[CS M51A Fall 15] Solutions for Midterm exam

Date: 11/3/15

Problem 1 (15 points)

1. (6 points) Given the following simplification of a boolean expression, identify all right and wrong steps and briefly explain what is wrong for each error.
   (For example, (10) → (11) wrong application of the Identity rule, (11) → (12) correct)

$$
\begin{aligned}
E_1(w,x,y,z) &= (((w+x+x'y')y+z)'+wx'+y')' && (1)\\
&= ((w+x+x'y')'y'z'+wx'+y')' && (2)\\
&= ((w+x+x'y')'y'z'+(w+y')(x'+y'))' && (3)\\
&= ((w+x'+y')'y'z'+(w+y')(x'+y'))' && (4)\\
&= (w'xyy'z'+(w+y')(x'+y'))' && (5)\\
&= (0+(w+y')(x'+y'))' && (6)\\
&= wy+xy && (7)
\end{aligned}
$$

   Solution
   (1) → (2) wrong application of DeMorgan's Law
   (2) → (3) correct
   (3) → (4) wrong application of Simplification rule
   (4) → (5) correct
   (5) → (6) correct
   (6) → (7) missing invert sign at wy, should be w'y

2. (5 points) Obtain the minimal sum of products form for $E_2(w,x,y,z)$ using the identities of Boolean algebra. Show all the steps in your derivation.

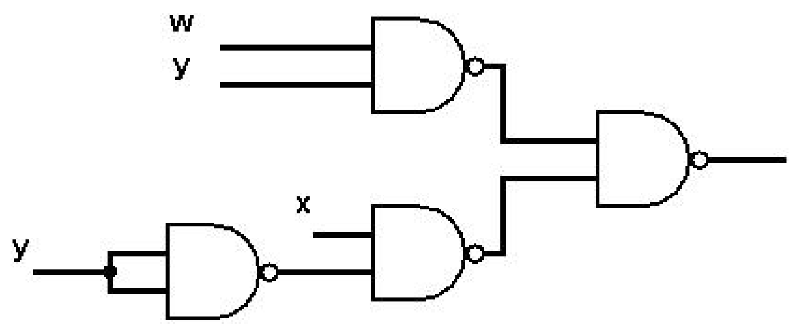
$$E_2 = xy' + xzw + yw$$

   Solution

$$
\begin{aligned}
E_2(w,x,y,z) &= xy' + xzw + yw\\
&= xy' + x(y+y')zw + yw\\
&= xy' + xyzw + xy'zw + yw\\
&= xy'(1+zw) + yw(xz+1)\\
&= xy' + yw
\end{aligned}
$$

3. (4 points) Using the expression obtained for E_2 from the previous step, obtain the NAND network that uses ONLY NAND gates. Inverted inputs are not available, and no constant inputs are allowed.

Solution



Problem 2 (15 points)

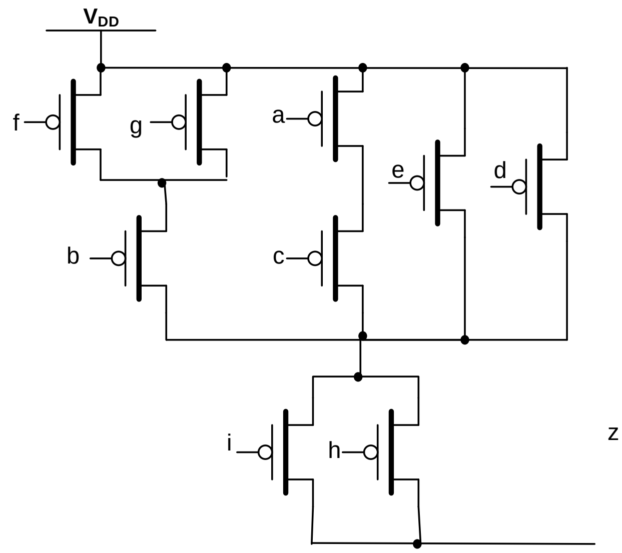
The following pull-down network is part of a complex CMOS gate that we want to implement.

1. **(8 points)** (a) Write the expression for the pull-down network.
   Solution Pull-down network expression : $z' = ih + (a + c)(b + fg)de$

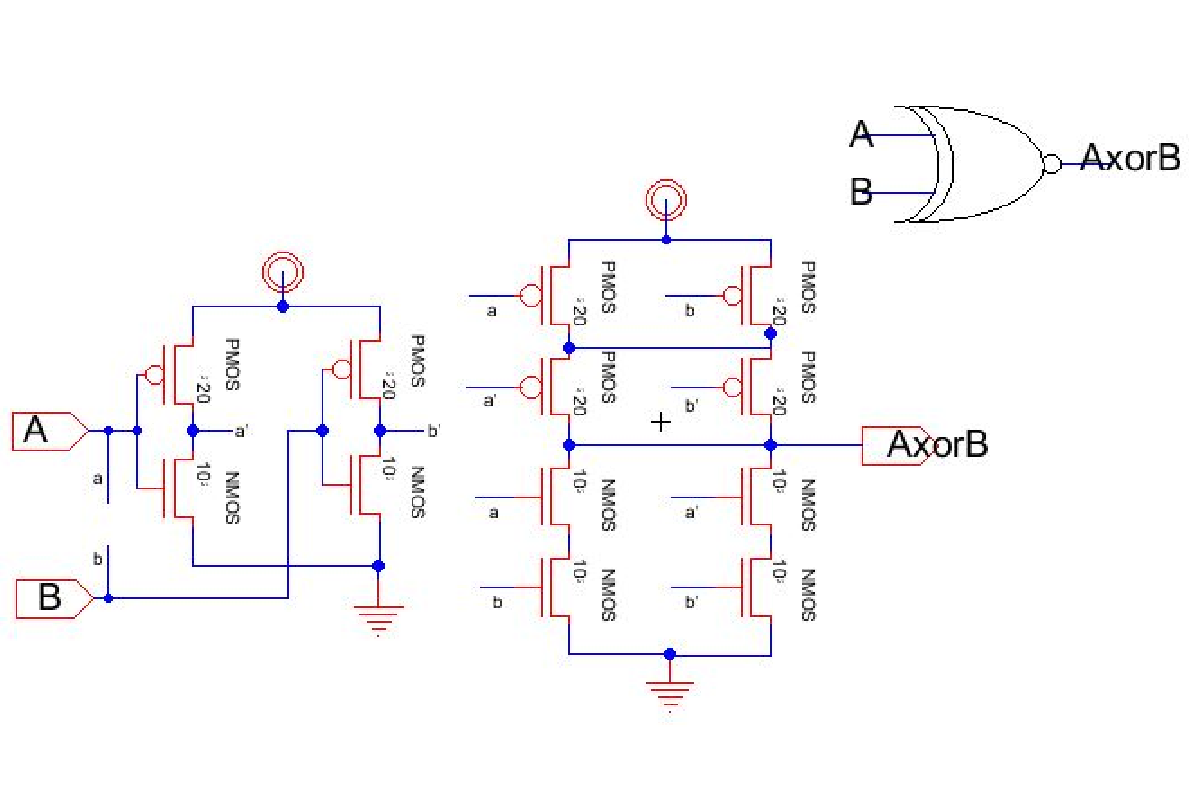
   (b) Obtain the expression for the corresponding pull-up network.
   Solution Pull-down network expression : $z = (i' + h')(a'c' + b'(f' + g') + d' + e')$

   (c) Draw the pull-up network.

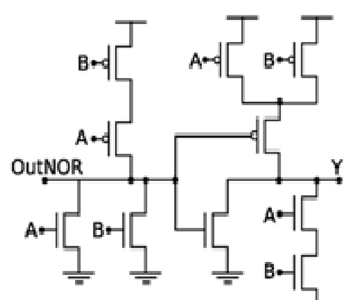
   Solution



2. **(7 points)** Draw a CMOS network that implements f(x,y) = xy'+x'y (2-input XOR). x' and y' are not available as inputs. Do not use transmission gates. How many transistors does your solution have? OPTIONAL: If you find another solution with fewer transistors, you get 3 extra points. Show your reduced design and explain why it works. ***Solution***
   If implemented correctly with two inverters and a XOR gate with a total of 12 transistors, you get 7 points.



but the minimum number of transistors is 10. Using boolean algebra:
The pull-down network expression is: $(XNOR)f' = x'y' + xy = (x+y)' + xy = NOR(x,y) + xy$
with this change, now pull-down network expression has no inverted input and has 10 transistors.



Ten-transistor XOR logic gate

Problem 3 (10 points)

1. **(5 points)** A 12-bit vector represents a set of positive integers {0,...,N}. Which of the following coding alternatives provides the largest range? Why? (Give N for each case).

   Solution

   (a) BCD: Max = 100110011001 = 999 → The range is 1,000
   (b) 2421 code: Max = 111111111111 = 999 → The range is 1,000
   (c) Excess-3 code: Max = 110011001100 = 999 → The range is 1,000
   (d) Octal: Max = 111111111111 = $7777_8 = 4,095$ → The range is 4,096
   (e) Binary: Max = 111111111111 = $111111111111_2 = 2^{12} - 1 = 4,095$ → The range is 4,096
   Thus, the answer is Octal and Binary.

2. **(5 points)** Let $a = (101110010110)$ and $b = (001110110101)$. If a represents a number in the Excess-3 code and b in the binary code, what is the value in decimal of their sum $a + b$? Show all your work.

Solution

$a = 101110010110 = 863$

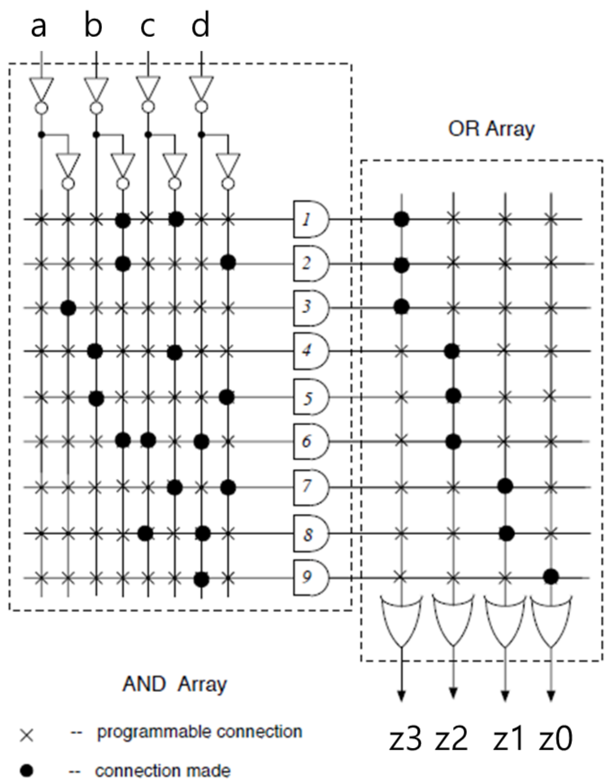
$b = 001110110101_2 = 949$

Thus, $a + b = 863 + 949 = 1,812$

Problem 4 (15 points)

We would like to verify that the PLA implementation shown here implements the following switching functions:

$$z3 = bc + bd + ac'$$
$$z2 = b'c + b'd + bc'd$$
$$z1 = 1$$
$$z0 = 0$$



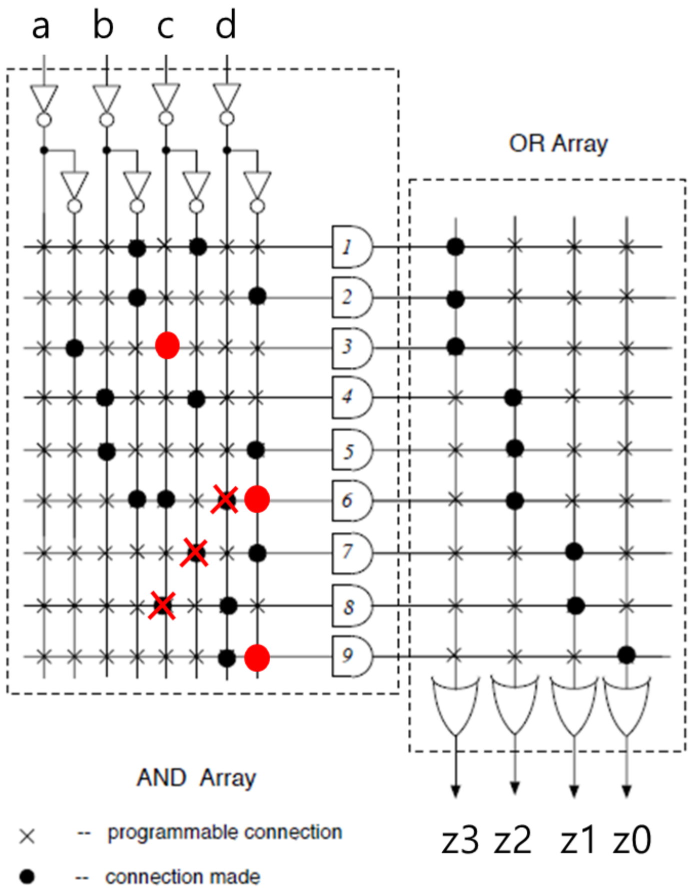
1. **(7 points)** Analyze the PLA shown above and show the output expressions.

   Solution

$$z3 = bc + bd + a$$
$$z2 = b'c + b'd + bc'd'$$
$$z1 = cd + c'd'$$
$$z0 = d'$$

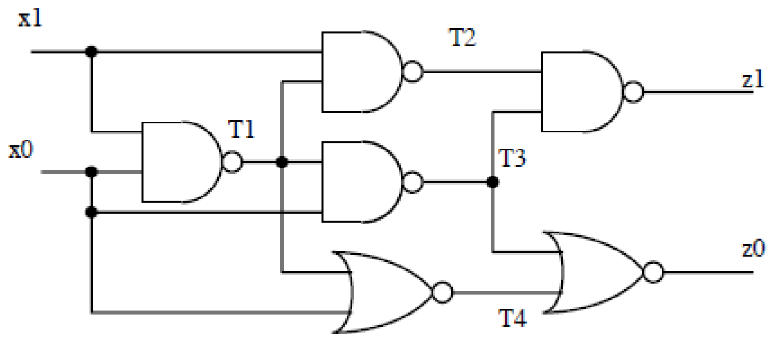
2. **(8 points)** Is the PLA implementation correct? If not, find errors and show the correct implementation (cross out wrong connections and insert correct ones)

Solution



Problem 5 (10 points)

Calculate the propagation delay $t_{pLH}(z0)$ when $x0$ changes. Assume that $z0$'s load value is 2. Fill in the blanks below with the appropriate values. You don't need to fill all the blanks.



Gate Type	Fan-in	Propagation Delays (ns) t_{pLH}	t_{pHL}	Load Factor I
NOT	1	$0.02 + 0.038L$	$0.05 + 0.017L$	1.0
AND	2	$0.15 + 0.037L$	$0.16 + 0.017L$	1.0
OR	2	$0.12 + 0.037L$	$0.20 + 0.019L$	1.0
NAND	2	$0.05 + 0.038L$	$0.08 + 0.027L$	1.0
NOR	2	$0.06 + 0.075L$	$0.07 + 0.016L$	1.0

Solution

Gate type & Fin-in: NAND2 $\rightarrow$ NAND2 $\rightarrow$ NOR2

LH / HL: LH $\rightarrow$ HL $\rightarrow$ LH

Output load L: $3.0 \rightarrow 2.0 \rightarrow 2.0$

Prop. Delay: $0.05 + 0.038 \times 3.0 \rightarrow 0.08 + 0.027 \times 2.0 \rightarrow 0.06 + 0.075 \times 2.0$

Problem 6 (15 points)

A gate G is defined by the following expression

$$E(a,b,c,d) = cd + a'b'c' + a'b'd' + bcd' + ab'c'd$$

Show that gate G forms a universal set assuming that constants 1 and 0 are available.

Specify a pre-established universal set you are using in the proof, and explicitly show the implementation for each element in the set using gate G with 1 and/or 0 as needed. For example, you can assign a=0 and b=1 in the expression E.

Solution Plugging in different input combinations, we can find an input vector that implements a NOR function.

$$E(a,b,0,0) = a'b' + a'b' = a'b' = (a+b)'$$

Since {NOR} forms a universal set and we can use $E(a,b,c,d)$ to implement any switching function and thus G is universal.

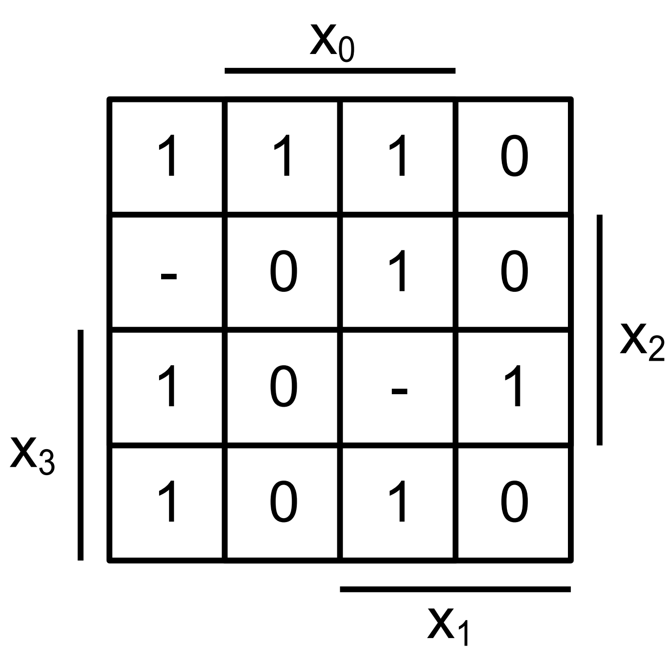
Problem 7 (20 points)

For the switching function $f(x_3, x_2, x_1, x_0)$, we are given the information below for the dc-set and zero-set.

$$\text{dc-set} = (4, 15)$$
$$\text{zero-set} = \text{zero-set of function}$$
$$(x_3+x_2+x_1'+x_0)(x_3+x_2'+x_1+x_0')(x_3+x_2'+x_1'+x_0)(x_3'+x_2+x_1+x_0')(x_3'+x_2+x_1'+x_0)(x_3'+x_2'+x_1+x_0')$$

1. **(2 points)** Fill out the following K-map.

   Solution The completed K-map is shown:



2. **(4 points)** Which of the given expressions are prime implicants of the function given above? Circle **all** that apply. Do not circle implicants that are not prime.

   Solution $x_1'x_0'$, x_1x_0, $x_3'x_2'x_1'$, $x_3'x_2'x_0$, $x_3x_2x_1$, $x_3x_2x_0'$

3. **(3 points)** Write down the complete set of **essential** prime implicants.

   Solution

   $x_1'x_0'$ and x_1x_0.

4. **(2 points)** Write down the minimal sum of products expressions for f. If there are multiple forms of minimal sum of products expressions, you only need to write down one of them.

   Solution $x_1'x_0' + x_1x_0 + x_3'x_2'x_0 + x_3x_2x_0'$.

5. **(4 points)** Which of the given expressions are prime implicates of the function given above? Circle **all** that apply. Do not circle implicates that are not prime.

   Solution $(x_3 + x_2' + x_1)$, $(x_2' + x_1 + x_0')$, $(x_3' + x_1 + x_0')$, $(x_3 + x_2' + x_0)$, $(x_3 + x_1' + x_0)$, $(x_2 + x_1' + x_0)$

6. **(3 points)** Write down the complete set of **essential** prime implicates.

   Solution $(x_3' + x_1 + x_0')$ and $(x_2 + x_1' + x_0)$

7. **(2 points)** Write down the minimal product of sums expressions for f. If there are multiple forms of minimal product of sums expressions, you only need to write down one of them.

   Solutionr $(x_3' + x_1 + x_0')(x_2' + x_1 + x_0')(x_2 + x_1' + x_0)(x_3 + x_1' + x_0)$.