CSM51A / EE16A Midterm
-closed books and notes-
-no programmable calculators/laptops-
-four pages of summary notes permitted-

THIS IS STRICTLY YOUR OWN WORK:
ANY FORM OF COLLABORATION WILL BE PENALIZED.

(6 problems, 110 minutes)
February 11, 2009

Name:______________________________________

ID No: ______________________________________

Problem	Points	Score
1	10	
2	20	
3	15	
4	15	
5	20	
6	20	
Total	100	

Problem 1 [10 points]

A full-adder (FA) has three inputs a, b and c and two outputs sum and $carry$. The module and truth table for a full-adder are shown Fig. 1 and Fig. 2, respectively.

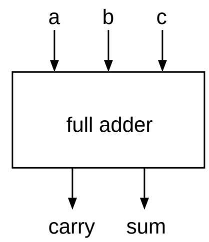


Figure 1: FA module

a	b	c	$carry$	sum
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

Figure 2: FA truth table.

a) [7 points] Show that a full-adder module forms a universal set. Assume that constants 0 and 1 are available.

$$\begin{aligned} cout(0, a, b) &= ab = AND(a, b) \\ cout(1, a, b) &= a + b = OR(a, b) \\ sum(0, 1, a) &= a' = NOT(a) \end{aligned}$$

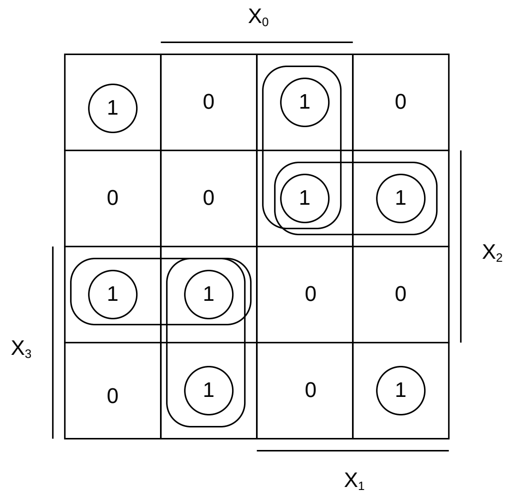
b) [3 points] Why doesn't $cout$ alone form a universal set?

$$cout(a, b, c) = bc + ac + ab$$

There is no way to form a complement so it's impossible for $cout$ to form a universal set.

Problem 2 [20 points]

The following Karnaugh map represents a function $z = f(X_3, X_2, X_1, X_0)$



a) [6 points]

- List all the implicants of the function:

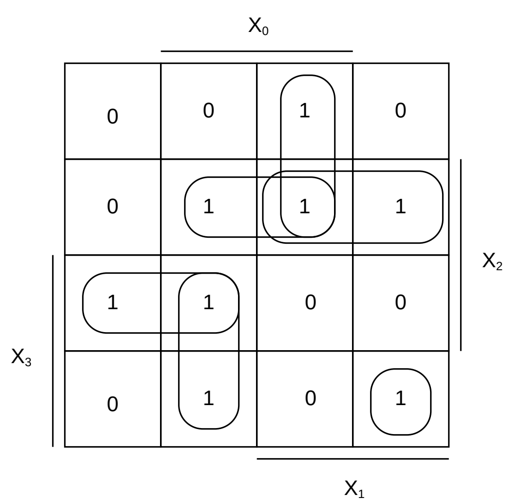
$$X_3'X_2'X_1'X_0',\ X_3'X_2'X_1X_0,\ X_3'X_2X_1X_0'$$
$$X_3'X_2X_1X_0,\ X_3X_2'X_1'X_0,\ X_3X_2'X_1X_0'$$
$$X_3X_2X_1'X_0',\ X_3X_2X_1'X_0,\ X_3X_2X_1',$$
$$X_3X_1'X_0,\ X_3'X_2X_1,\ X_3'X_1X_0$$

- List all the prime implicants:

$$X_3'X_2'X_1'X_0', X_3X_2X_1',\ X_3X_1'X_0,\ X_3'X_2X_1,\ X_3'X_1X_0,\ X_3X_2'X_1X_0'$$

- Indicate which of these prime implicants are essential.
  All of them.

b) [5 points]Determine the minimal sum of products expression for the following Karnaugh map.

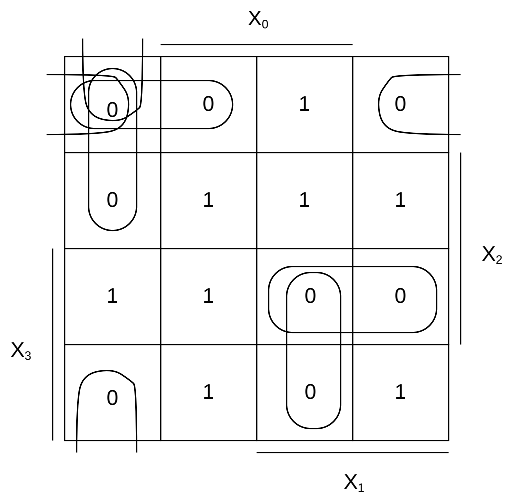


$$z = X_3X_2X_1' + X_3X_0X_1' + X_3'X_2X_1 + X_3'X_2X_0 + X_3'X_1X_0$$

c) [2 points] Is the minimal expression for the sum of products obtained in part (b) unique? Explain.

Yes. All prime implicants are essential prime implicants.

d) [5 points] Determine the minimal product of sums expression for the following Karnaugh map.



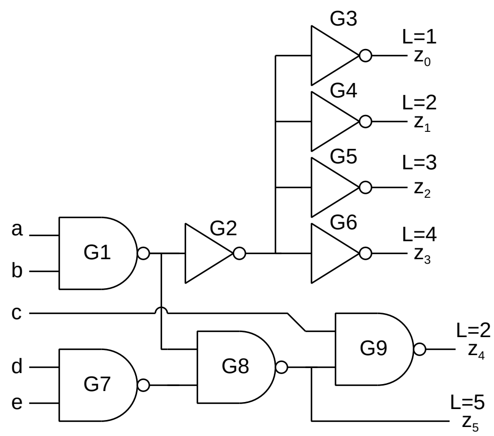
$$z = (X_3+X_1+X_0)(X_3+X_2+X_1)(X_3+X_2+X_0)(X_2+X_1+X_0)(X_3'+X_1'+X_0')(X_3'+X_2'+X_1')$$

e) [2 points] Is the minimal expression for the product of sums obtained in part (d) unique? Explain.

Yes. All prime implicates are essential prime implicates

Problem 3 [15 points]

Given the following gate network, determine the path with the longest delay between the network inputs and the network outputs.



The characteristics of the gates are given in the following table.

Gate type	Fan-in	Propagation delays t_{pLH} (ns)	t_{pHL} (ns)	Load factor (standard loads)	Size (equiv. gates)
NAND	2	$0.05 + 0.038L$	$0.08 + 0.027L$	1.0	1
NOT	1	$0.02 + 0.038L$	$0.05 + 0.017L$	1.0	1

a) [7 points] Find max(t_{pLH}): show the path you will be performing timing analysis on, followed by your analysis.

For path $G1 \rightarrow G8 \rightarrow G9$,

$$\begin{aligned} t_{pLH}(z_4) &= t_{pLH}(G9) + t_{pHL}(G8) + t_{pLH}(G1) \\ &= (0.05 + 0.038(2)) + (0.08 + 0.027(6)) + (0.05 + 0.038(2)) \\ &= 0.494\, ns \end{aligned}$$

b) [7 points] Find max(t_{pHL}): show the path you will be performing timing analysis on, followed by your analysis.

For path $G1 \rightarrow G8 \rightarrow G9$,

$$\begin{aligned} t_{pHL}(z_4) &= t_{pHL}(G9) + t_{pLH}(G8) + t_{pHL}(G1) \\ &= (0.08 + 0.027(2)) + (0.05 + 0.038(6)) + (0.08 + 0.027(2)) \\ &= 0.546\, ns \end{aligned}$$

c) [1 point] What is the maximum delay of the longest path?

$t_{crit} = 0.546\, ns$

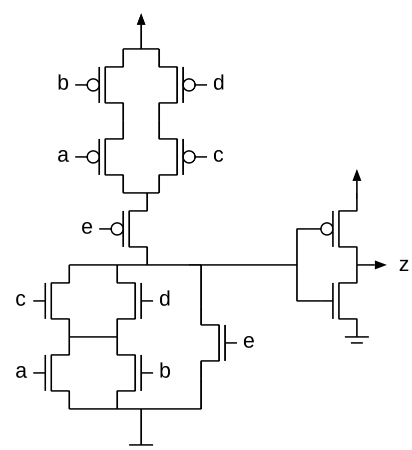
Problem 4 [15 points]

Design a CMOS circuit which implements the function

$$z = (a + b)(c + d) + e$$

using a minimum number of transistors. You may **not** assume that inverted inputs are available.

$$\begin{aligned} f_{PUP} &= z' = ((a + b)' + (c + d)')e' \\ &= (a'b' + c'd')e' \\ f_{PDN} &= (a + b)(c + d) + e \end{aligned}$$



Problem 5 [20 points]

Design a system with the following specification

$$\textbf{Input:} \quad x = (x_3, x_2, x_1, x_0)$$

$$\textbf{Output:} \quad (y_4, y_3, y_2, y_1, y_0) = \begin{cases} (P_e(x), x_0, x_3, x_2, x_1) & \text{, if } P_e(x) = 1 \\ (P_e(x), x_3, x_2, x_1, x_0) & \text{, otherwise} \end{cases}$$

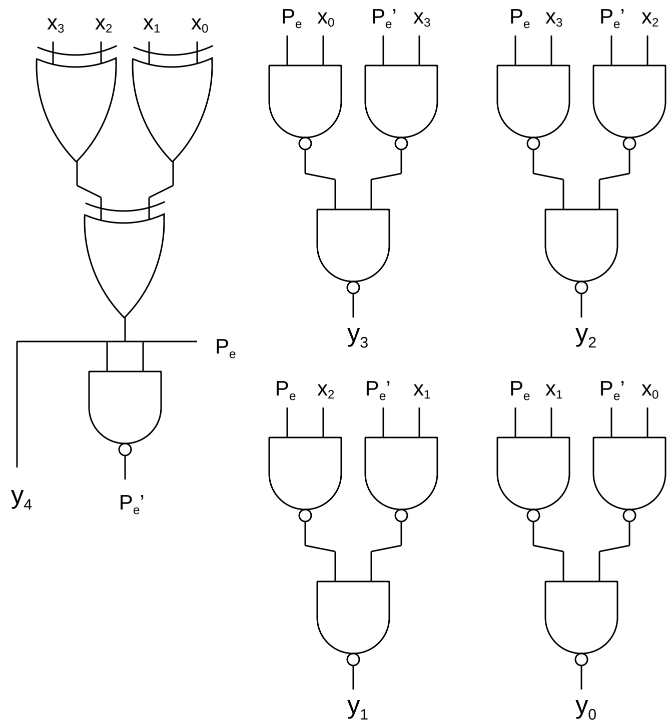
where $P_e(x)$ is an even parity function defined in the box:

> An even parity function $z = P_e(x_{n-1}, x_{n-2}, \ldots, x_1, x_0)$ has a value 1 if the number of 1s in the bit-vector $(x_{n-1}, x_{n-2}, \ldots, x_1, x_0)$ is odd, and 0 otherwise.
> For example, $P_e(x) = 0$ because the input vector $x = (1, 0, 0, 1, 0)$ has 2 ones. For $x = (1, 1, 1, 0, 0)$, $P_e(x) = 1$ since there are 3 ones in x. In other words, $P_e(x)$ is assigned a value such that $(P_e(x), x)$ has an even number of ones.
> The even parity function $P_e(x)$ can be implemented with XOR gates as
>
> $$P_e(x_{n-1}, x_{n-2}, \ldots, x_1, x_0) = x_{n-1} \oplus x_{n-2} \oplus \ldots \oplus x_1 \oplus x_0$$

a) [10 points] Write the switching expression for each y_i (don't worry about minimization.)

$$\begin{aligned} y_4 &= P_e(x) = x_3 \oplus x_2 \oplus x_1 \oplus x_0 \\ y_3 &= P_e(x)x_0 + Pe(x)'x_3 \\ y_2 &= P_e(x)x_3 + Pe(x)'x_2 \\ y_1 &= P_e(x)x_2 + Pe(x)'x_1 \\ y_0 &= P_e(x)x_1 + Pe(x)'x_0 \end{aligned}$$

b) [10 points] Draw the gate network which implements the system. You may only use XOR and NAND gates.


x3 x2 x1 x0
Pe
y4
Pe'
Pe x0 Pe' x3
y3
Pe x3 Pe' x2
y2
Pe x2 Pe' x1
y1
Pe x1 Pe' x0
y0

Problem 6 [20 points]

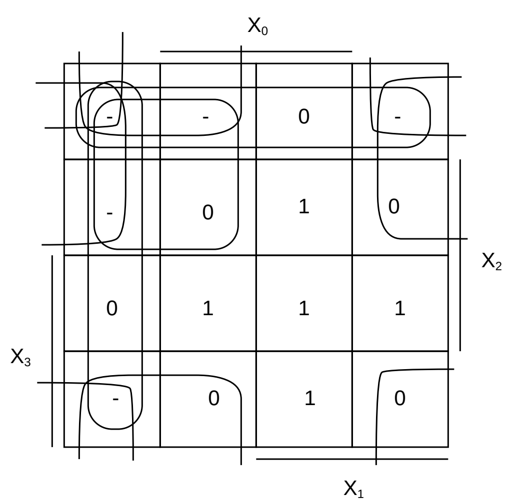
Design a system to detect if an input vector (x_3, x_2, x_1, x_0) has a sequence of 3 consecutive ones, assuming that vector wraps around on the ends.. For example, **111**0, **1**0**11**, **11**0**1**, etc. are examples of the input where a sequence of 3 consecutive ones occur (without and with wrap around).

Assume that at least two of the input bits (x_i) are always 1.

a) [6 points] Describe the system using a truth table.

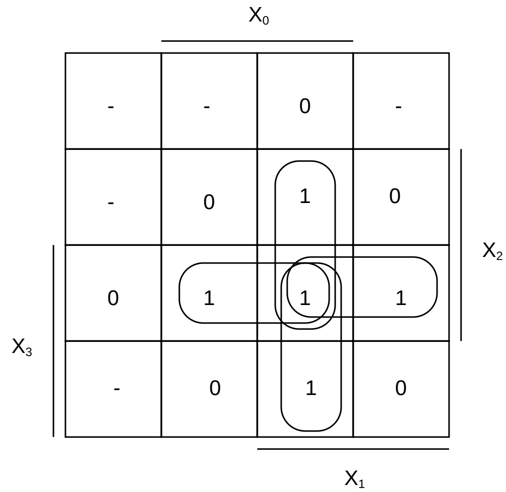
x_3	x_2	x_1	x_0	z
0	0	0	0	-
0	0	0	1	-
0	0	1	0	-
0	0	1	1	0
0	1	0	0	-
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	-
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

b) [5 points] Find the minimal product of sums expression.



$$POS = (x_1 + x_0)(x_3 + x_2)(x_0 + x_2)(x_3 + x_1)(x_3 + x_0)(x_2 + x_1)$$

c) [5 points] Find the minimal sum of products expression.



$$SOP = x_3x_2x_1 + x_3x_2x_0 + x_3x_1x_0 + x_2x_1x_0$$

d) [4 points] Consider a minimal two level NAND-NAND network. Which minimal expression does it correspond to? Draw the network.