[CS M51A FALL 18] MIDTERM EXAM

Date: 10/30/18

- The midterm is closed book, and up to 4 sheets (= 8 pages) of summary notes are allowed. You can use a calculator but not smart phones.
- For multiple choice questions, wrong answers may have a negative score value so choose carefully. It is possible that questions can have multiple answers.
- Please show all your work and write legibly, otherwise no partial credit will be given.
- **This should strictly be your own work; any form of collaboration will be penalized.**

Name : [redacted]

Student ID : [redacted]

Problem	Points	Score
1	10	10
2	15	15
3	15	15
4	15	7
5	20	16
6	10	10
7	~~15~~ 9	5
Total	~~100~~ 94	78

Problem 1 (10 points)

1. **(4 points)** How many bits are required to encode a color spectrum capable of supporting 16 million colors using:

   a. Decimal digits in BCD

   10 colors for every 4 bits.

   $8 \cdot 4 = \boxed{32 \text{ bits}}$

   $10^n = 16\text{mil}$

   $n = \log 16\text{mil} = 7.2 \to 8.$

   b. Hexadecimal representation

   $16^n = 16{,}000{,}000$

   $\boxed{6 \text{ bits}}$

   $n = \log_{16} 16m = 5.95 \to 6$

   Which representation is more efficient? Why?

   Hexadecimal is more efficient because it requires 26 less bits.

2. **(6 points)** Fill in the missing entries in the table.

Radix	Digit vector $\underline{x}$	Value x in decimal
16	(5, 1, 7)	1303.
8	(5, 1, 7)	335
7	(4, 5, 3, 2)	1640

10

$5 \cdot 16^2 + 1 \cdot 16^1 + 7 \cdot 16^0$

$5 \cdot 8^2 + 1 \cdot 8^1 + 7 \cdot 8^0$

7 | 1640 2
7 | 234 3
7 | 33 5
| 4

$\to 4532$

check

$4 \cdot 7^3 + 5 \cdot 7^2 + 3 \cdot 7 + 2.$

Problem 2 (15 points)

$a+b=b+a$	$ab=ba$	Commutativity
$a+(bc)=(a+b)(a+c)$	$a(b+c)=(ab)+(ac)$	Distributivity
$a+(b+c)=(a+b)+c=a+b+c$	$a(bc)=(ab)c=abc$	Associativity
$a+a=a$	$aa=a$	Idempotency
$a+a'=1$	$aa'=0$	Complement
$1+a=1$	$0a=0$	
$0+a=a$	$1a=a$	Identity
$(a')'=a$		Involution
$a+ab=a$	$a(a+b)=a$	Absorption
$a+a'b=a+b$	$a(a'+b)=ab$	Simplification
$(a+b)'=a'b'$	$(ab)'=a'+b'$	DeMorgan's law

Given $E(a,b,c,d)=(ab+c)'(ac+(b'+c'+a'cd)')+a((b+c)(b+d)+c)'$, which of the following represents the same function as $E(a,b,c,d)$? Show all your work.

1. $a+b+c+d'$
2. $a'+b+c$
3. $b+c'+d$
4. $a'b'c'd$
5. $\boxed{ab'c'}$
6. $b'cd'$

$E(a,b,c,d) = \underbrace{(ab+c)'(ac+(b'+c'+a'cd)')}_{1} + \underbrace{a((b+c)(b+d)+c)'}_{2}$

1

$(ab+c)'(ac+(b'+c'+a'cd)')$

$((a'+b')c')(ac+(bc(a+c'+d')))$ demorgan.

$(a'c'+b'c')(ac+abc+bcc'+bcd')$ distributivity & complement.

$a'c'ac + a'c'abc + a'c'bcd' + b'c'ac + b'c'abc + b'c'bcd'$ distributivity

$= 0$ complement simplification.

2

$a((b+c)(b+d)+c)'$

$a((b'c'+b'd')c')$ demorgans

$a(b'c'(c'+d'))$ distributivity

$a(b'(c'c'+c'd'))$ distributivity

$a(b'(c'+c'd'))$ Idempotency.

$ab'c' + ab'c'd'$ distributivity.

$ab'c'(1+d')$ associativity

$= ab'c'$

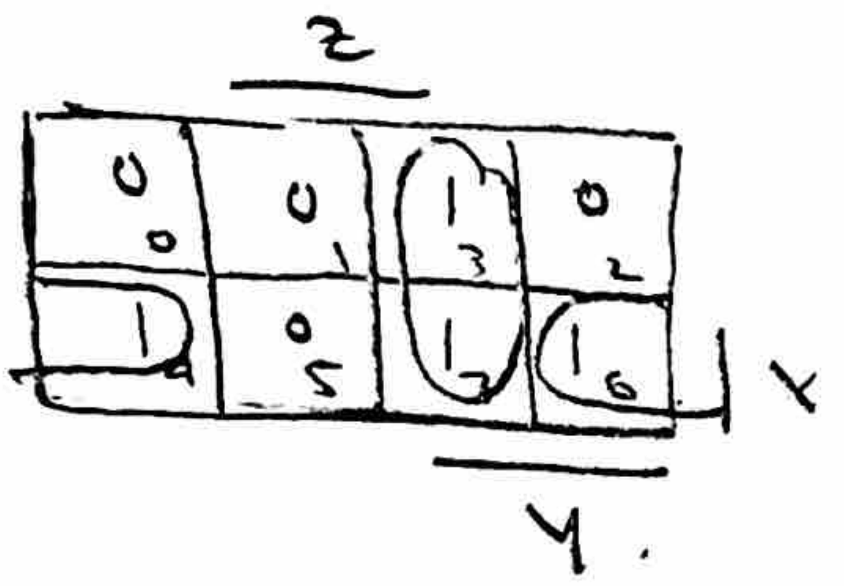
$\boxed{E(a,b,c,d) = ab'c'}$

15

Problem 3 (15 points)

Show if the gate G, described by $G(x, y, z) = one - set\{3, 4, 6, 7\}$, can implement NOT and AND gates. Assume that 0 and 1 are available. If it can, then use G gates to implement the following expression and show the corresponding network of G gates

$$E(a, b, c) = (a + b')(b + c')$$



$G(x,y,z) = yz + xz'$

NOT

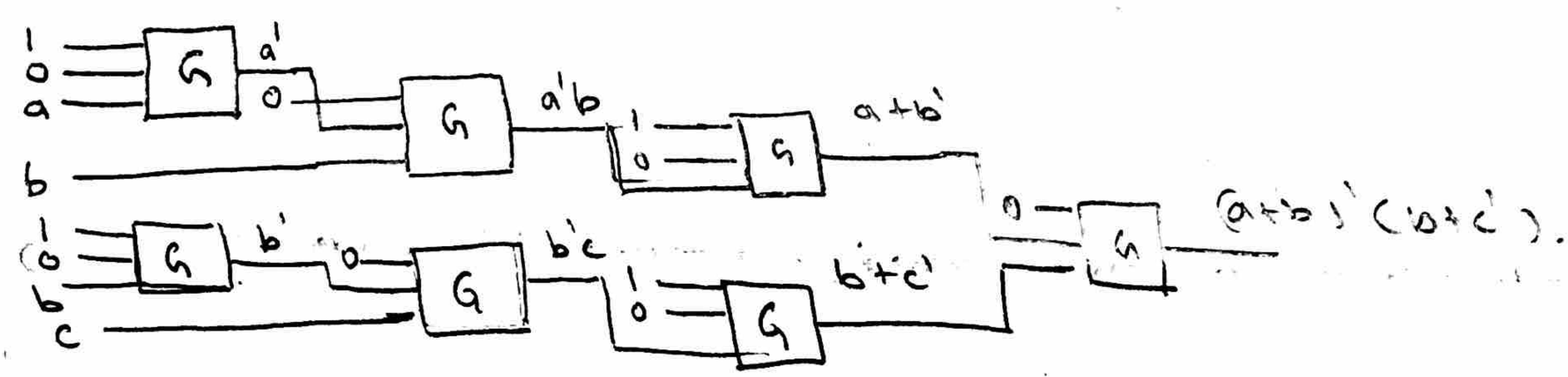
$G(1,0,a) = 0 + 1a' = a'$

AND

$G(0,a,b) = ab + 0 = ab.$

$E(a,b,c) = ((a+b')')'((b+c')')'$

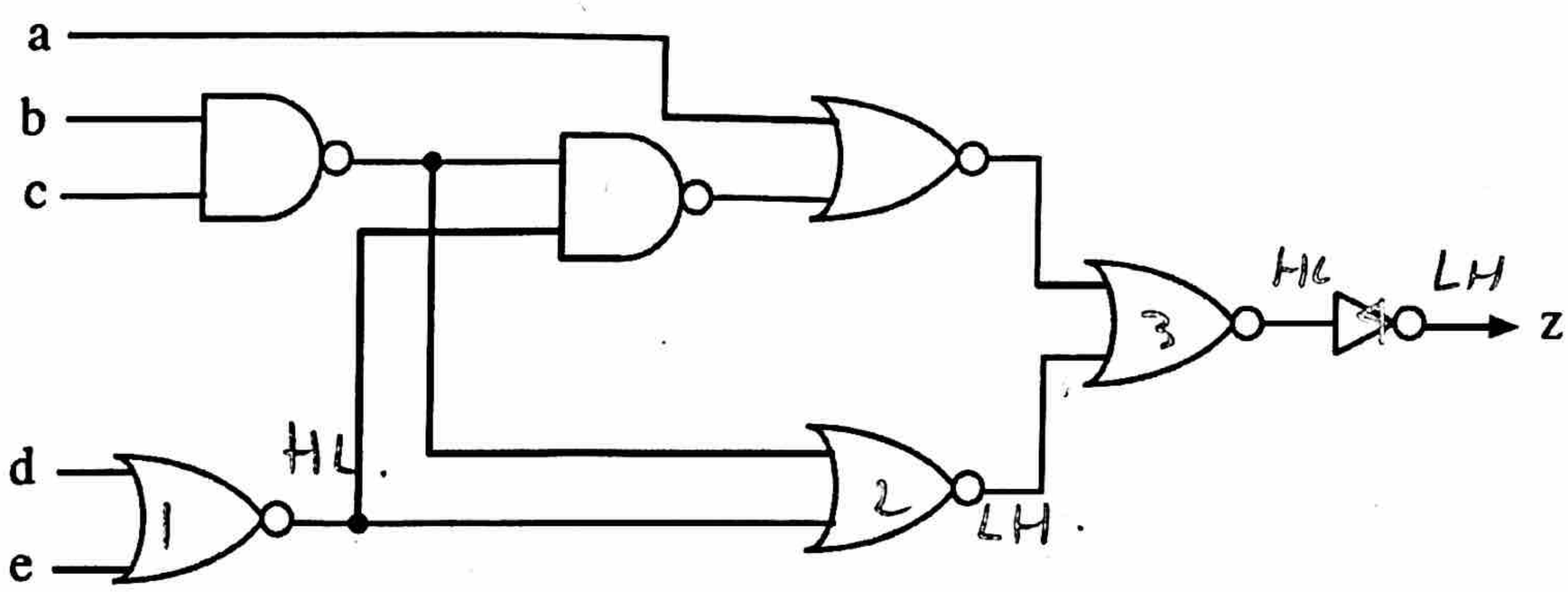
$= (a'b)'(b'c)'$



15

Problem 4 (15 points)

With the help of the table below, determine the low to high propagation delay $t_{pLH}(d, z)$ of the output z of the network shown below. Assume the network output has a load of 6.



Gate Type	Fan-in	Propagation Delays (ns) t_{pLH}	t_{pHL}	Load Factor
NOT	1	$0.02 + 0.038L$	$0.05 + 0.017L$	1.0
NAND	2	$0.05 + 0.038L$	$0.08 + 0.027L$	1.0
NOR	2	$0.06 + 0.075L$	$0.07 + 0.016L$	1.0

GATE: G1: NOR2 → G2: NOR2 → G3: NOR2 → G4: NOT1.

LH/HL. G1: HL → G2: LH → G3: HL → G4: LH.

LOAD. G1: 2 → G2: 1 → G3: 1 → G4: 6.

$$t_{pLH}(d,z) = \underbrace{0.07 + 0.016(2)}_{\text{NOR2, HL, L=2}} + \underbrace{0.06 + 0.075(1)}_{\text{NOR2, LH, L=1}} + \underbrace{0.07 + 0.016(1)}_{\text{NOR2, HL, L=1}} + \underbrace{0.02 + 0.038(6)}_{\text{NOT1, LH, L=6}}$$

$$= \boxed{0.571\text{ ns}}$$

wrong critical path

7

Problem 5 (20 points)

Obtain a two-level gate network of the following system.

Inputs:	$x, y \in \{0, 1, 2, 3\}$
Outputs:	$z \in \{0, 1, 2, 3\}$
Function:	$z = \{3xy + 1\}$ mod 4

1. **(2 points)** Complete the switching table using binary encoding for all values.

0	0	1
0	1	1
0	2	1
0	3	1
1	0	1
1	1	0
1	2	3
1	3	2
2	0	1
2	1	3
2	2	1
2	3	3
3	0	1
3	1	2
3	2	3
3	3	0

	x_1	x_0	y_1	y_0	z_1	z_0
0	0	0	0	0	0	1
1	0	0	0	1	0	1
2	0	0	1	0	0	1
3	0	0	1	1	0	1
4	0	1	0	0	0	1
5	0	1	0	1	0	0
6	0	1	1	0	1	1
7	0	1	1	1	1	0
8	1	0	0	0	0	1
9	1	0	0	1	1	1
10	1	0	1	0	0	1
11	1	0	1	1	1	1
12	1	1	0	0	0	1
13	1	1	0	1	1	0
14	1	1	1	0	1	1
15	1	1	1	1	0	0

2. **(5 points)** Show the switching expressions of z_1 and z_0 in sum of minterms form.

$z_1 = \sum m(6, 7, 9, 11, 13, 14)$

$z_0 = \sum m(0, 1, 2, 3, 4, 6, 8, 9, 10, \cancel{11}, 12, 14)$

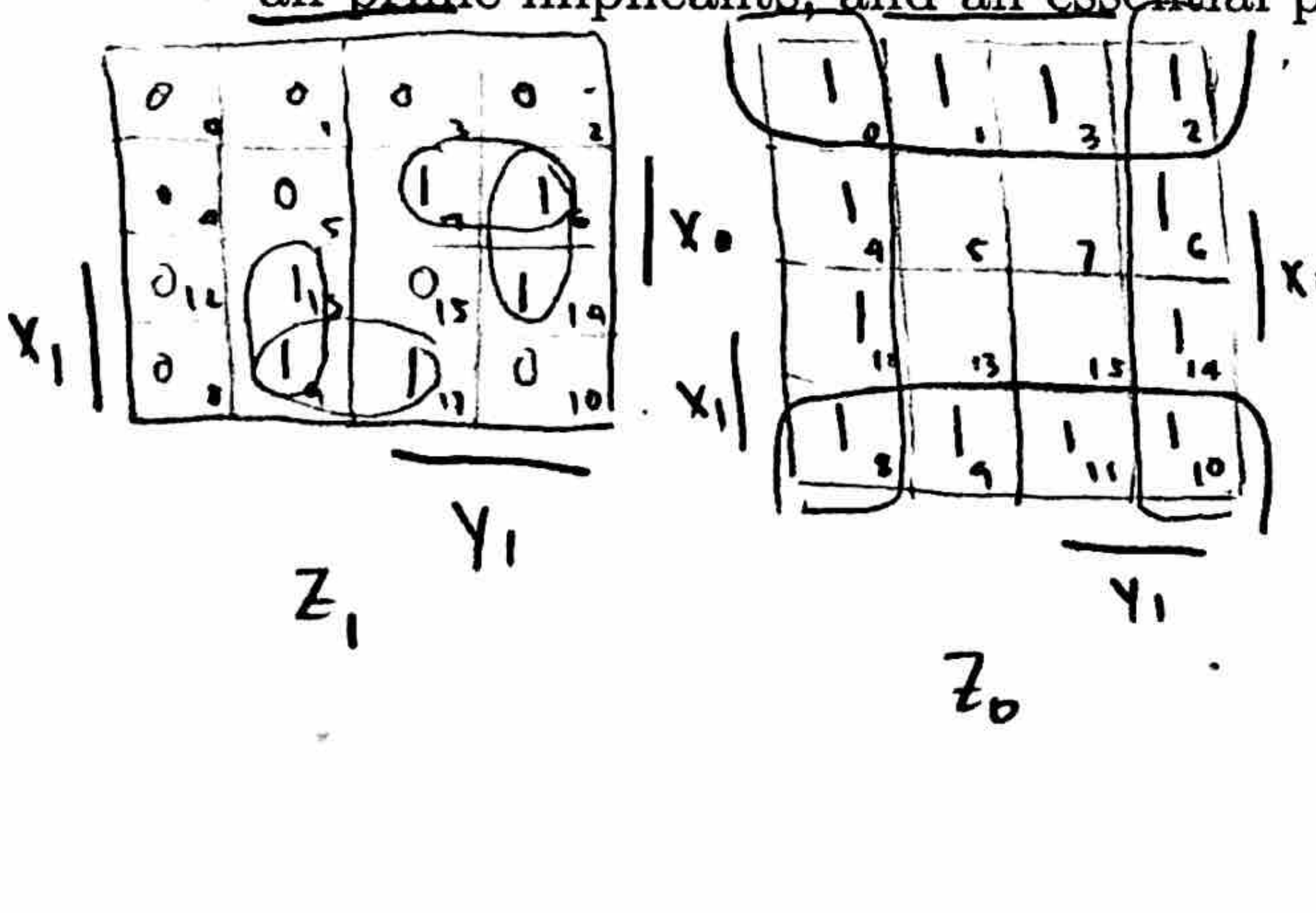
6

$z_1 = x_1' x_0 y_1 y_0' + x_1' x_0 y_1 y_0 + x_1 x_0' y_1' y_0 + x_1 x_0' y_1 y_0 + x_1 x_0 y_1' y_0 + x_1 x_0 y_1 y_0'$

$z_0 = x_1' x_0' y_1' y_0' + x_1' x_0' y_1' y_0 + x_1' x_0' y_1 y_0' + x_1' x_0' y_1 y_0 + x_1' x_0 y_1' y_0' + x_1' x_0 y_1 y_0' + x_1 x_0' y_1' y_0' +$
$x_1 x_0' y_1' y_0 + x_1 x_0' y_1 y_0' + \cancel{x_1 x_0' y_1 y_0} + x_1 x_0 y_1' y_0' + x_1 x_0 y_1 y_0'$.

3. **(8 points)** Show the minimal sum of products expressions of z_1 and z_0. In each case, show a K-map, indicate all prime implicants, and all essential prime implicants. Show NAND-NAND networks.

y_0



$z_1 \rightarrow$ PIs: $x_1 y_1' y_0$, $x_1 x_0' y_0$, $y_1 y_0' x_0$, $y_1 x_1' x_0$

all are essential!

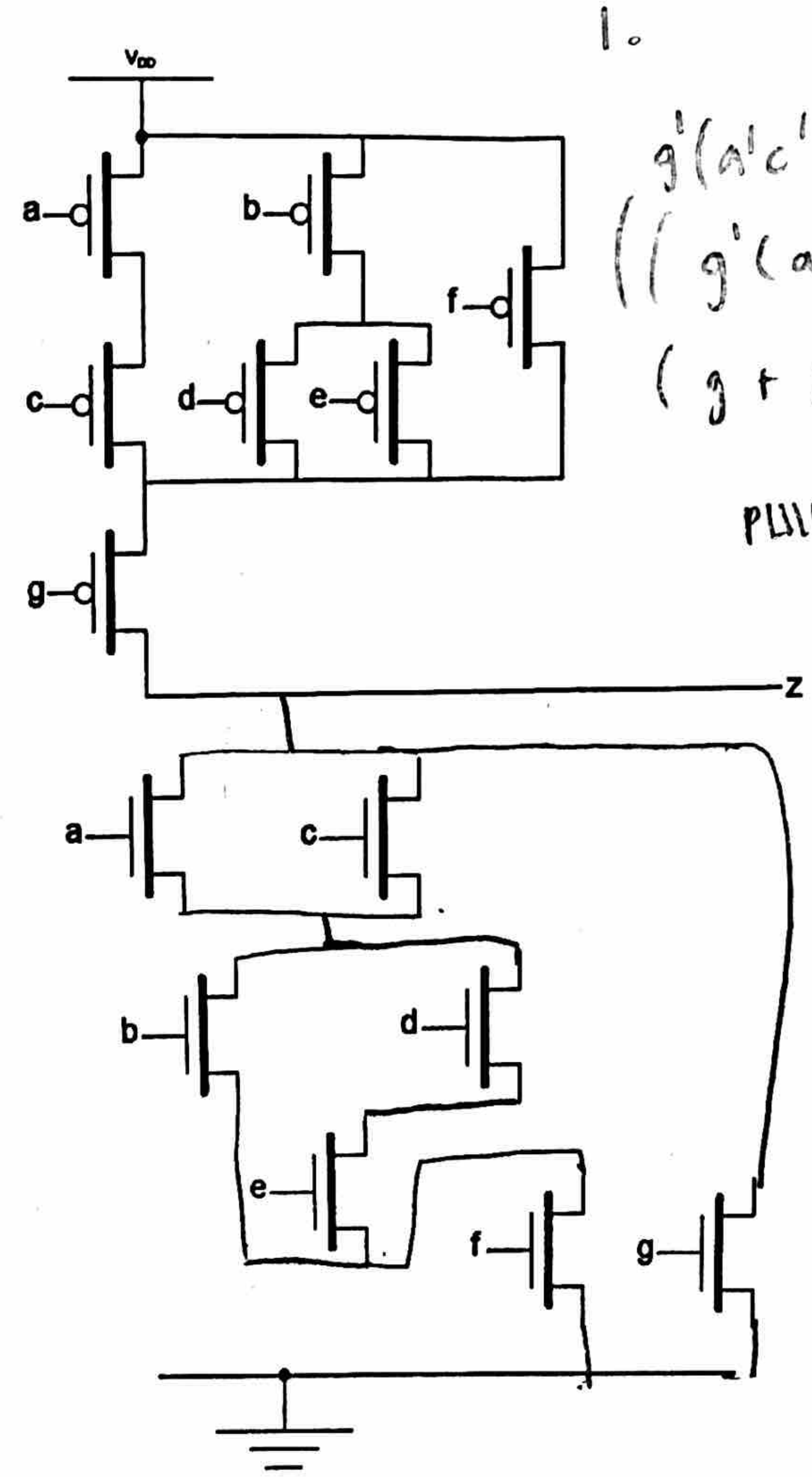
$z_0 \rightarrow$ PIs: $x_0' + y_0'$

$z_1 =$ NAND(NAND(x_1, y_1', y_0), NAND(x_1, x_0', y_0), NAND(y_1, y_0', x_0), NAND(y_1, x_1', x_0))

$z_0 =$ NAND(x, y)

Problem 6 (10 points)

We are given the following partial CMOS network.



1.

$g'(a'c' + b'(d'+e') + f')$

$((g'(a'c'+b'(d'+e')+f')))')'$

$(g + ((a+c)(b+(de))f))'$

PULL DOWN

$\boxed{z' = g + ((a+c)(b+de)f}$

1. **(5 points)**
   Write the expression for the pull-up network. From this, derive the expression for the pull-down network using switching algebra.

2. **(5 points)** Connect NMOS transistors to complete the circuit according to the pull-down expression. Please only add missing wires.

Problem 7 (15 points)

For $f(x_3, x_2, x_1, x_0) = (x_3 + x_2' + x_1 + x_0)(x_3 + x_2' + x_1' + x_0)(x_3' + x_2 + x_1 + x_0')(x_3' + x_2' + x_1 + x_0)(x_3' + x_2' + x_1 + x_0')(x_3' + x_2' + x_1' + x_0)$

1. **(2 points)** Fill out the following K-map.

$f(x_3,x_2,x_1,x_0) = \prod M(4,6,9,12,13,14)$

		X_0	X_0		
	1	1	1	1	
	0	1	1	0	X_2
X_3	0	0	1	0	X_2
X_3	1	0	1	1	
			X_1	X_1	

big boys

$x_3'x_0$, x_1x_0, $x_3'x_2'$, $x_2'x_1$

small boys.

$x_3x_2'x_1$, $x_3x_2'x_0'$, $x_3x_1x_0$, $x_3'x_2x_0$, $x_3'x_1x_0$, $x_3'x_1'x_0$, $x_3'x_2'x_0$, $x_3'x_2'x_0'$, $x_3'x_2'x_1$

2. **(4 points)** Which of the given expressions are prime implicants of the function given above? Circle **all** that apply. Write down any prime implicants that are missing.

(a) $\cancel{x_1}$ (b) $\cancel{x_3x_1}$ (c) $x_3'x_2'$

(d) $\cancel{x_3'x_1}$ (e) $x_3'x_0$ (f) $\cancel{x_2x_1}$

(g) $\cancel{x_2'x_0}$ (h) x_1x_0 (i) $\cancel{x_1x_0'}$

(j) $x_3'x_2'x_1$ (k) $x_2x_1x_0$ (l) $\cancel{x_3x_2x_1x_0}$

$x_2'x_1'x_0'$ $x_3'x_2x_0$

$x_3'x_2'x_1'$ $x_3'x_1x_0$

$x_3'x_2'x_0'$ $x_2'x_1$

$x_3'x_1'x_0$ $x_2'x_1x_0$

$x_3'x_2'x_0$ $x_2'x_1x_0'$

$x_3x_1x_0$

$x_3x_2'x_1$

$x_3x_2'x_0'$

-2

3. **(2 points)** Write down the complete set of **essential** prime implicants.

$x_3'x_0$, x_1x_0, $x_3'x_2'$, $x_3x_2'x_0'$

-1

4. **(1 point)** Write the minimal sum of products expression for f. Is it unique?

$f = x_3'x_0 + x_1x_0 + x_3'x_2' + x_3x_2'x_0'$

-1