[CS M51A Fall 15] Midterm exam

Date: 11/3/15

2C

- The midterm is closed books and notes. Tablets and smartphone are not allowed.
- You can use calculators and have up to 2 sheets (= 4 pages) of summary notes.
- Please show all your work and write legibly, otherwise no partial credit will be given.
- **This should strictly be your own work; any form of collaboration will be penalized.**

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Problem	Points	Score
1	15	15
2	15	15
3	10	8
4	15	15
5	10	10
6	15	15
7	20	18
Total	100	96

Problem 1 (15 points)

1. (6 points) Given the following simplification of a boolean expression, identify all right and wrong steps and briefly explain what is wrong for each error.
(For example, (10) → (11) wrong application of the Identity rule, (11) → (12) correct)

$$
\begin{aligned}
E_1(w,x,y,z) &= (((w+x+x'y')y+z)'+wx'+y')' && (1)\\
&= ((w+x+x'y')'y'z'+wx'+y')' && (2)\\
&= ((w+x+x'y')'y'z'+(w+y')(x'+y'))' && (3)\\
&= ((w+x'+y')'y'z'+(w+y')(x'+y'))' && (4)\\
&= (w'xyy'z'+(w+y')(x'+y'))' && (5)\\
&= (0+(w+y')(x'+y'))' && (6)\\
&= wy+xy && (7)
\end{aligned}
$$

(1) → (2) Wrong application of De Morgan's Law (should be $[(w+x+x'y')y]'z'$)

(2) → (3) Correct

(3) → (4) Wrong application of Simplification rule (should be $(w+x+y')$)

(4) → (5) Correct

(5) → (6) Correct

(6) → (7) Wrong application of De Morgan's Law (should be $w'y+xy$)

✓

2. (5 points) Obtain the minimal sum of products form for $E_2(w, x, y, z)$ using the identities of Boolean algebra. Show all the steps in your derivation.

$$E_2 = xy' + xzw + yw$$

$E_2 = xy' + xzw + yw$

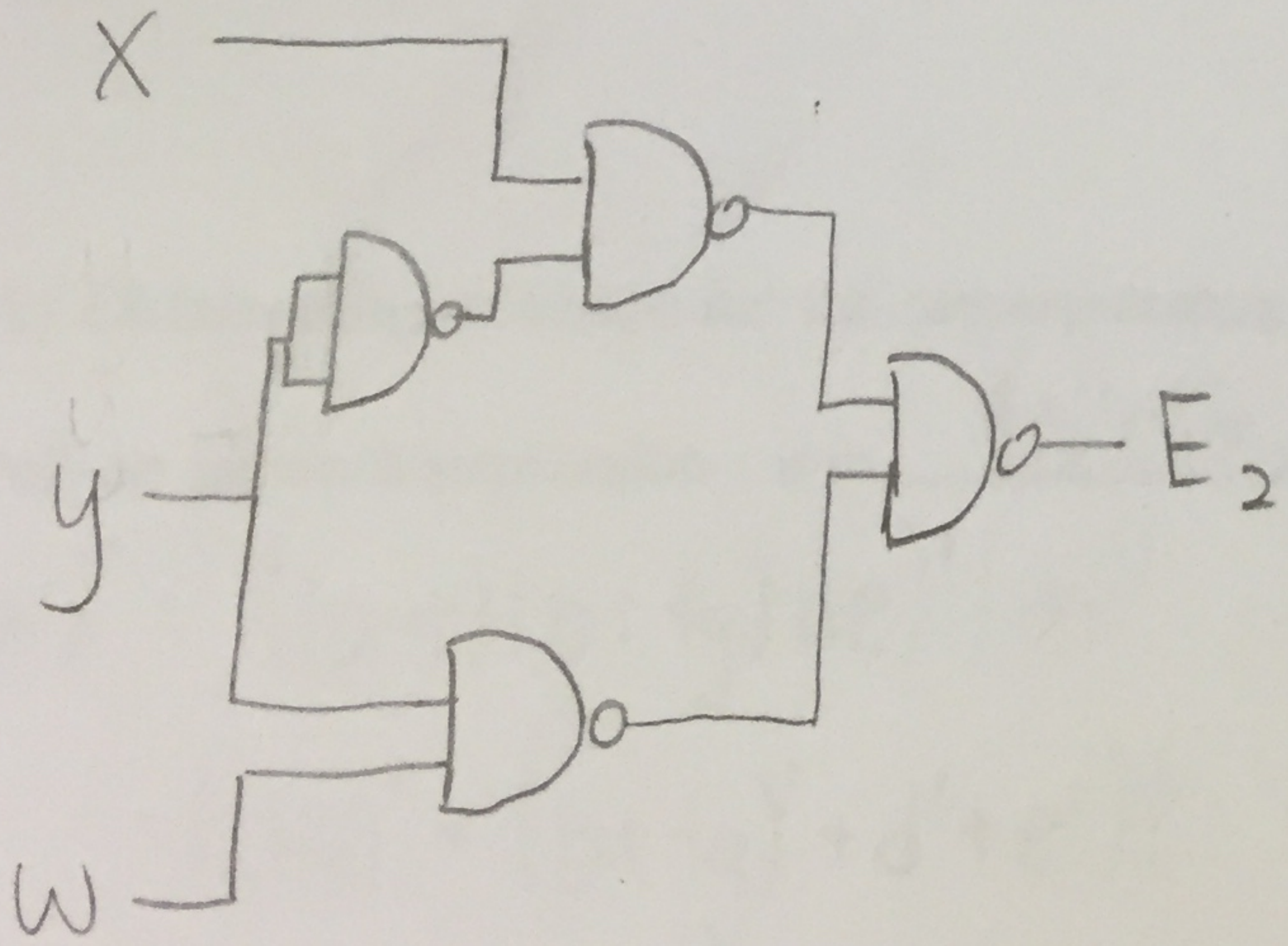
$= xy' + xzwy + xzwy' + yw$

$= xy' + xy'zw + yw + ywxz$

$= xy'(1 + zw) + yw(1 + xz)$

$= xy' + yw$

3. (4 points) Using the expression obtained for E_2 from the previous step, obtain the NAND network that uses ONLY NAND gates. Inverted inputs are not available, and no constant inputs are allowed.



$((x(yy)')'(yw)')'$

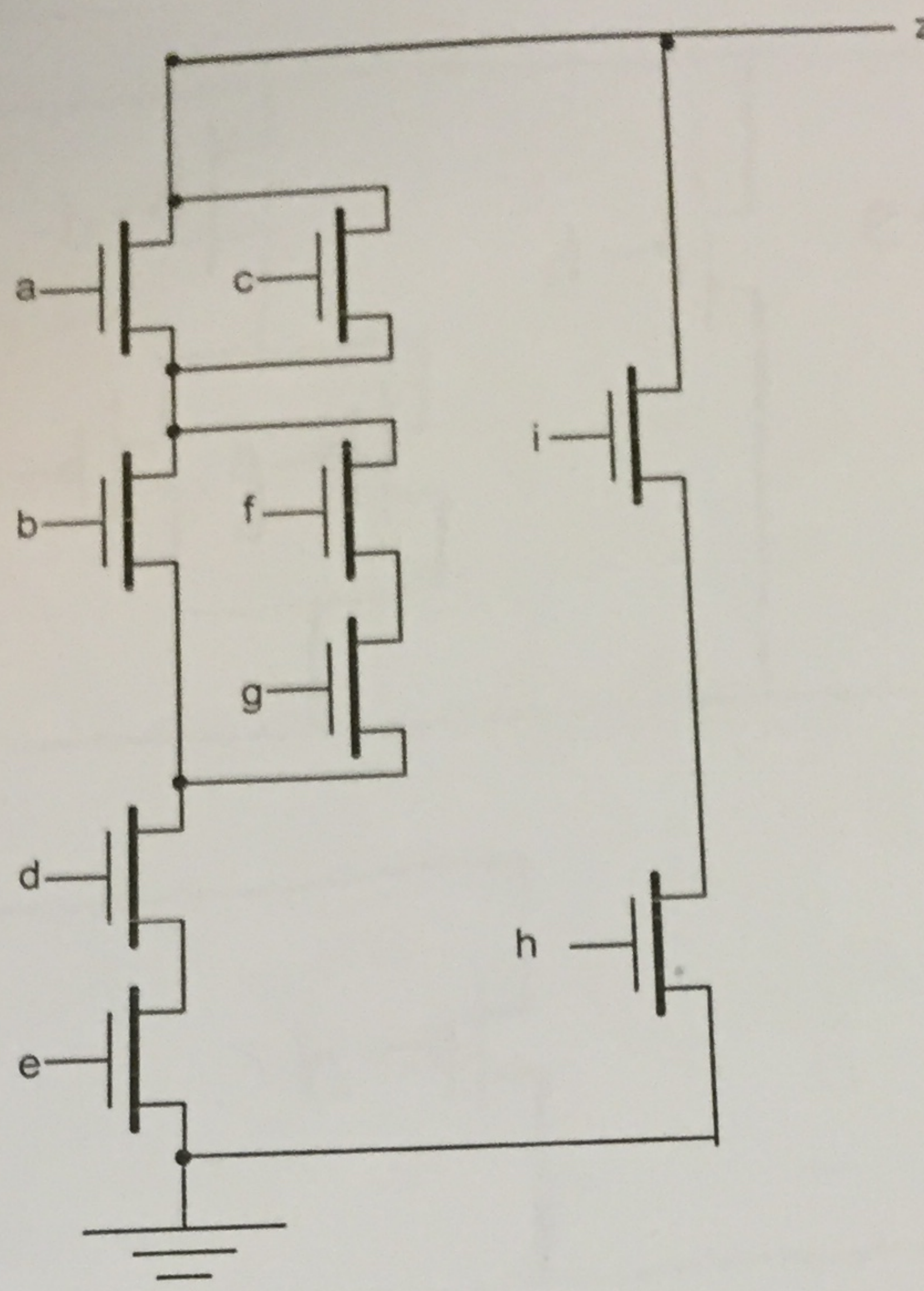
$= (x(yy)') + (yw)$

$= (x(y' + y')) + (yw)$

$= xy' + yw$

Problem 2 (15 points)

The following pull-down network is part of a complex CMOS gate that we want to implement.



1. **(8 points)** (a) Write the expression for the pull-down network.

Pull-down network expression : z' = $((a+c)(b+fg)de) + ih$

✓

(b) Obtain the expression for the corresponding pull-up network.

Pull-up network expression : z = $(a'c' + b'(f'+g') + d' + e')(i'+h')$

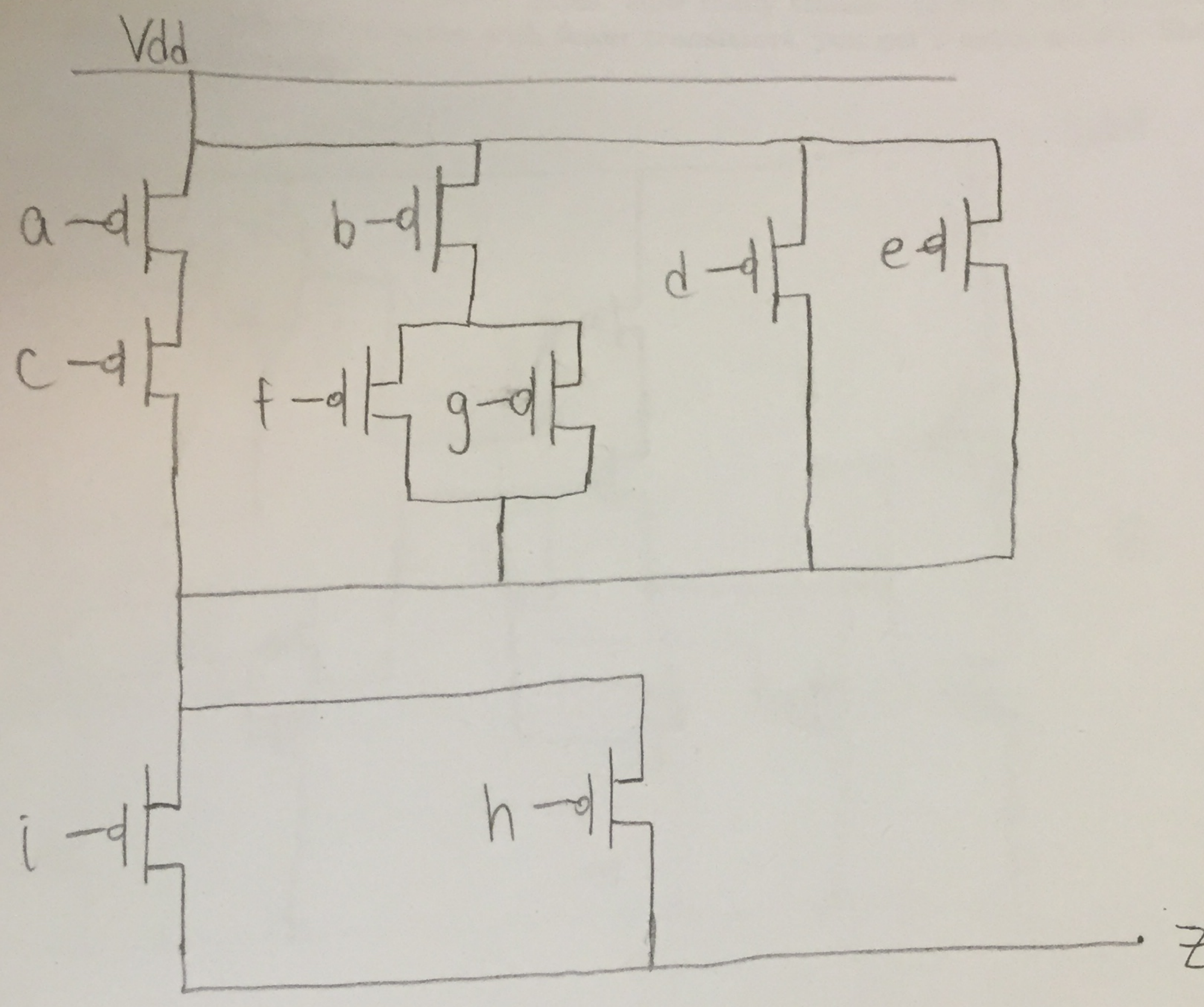
$$(z')' = ((a+c)(b+fg)de)'(ih)'$$

$$= ((a+c)' + (b+fg)' + d' + e')(i'+h')$$

$$= (a'c' + b'(f'+g') + d' + e')(i'+h')$$

✓

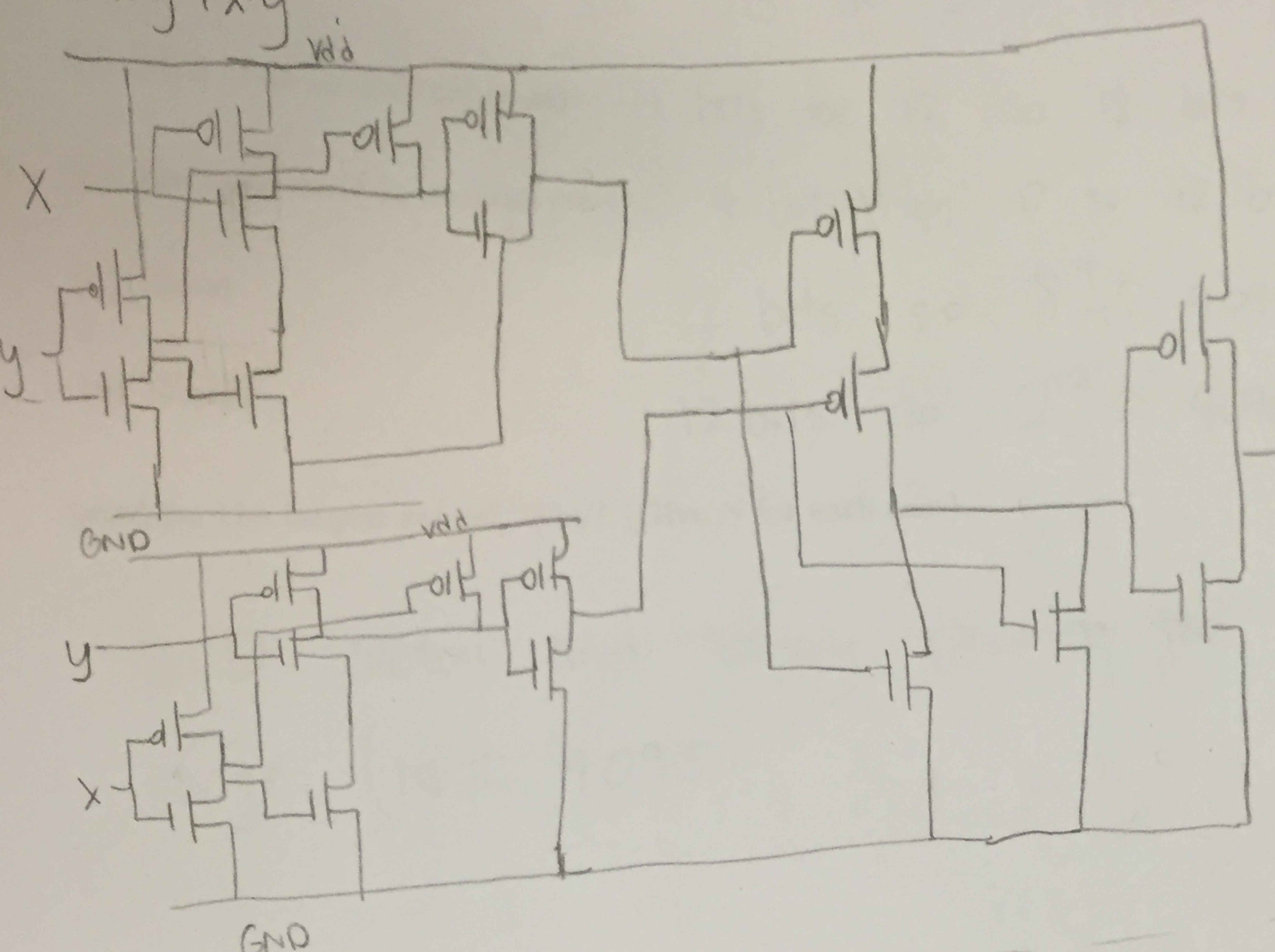
(c) Draw the pull-up network.



2. **(7 points)** Draw a CMOS network that implements f(x,y) = xy'+x'y (2-input XOR). x' and y' are not available as inputs. Do not use transmission gates. How many transistors does your solution have? OPTIONAL: If you find another solution with fewer transistors, you get 3 extra points. Show your reduced design and explain why it works.

xy'+x'y

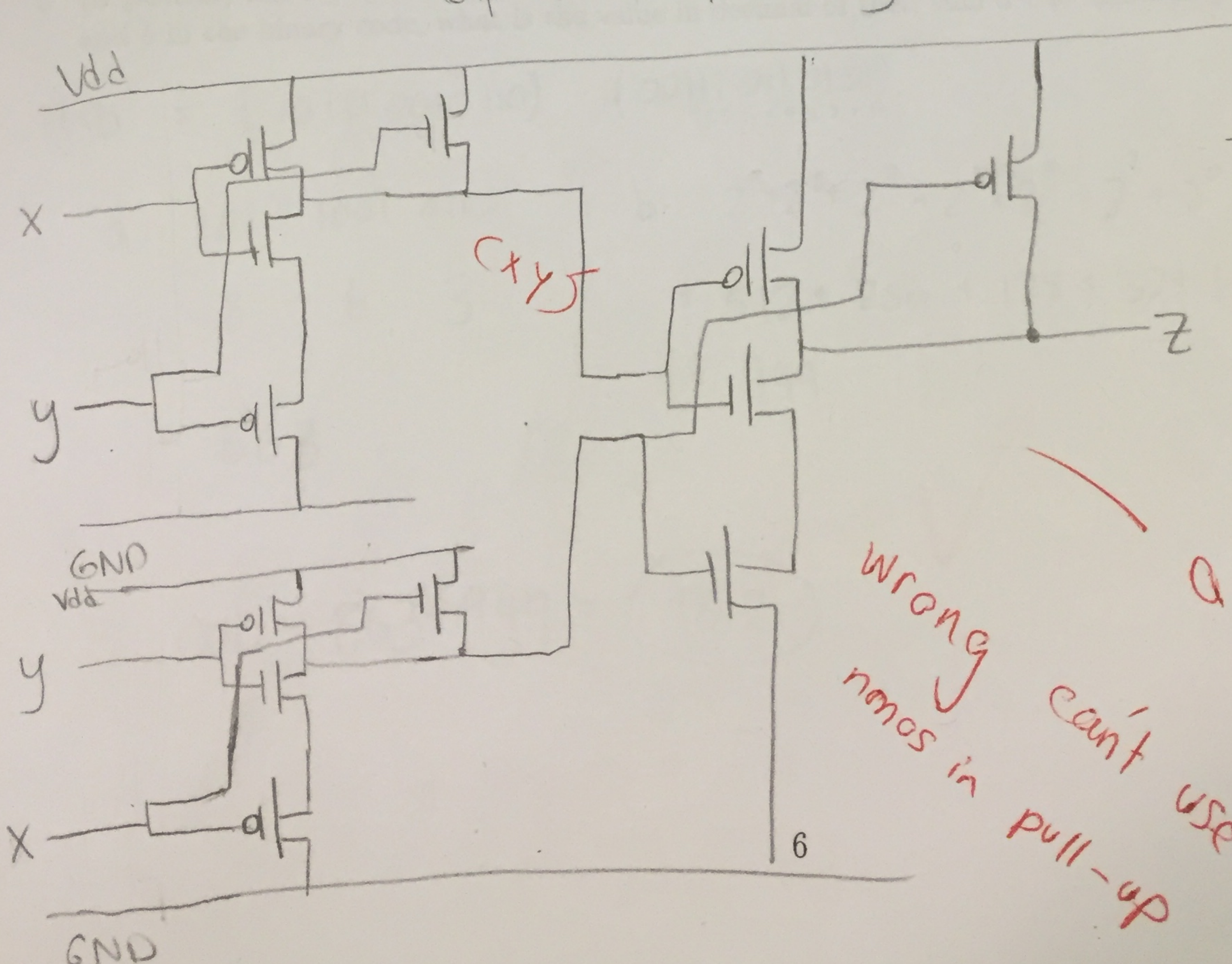
Total: 22 transistors



-7

Better Solution: works because we don't use double NOT's exploit complementary logic

Total: 12 transistors



wrong can't use nmos in pull-up

a

8

Problem 3 (10 points)

1. **(5 points)** A 12-bit vector represents a set of positive integers {0,...,N}. Which of the following coding alternatives

   (a) BCD — 12 bits so $2^{12} = 4096-1$ X −1

   (b) 2421 code (a decimal code) — 4 bits for 10, so 12 bits for $10^3 = 1000 - 1$

   (c) Excess-3 code (a decimal code) — 4 bits for 10 so 12 bits for $10^3 = 1000 - 1$

   (d) Octal — 12 bits so $8^4 \approx 4096 - 1$

   (e) Binary — 12 bits so $2^{12} \approx 4096 - 1$

   −1

   provides the largest range? Why? (Give N for each case).

   BCD Octal and binary provide the largest range (N = 4095)

2. **(5 points)** Let $a = (101110010110)$ and $b = (001110110101)$. If a represents a number in the Excess-3 code and b in the binary code, what is the value in decimal of their sum $a + b$? Show all your work.

a+b = (101110010110) (001110110101)

a: 1011 1001 0110
   8 6 3
   = 863

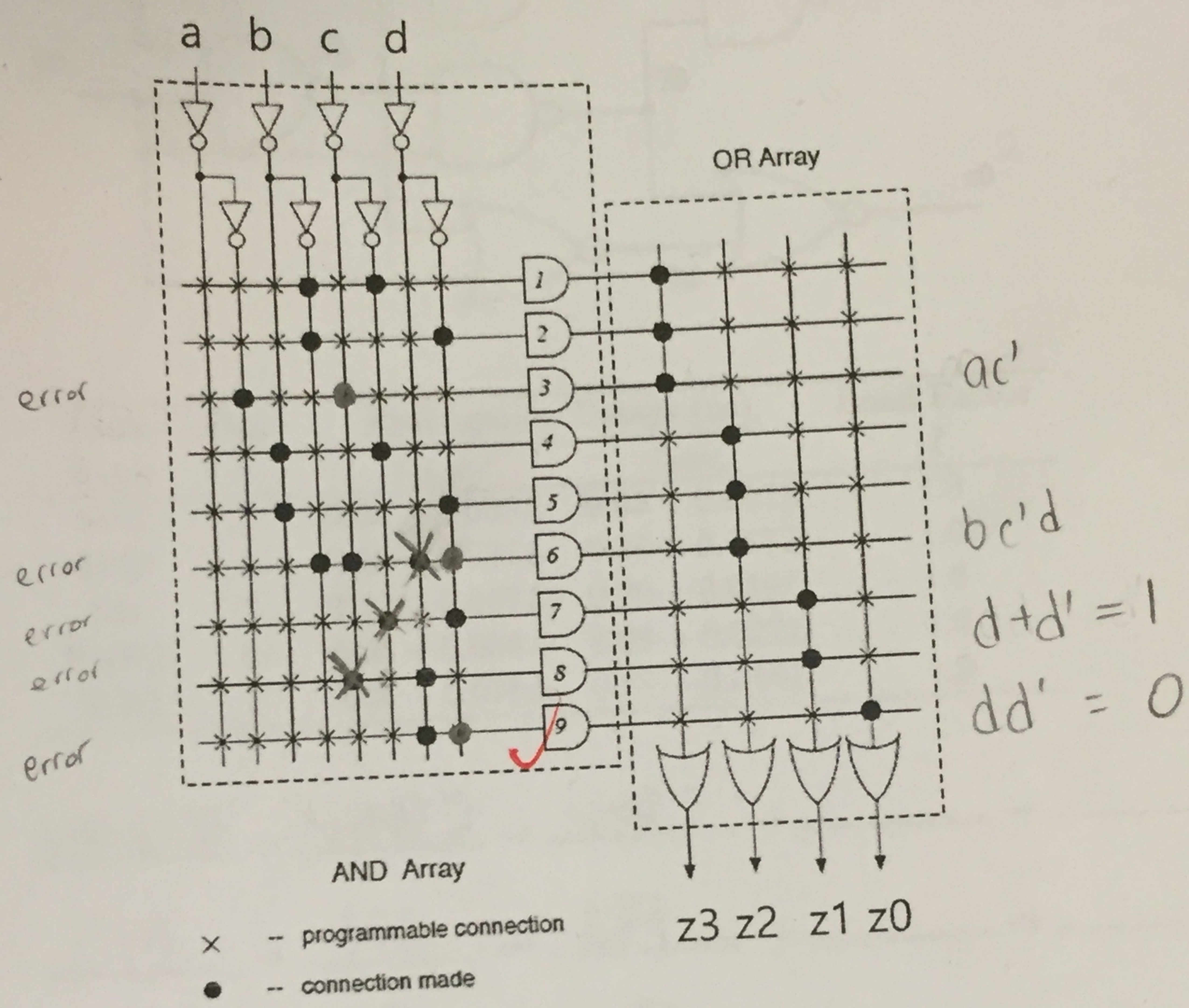
b: $2^9 + 2^8 + 2^7 + 2^5 + 2^4 + 2^2 + 2^0$
   = 512 + 256 + 128 + 32 + 16 + 4 + 1
   = 949

a+b = 863 + 949 = 1812

Problem 4 (15 points) 15

We would like to verify that the PLA implementation shown here implements the following switching functions:

$$z3 = bc + bd + ac'$$
$$z2 = b'c + b'd + bc'd$$
$$z1 = 1$$
$$z0 = 0$$



1. **(7 points)** Analyze the PLA shown above and show the output expressions.

$Z_3 = bc + bd + a$

$Z_2 = b'c + b'd + bc'd'$

$Z_1 = cd + c'd'$

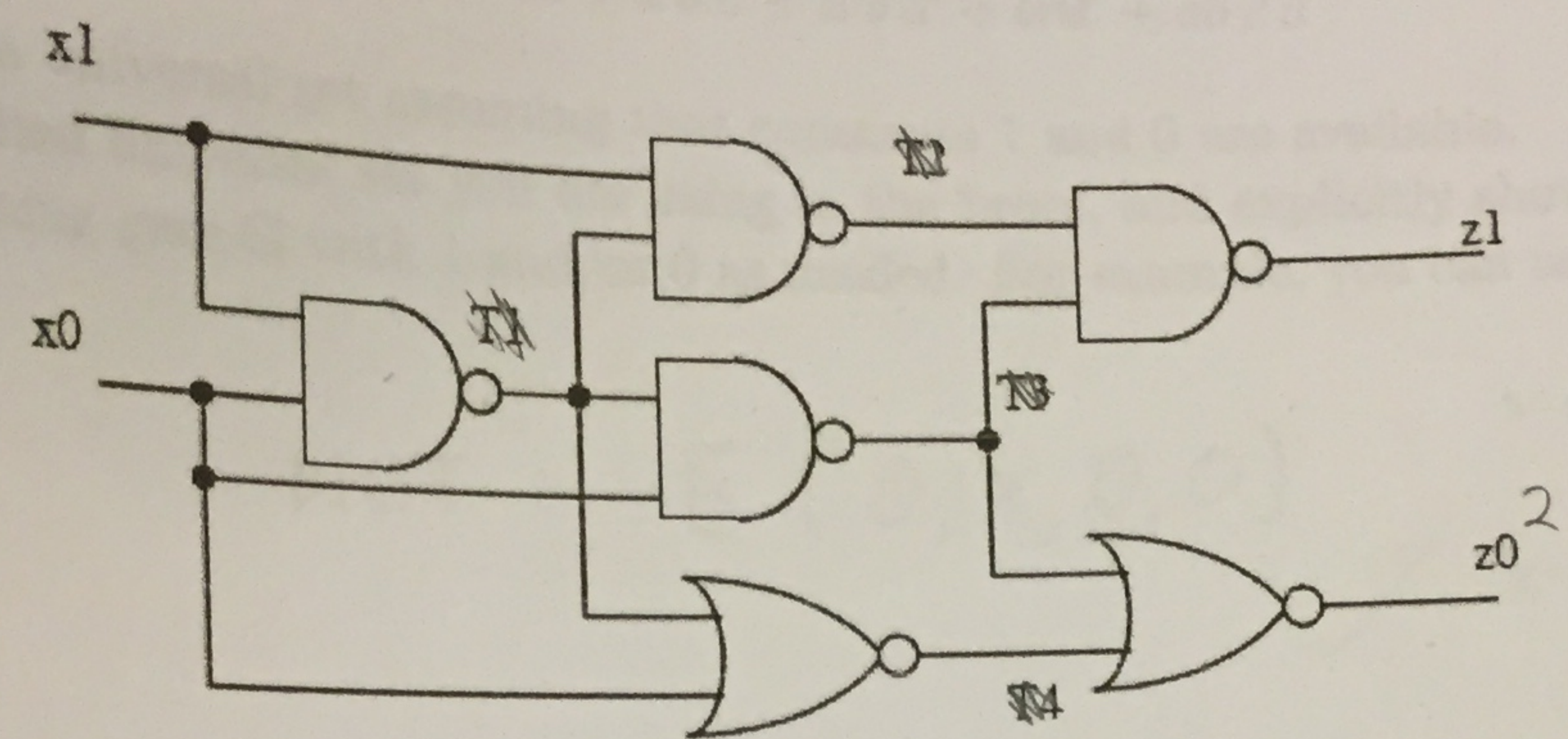
$Z_0 = d'$

2. **(8 points)** Is the PLA implementation correct? If not, find errors and show the correct implementa
(cross out wrong connections and insert correct ones)

No, errors on line 3, 6, 7, 8, 9

Problem 5 (10 points) 10

Calculate the propagation delay $t_{pLH}(z0)$ when $x0$ changes. Assume that $z0$'s load value is 2. Fill in the blanks below with the appropriate values. You don't need to fill all the blanks.



Gate Type	Fan-in	Propagation Delays (ns) t_{pLH}	t_{pHL}	Load Factor I
NOT	1	$0.02 + 0.038L$	$0.05 + 0.017L$	1.0
AND	2	$0.15 + 0.037L$	$0.16 + 0.017L$	1.0
OR	2	$0.12 + 0.037L$	$0.20 + 0.019L$	1.0
NAND	2	$0.05 + 0.038L$	$0.08 + 0.027L$	1.0
NOR	2	$0.06 + 0.075L$	$0.07 + 0.016L$	1.0

Gate type & Fan-in	NAND 2 →	NAND 2 →	NOR 2 →	→	→	
LH / HL	LH →	HL →	LH →	→	→	
Output load L	3 →	2 →	2 →	→	→	
Prop. Delay	0.05+ 0.038(3) →	0.08+0.027(2) →	0.06+0.075(2) →	✓	→	→

Problem 6 (15 points)

A gate G is defined by the following expression

15

$$E(a, b, c, d) = cd + a'b'c' + a'b'd' + bcd' + ab'c'd$$

Show that gate G forms a universal set assuming that constants 1 and 0 are available.

Specify a pre-established universal set you are using in the proof, and explicitly show the implementation for each element in the set using gate G with 1 and/or 0 as needed. For example, you can assign a=0 and b=1 in the expression E.

abcd	
0000	1
0001	1
0010	1
0011	1
0100	0
0101	0
0110	1
0111	1
1000	0
1001	1
1010	0
1011	1
1100	0
1101	0
1110	1
1111	1

NOT : $E(0, x, 0, 0)$

$x=0$ $E(0,0,0,0) = 1$

$x=1$ $E(0,1,0,0) = 0$

OR : $E(E(0,x,0,0), y, 1, 0) = E(x', y, 1, 0)$

$x=0$ $y=0$ $E(1,0,1,0) = 0$

$x=0$ $y=1$ $E(1,1,1,0) = 1$

$x=1$ $y=0$ $E(0,0,1,0) = 1$

$x=1$ $y=1$ $E(0,1,1,0) = 1$

Since gate G can implement NOT and OR,

G forms a universal set

Problem 7 (20 points)

18

For the switching function $f(x_3, x_2, x_1, x_0)$, we are given the information below for the dc-set and zero-set.

$$\text{dc-set} = (4, 15)$$

$$\text{zero-set} = \text{zero-set of function}$$

$$\underset{13}{(x_3+x_2+x_1'+x_0)}\underset{10}{(x_3+x_2'+x_1+x_0')}\underset{9}{(x_3+x_2'+x_1'+x_0)}\underset{6}{(x_3'+x_2+x_1+x_0')}\underset{5}{(x_3'+x_2+x_1'+x_0)}\underset{2}{(x_3'+x_2'+x_1+x_0')}$$

1. **(2 points)** Fill out the following K-map.

		x_0	x_0		
	1	1	1	0	
	–	0	1	0	x_2
x_3	1	0	–	1	x_2
x_3	1	0	1	0	
			x_1	x_1	

$X_1 + X_0' + X_2'$

$X_1 + X_0' + X_3'$

$X_1' + X_0 + X_3$

$X_1' + X_0 + X_2$

$X_0'X_1'$ | X_0X_1

$X_2'X_3'X_1'$ | $X_2'X_3'X_0$

$X_2X_3X_1$

2. **(4 points)** Which of the given expressions are prime implicants of the function given above? Circle **all** that apply. Do not circle implicants that are not prime.

(a) x_3x_1
(b) x_3x_2'
(c) $x_3'x_1$
(d) $x_1'x_0'$
(e) x_1x_0
(f) $x_3'x_2'x_1'$
(g) $x_3'x_2'x_0$
(h) $x_3x_2x_1$
(i) $x_3x_2x_0'$
(j) $x_3'x_2x_1'x_0$
(k) $x_3x_2'x_1x_0$
(l) $x_3'x_2x_1x_0'$

-1

3. **(3 points)** Write down the complete set of **essential** prime implicants.

$X_0'X_1'\ ,\ X_0X_1\ ,\ X_1X_2X_3$

-1

4. **(2 points)** Write down the minimal sum of products expressions for f. If there are multiple forms of minimal sum of products expressions, you only need to write down one of them.

$f = X_0'X_1' + X_0X_1 + X_1X_2X_3 + X_1'X_2'X_3'$

5. **(4 points)** Which of the given expressions are prime implicates of the function given above? Circle all that apply. Do not circle implicates that are not prime.

(a) $(x_3 + x_2' + x_1)$
(b) $(x_3' + x_2')$
(c) $(x_2' + x_1 + x_0')$
(d) $(x_3' + x_1 + x_0')$
(e) $(x_3 + x_2' + x_0)$
(f) $(x_3 + x_1' + x_0)$
(g) $(x_2 + x_1' + x_0)$
(h) $(x_3 + x_1 + x_0)$
(i) $(x_3' + x_2' + x_1)$
(j) $(x_3' + x_1')$
(k) $(x_3 + x_2 + x_1 + x_0')$
(l) $(x_3 + x_2' + x_1' + x_0)$

6. **(3 points)** Write down the complete set of **essential** prime implicates.

$(X_2' + X_1 + X_0')$, $(X_3' + X_1 + X_0')$, $(X_3 + X_1' + X_0)$, $(X_2 + X_1' + X_0)$

7. **(2 points)** Write down the minimal product of sums expressions for f. If there are multiple forms of minimal product of sums expressions, you only need to write down one of them.

$(X_2' + X_1 + X_0')(X_3' + X_1 + X_0')(X_3 + X_1' + X_0)(X_2 + X_1' + X_0)$