# [CS M51A Fall 18] Midterm exam

Date: 10/30/18

- The midterm is closed book, and up to 4 sheets (= 8 pages) of summary notes are allowed. You can use a calculator but not smart phones.
- For multiple choice questions, wrong answers may have a negative score value so choose carefully. It is possible that questions can have multiple answers.
- Please show all your work and write legibly, otherwise no partial credit will be given.
- **This should strictly be your own work; any form of collaboration will be penalized.**

Na[redacted] ________________

Student [redacted] ________________

| Problem | Points | Score |
|---|---|---|
| 1 | 10 | 7 |
| 2 | 15 | 15 |
| 3 | 15 | 7 |
| 4 | 15 | 15 |
| 5 | 20 | 16 |
| 6 | 10 | 7 |
| 7 | ~~15~~ 9 | 9 |
| Total | ~~100~~ 94 | 76 |

## Problem 1 (10 points)

1. **(4 points)** How many bits are required to encode a color spectrum capable of supporting 16 million colors using:

   **a.** Decimal digits in BCD

   24 bits

   **b.** Hexadecimal representation

   6 hexadecimal digits → 24 bits

   Which representation is more efficient? Why?

   Neither representation is more efficient because loosing 7 possibilities per digit (16 ~~8~~ possibilities) is relatively insignificant compared to the overall 16 million possibilities

2. **(6 points)** Fill in the missing entries in the table.

| Radix | Digit vector $\underline{x}$ | Value x in decimal |
|---|---|---|
| 16 | (5, 1, 7) | 1303 |
| 8 | (5, 1, 7) | 335 |
| 7 | (4, 5, 3, 2) | 1640 |

## Problem 2 (15 points)

| | | |
|---|---|---|
| $a+b=b+a$ | $ab=ba$ | Commutativity |
| $a+(bc)=(a+b)(a+c)$ | $a(b+c)=(ab)+(ac)$ | Distributivity |
| $a+(b+c)=(a+b)+c=a+b+c$ | $a(bc)=(ab)c=abc$ | Associativity |
| $a+a=a$ | $aa=a$ | Idempotency |
| $a+a'=1$ | $aa'=0$ | Complement |
| $1+a=1$ | $0a=0$ | |
| $0+a=a$ | $1a=a$ | Identity |
| $(a')'=a$ | | Involution |
| $a+ab=a$ | $a(a+b)=a$ | Absorption |
| $a+a'b=a+b$ | $a(a'+b)=ab$ | Simplification |
| $(a+b)'=a'b'$ | $(ab)'=a'+b'$ | DeMorgan's law |

Given $E(a,b,c,d) = (ab+c)'(ac+(b'+c'+a'cd)') + a((b+c)(b+d)+c)'$, which of the following represents the same function as $E(a,b,c,d)$? Show all your work.

1. $a+b+c+d'$
2. $a'+b+c$
3. $b+c'+d$
4. $a'b'c'd$
5. $ab'c'$
6. $b'cd'$

$= ((a'+b')c')(ac+(bc(a+c'+d')) + a((b'c'+b'd')c')$ Apply DeMorgan Law

$(cc'=0)$

$(a'c'+b'c')(ac+abc+bcd') + ac'b' + ab'c'd'$ Distributivity

All terms cancel out due to $aa=0$

$ab'c'\cancel{(1+d')}$ identity

I showed work on back page

## Problem 3 (15 points)

Show if the gate G, described by $G(x, y, z) = one - set\{3, 4, 6, 7\}$, can implement NOT and AND gates. Assume that 0 and 1 are available. If it can, then use G gates to implement the following expression and show the corresponding network of G gates

$$E(a, b, c) = (a + b')(b + c')$$

$G = \Sigma m(0,1,2,5)$ wrong expression

$G(x,y,z) = x'y'z' + x'y'z + x'yz' + xy'z$

$G(1,y,z) = y'z$

$G(x,1,z) = x'z'$

$G(x,y,1) = x'y' + xy'$

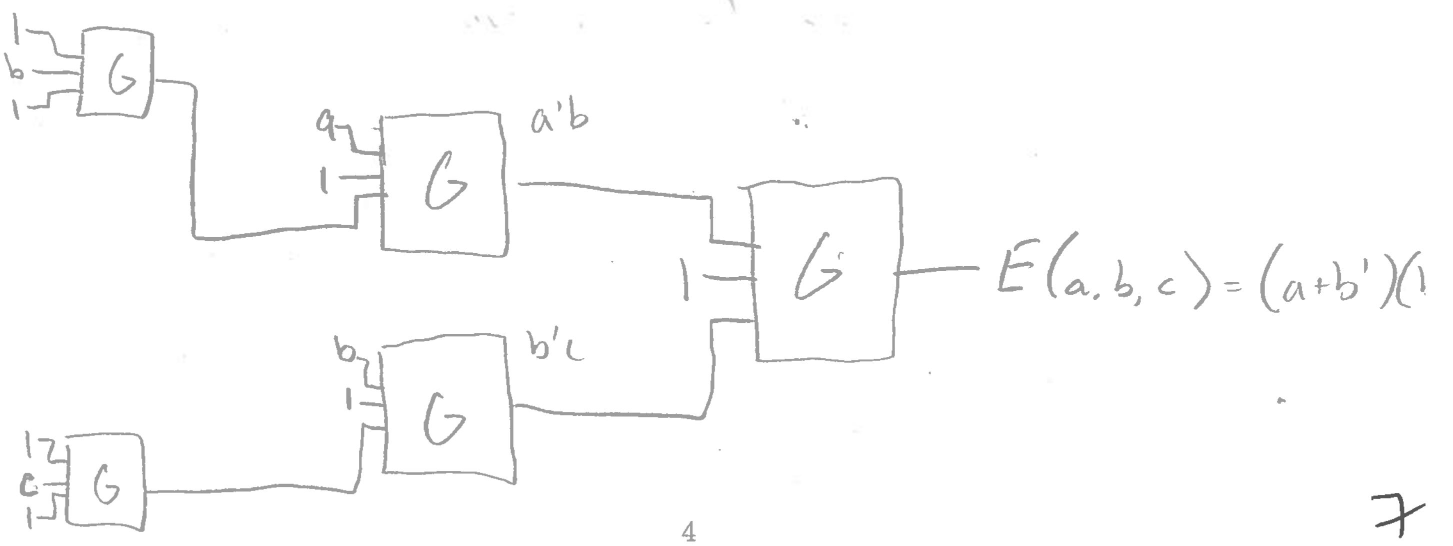
$G(x,y,0) = x'y' + x'y$

Not: $G(1,y,1) = y'$

AND: $G(G(1,x,1), 1, G(1,z,1))$

$= G(x', 1, z') = xz$

$E(a,b,c) = G(G(a,1,G(1,b,1)),1,G(b,1,G(1,c,1)))$



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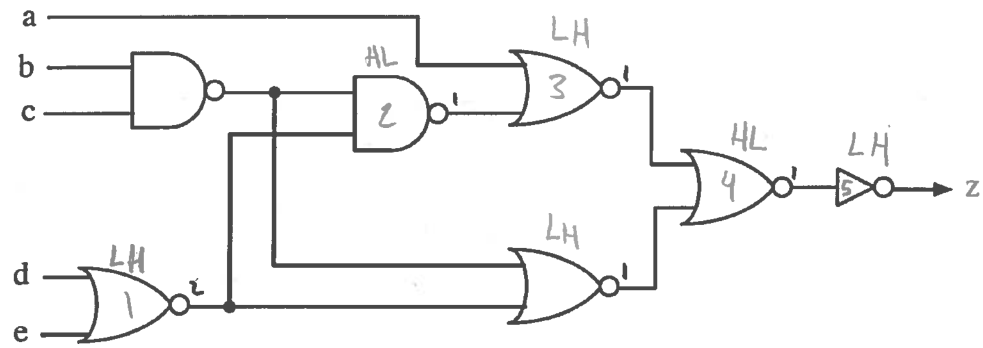
## Problem 4 (15 points)

With the help of the table below, determine the low to high propagation delay $t_{pLH}(d, z)$ of the output $z$ of the network shown below. Assume the network output has a load of 6.




| Gate Type | Fan-in | Propagation Delays (ns) $t_{pLH}$ | Propagation Delays (ns) $t_{pHL}$ | Load Factor |
|---|---|---|---|---|
| NOT | 1 | $0.02 + 0.038L$ | $0.05 + 0.017L$ | 1.0 |
| NAND | 2 | $0.05 + 0.038L$ | $0.08 + 0.027L$ | 1.0 |
| NOR | 2 | $0.06 + 0.075L$ | $0.07 + 0.016L$ | 1.0 |

| | | L | $t_p$ | |
|---|---|---|---|---|
| 1 | LH | 2 | 0.06 + 0.075(2) | 0.210 |
| 2 | HL | 1 | 0.08 + 0.027(1) | 0.107 |
| 3 | LH | 1 | 0.06 + 0.075(1) | 0.135 |
| 4 | HL | 1 | 0.07 + 0.016(1) | 0.086 |
| 5 | LH | 6 | 0.02 + 0.038(6) | 0.248 |

$\boxed{t_{pLH}(d,z) = 0.786 \text{ ns}}$

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## Problem 5 (20 points)

Obtain a two-level gate network of the following system.

| | |
|---|---|
| Inputs: | $x, y \in \{0,1,2,3\}$ |
| Outputs: | $z \in \{0,1,2,3\}$ |
| Function: | $z = \{3xy+1\} \bmod 4$ |

1. **(2 points)** Complete the switching table using binary encoding for all values.

| 3xy | x | y | $x_1$ | $x_0$ | $y_1$ | $y_0$ | $z_1$ | $z_0$ | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| | | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 |
| | | 2 | 0 | 0 | 1 | 0 | 0 | 1 | 2 |
| | | 3 | 0 | 0 | 1 | 1 | 0 | 1 | 3 |
| 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 4 |
| 3 | | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 5 |
| 6 | | 2 | 0 | 1 | 1 | 0 | 1 | 1 | 6 |
| 9 | | 3 | 0 | 1 | 1 | 1 | 1 | 0 | 7 |
| 0 | 2 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 8 |
| 6 | | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 9 |
| 12 | | 2 | 1 | 0 | 1 | 0 | 0 | 1 | 10 |
| 18 | | 3 | 1 | 0 | 1 | 1 | 1 | 1 | 11 |
| 0 | 3 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 12 |
| 9 | | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 13 |
| 18 | | 2 | 1 | 1 | 1 | 0 | 1 | 1 | 14 |
| 24 | | 3 | 1 | 1 | 1 | 1 | 0 | 1 | 15 |

2. **(5 points)** Show the switching expressions of $z_1$ and $z_0$ in sum of minterms form.

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$Z_1 = x_1' x_0 y_1 y_0' + x_1' x_0 y_1 y_0 + x_1 x_0' y_1' y_0 + x_1 x_0' y_1 y_0 + x_1 x_0 y_1' y_0 + x_1 x_0 y_1 y_0'$

$Z_1 = \Sigma m(6,7,9,11,13,14)$

$Z_0 = \Sigma m(0,1,2,3,4,6,8,9,10,11,12,14,15)$

3. **(8 points)** Show the minimal sum of products expressions of $z_1$ and $z_0$. In each case, show a K-map, indicate all prime implicants, and all essential prime implicants. Show NAND-NAND networks.

$Z_1$

| | | $y_0$ | $y_0$ / $y_1$ | $y_1$ |
|---|---|---|---|---|
| | 0 | 0 | 0 | 0 |
| $x_0$ | 0 | 0 | 1 | 1 |
| $x_0$, $x_1$ | 0 | 1 | 0 | 1 |
| $x_1$ | 0 | 1 | 1 | 0 |

$Z_0$

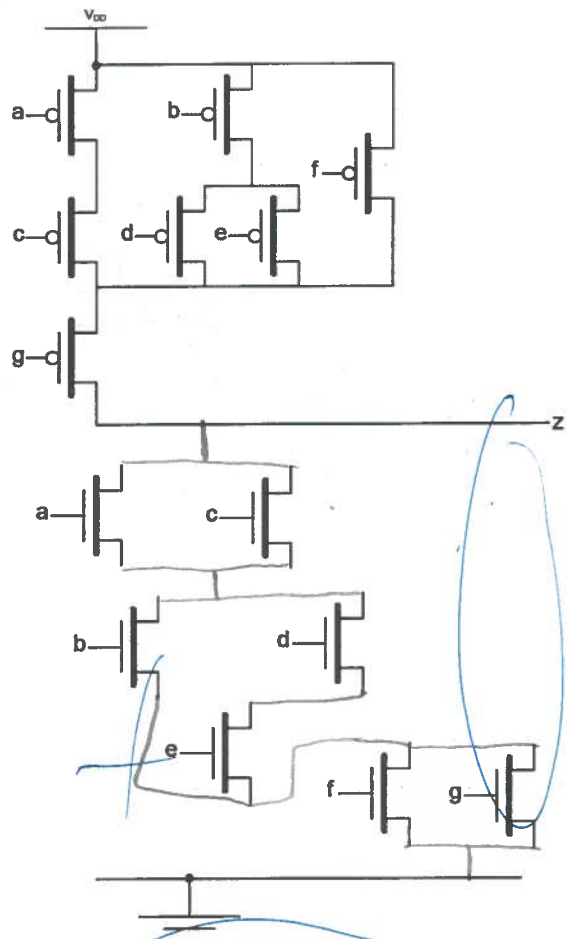
| | | $y_0$ | $y_0$ / $y_1$ | $y_1$ |
|---|---|---|---|---|
| | 1 | 1 | 1 | 1 |
| $x_0$ | 1 | 0 | 0 | 1 |
| $x_0$, $x_1$ | 1 | 0 | 1 | 1 |
| $x_1$ | 1 | 1 | 1 | 1 |

$Z_1 = x_0 y_1 y_0' + x_1' x_0 y_1 + x_1 y_1' y_0 + x_1 x_0' y_0$

$Z_0 = y_0' + x_0' + x_1 y_1$

## Problem 6 (10 points)

We are given the following partial CMOS network.




1. **(5 points)** $Z = ((a+c)(b+de)f) + g$

   Write the expression for the pull-up network. From this, derive the expression for the pull-down network using switching algebra. $Z = (a'c' + b'(d'+e) + f')g' = a'c'g' + b'd'g' + b'e'g' + f'g'$

2. **(5 points)** Connect NMOS transistors to complete the circuit according to the pull-down expression. Please only add missing wires.

**Problem 7 (15 points)**

4 6 9 12 13

For $f(x_3, x_2, x_1, x_0) = (x_3 + x_2' + x_1 + x_0)(x_3 + x_2' + x_1' + x_0)(x_3' + x_2 + x_1 + x_0')(x_3' + x_2' + x_1 + x_0)(x_3' + x_2' + x_1 + x_0')(x_3' + x_2' + x_1' + x_0)$

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1. **(2 points)** Fill out the following K-map.

X0

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 |
| 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 |

X2

X3

X1

2. **(4 points)** Which of the given expressions are prime implicants of the function given above? Circle **all** that apply. Write down any prime implicants that are missing.

~~(a) $x_1$~~ ~~(d) $x_3'x_1$~~ (g) $x_2'x_0$ ~~(j) $x_3'x_2'x_1$~~

~~(b) $x_3x_1$~~ (e) $x_3'x_0$ (h) $x_1x_0$ ~~(k) $x_2x_1x_0$~~

(c) $x_3'x_2'$ (f) $x_2x_1$ ~~(i) $x_1x_0'$~~ (l) $x_3x_2x_1x_0$

$X_2'X_0'$ $X_2'X_1$

3. **(2 points)** Write down the complete set of **essential** prime implicants.

$X_3'X_0$, $X_1X_0$, $X_2'X_0'$

4. **(1 point)** Write the minimal sum of products expression for $f$. Is it unique?

$f = X_3'X_0 + X_1X_0 + X_2'X_0'$ This is unique.

Use of any other implicants would not qualify as the minimal sum of products.

5.3) $Z_1$ prime Implicants

$X_0 Y_1 Y_0'$

$X_1' X_0 Y_1$

$X_1 Y_1' Y_0$

$X_1 X_0' Y_0$

essential prime implicants

$X_0 Y_1 Y_0'$

$X_1' X_0 Y_1$

$X_1 Y_1' Y_0$

$X_1 X_0' Y_0$

$Z_0$ prime Implicants

$Y_0'$

$X_0'$

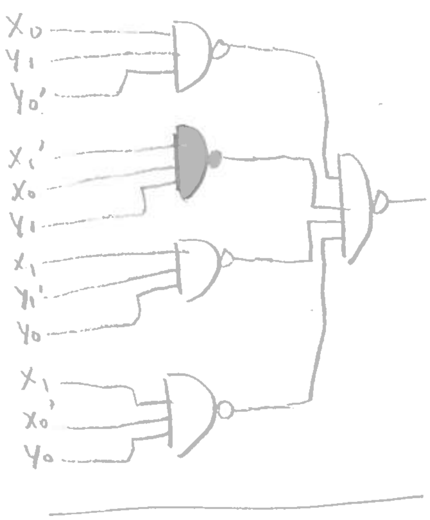
$X_1 Y_1$

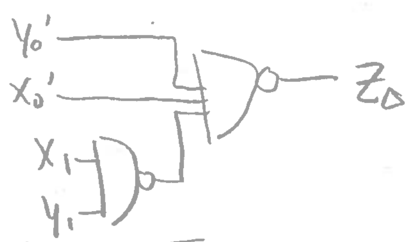
essential prime implicants

$Y_0'$

$X_0'$

$X_1 Y_1$



2)

$(ab+c)'(ac+(b'+c'+a'cd)')+a((b+c)(b+d)+c)'$ Given

$((ab)'\cdot c')(ac+((b'+c')'(a'cd)')+a(((b+c)(b+d))'\cdot c')$ DeMorgan's Law

$((a'+b')\cdot c')(ac+((bc)(a+c'+d'))+a(((b+c)'+(b+d)')\cdot c')$ DeMorgan's Law

$((a'+b')\cdot c')(ac+((bc)(a+c'+d'))+a((b'c')+(b'd'))\cdot c')$ DeMorgan's Law

$(a'c'+b'c')(ac+abc+bcc'+bcd')+a(c'c'b'+c'b'd')$ distributivity

$(a'c'+b'c')(ac+abc+0+bcd')+a(c'b'+c'b'd')$ Complement & idempotency

$(a'c'+b'c')(ac+abc+bcd')+a(c'b'+c'b'd')$ Identity

$a'ac'c+a'abc'c+abc'cd'+ab'cc'+abb'cc'+bb'cc'd'$
$+ac'b'(1+d)$

distributivity

$0+0+0+0+0+0+ac'b'(1)$ Complement

$ac'b'$ identity

$ab'c'$ Commutativity ∎