as $f(a, b, c) = ac' + a'b$ and the Boolean function g is defined as $g(a, b, c) = ac + b'c + a'b'$. Show that $g(a, b, c)' = f(a, b, c)$.

$g' = f \Rightarrow (ac + b'c + a'b')' = ac' + a'b$

$(ac)'(b'c)'(a'b')' =$

$(a' + c')(b + c')(a + b) =$

$(a'b + a'c' + bc' + c'c')(a+b) =$

$(\cancel{aa'b} + \underbrace{a'bb}_{a'b} + \cancel{aa'c'} + a'bc' + abc' + \underbrace{bbc'}_{bc'} + \underbrace{acc'}_{ac'} + \underbrace{bc'c'}_{bc'}) =$

$= a'b + a'bc' + abc' + \underbrace{bc' + ac' + bc'}_{bc'}$

$= a'b(1 + c') + abc' + ac' + bc' \qquad = a'b + abc' + ac' + bc'$

$= a'b + bc'(1+a) + ac' = a'b + bc' + ac' = \boxed{ac' + a'b}$

-2

$a + a'b = a + b$

2. **(8 points)** Simplify the following expression.

$$xyzw' + xyz' + xy' + x'$$

$xyzw' + xyz' + xy' + x' = xyzw' + xyz' + x' + y'$

$= xyzw' + yz' + x' + y' = xyzw' + x' + y' + z' = yzw' + x' + y' + z'$

$= zw' + x' + y' + z' = \boxed{x' + y' + z' + w'}$

Problem 3 (24 points)

F is a function that accepts inputs $x \in \{0, 1, 2\}$, $y \in \{1, 2, 3\}$, and outputs $z = max(x^2, y)$. Suppose you use binary code to encode x, y, and z. x is encoded as x_1x_0, y is encoded as y_1y_0, z is encoded as $z_2z_1z_0$.

1. **(16 points)** Fill in the table below.

X: no 3 → dc set

Y: no 0 → dc set

x_1	x_0	y_1	y_0	z_2	z_1	z_0
0	0	0	0	-	-	-
0	0	0	1	0	0	1
0	0	1	0	0	1	0
0	0	1	1	0	1	1
0	1	0	0	-	-	-
0	1	0	1	0	0	1
0	1	1	0	0	1	0
0	1	1	1	0	1	1
1	0	0	0	-	-	-
1	0	0	1	1	0	0
1	0	1	0	1	0	0
1	0	1	1	1	0	0
1	1	0	0	-	-	-
1	1	0	1	-	-	-
1	1	1	0	-	-	-
1	1	1	1	-	-	-

2. **(8 points)** Fill in the sets in the forms specified below.

$$z_2 = \sum m(\ 9,\ 10,\ 11\ )$$
$$z_1 = \sum m(\ 2,\ 3,\ 6,\ 7\ )$$
$$z_0 = \prod M(\ 2,\ 6,\ 9,\ 10,\ 11\ )$$
$$dc-set\ of\ z_1 = dc(\ 0,\ 4,\ 8,\ 12,\ 13,\ 14,\ 15)$$