[CS M51A Fall 15] Midterm exam

Date: 11/3/15

- The midterm is closed books and notes. Tablets and smartphone are not allowed.
- You can use calculators and have up to 2 sheets (= 4 pages) of summary notes.
- Please show all your work and write legibly, otherwise no partial credit will be given.
- **This should strictly be your own work; any form of collaboration will be penalized.**

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Discussion 2B

Problem	Points	Score
1	15	15
2	15	17
3	10	3
4	15	11
5	10	10
6	15	15
7	20	20
Total	100	91

Problem 1 (15 points)

1. (6 points) Given the following simplification of a boolean expression, identify all right and wrong steps and briefly explain what is wrong for each error.
   (For example, (10) → (11) wrong application of the Identity rule, (11) → (12) correct)

$$
\begin{aligned}
E_1(w,x,y,z) &= (((w+x+x'y')y+z)' + wx' + y')' && (1)\\
&= ((w+x+x'y')'y'z' + wx' + y')' && (2)\\
&= ((w+x+x'y')'y'z' + (w+y')(x'+y'))' && (3)\\
&= ((w+x'+y')'y'z' + (w+y')(x'+y'))' && (4)\\
&= (w'xyy'z' + (w+y')(x'+y'))' && (5)\\
&= (0 + (w+y')(x'+y'))' && (6)\\
&= wy + xy && (7)
\end{aligned}
$$

$(w+y')'+(x'+y')'$

$w'y + x'y$

(1) → (2) Wrong application of DeMorgan's Law

(2) → (3) Correct

(3) → (4) wrong application of ~~absorption~~ simplification rule

(4) → (5) Correct

(5) → (6) Correct

(6) → (7) wrong application of DeMorgan's Law

2. (5 points) Obtain the minimal sum of products form for $E_2(w, x, y, z)$ using the identities of Boolean algebra. Show all the steps in your derivation.

$$E_2 = xy' + xzw + yw$$

$xzw = xzwy' + xzwy$

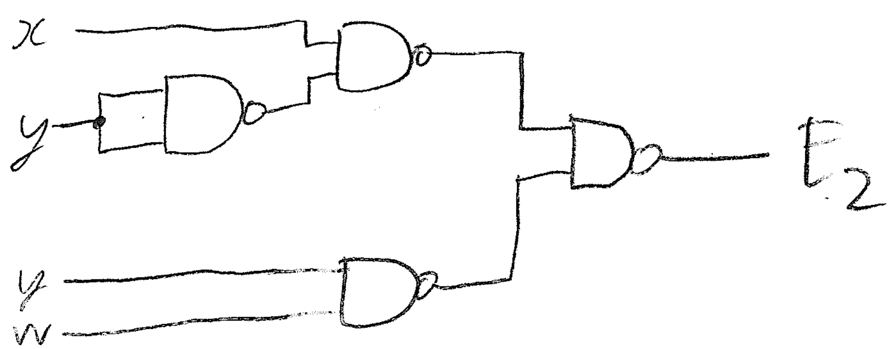
$E_2 = xy' + xzwy' + xzwy + yw$

$E_2 = (xy') + (xy')zw + (yw) + (yw)xz$

$E_2 = xy' + yw$

✓

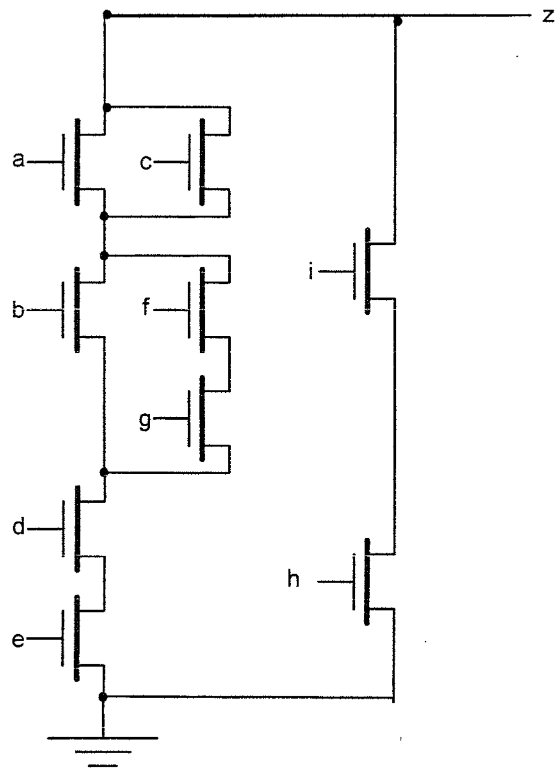
3. (4 points) Using the expression obtained for E_2 from the previous step, obtain the NAND network that uses ONLY NAND gates. Inverted inputs are not available, and no constant inputs are allowed.



✓

Problem 2 (15 points)

The following pull-down network is part of a complex CMOS gate that we want to implement.



1. **(8 points)** (a) Write the expression for the pull-down network.

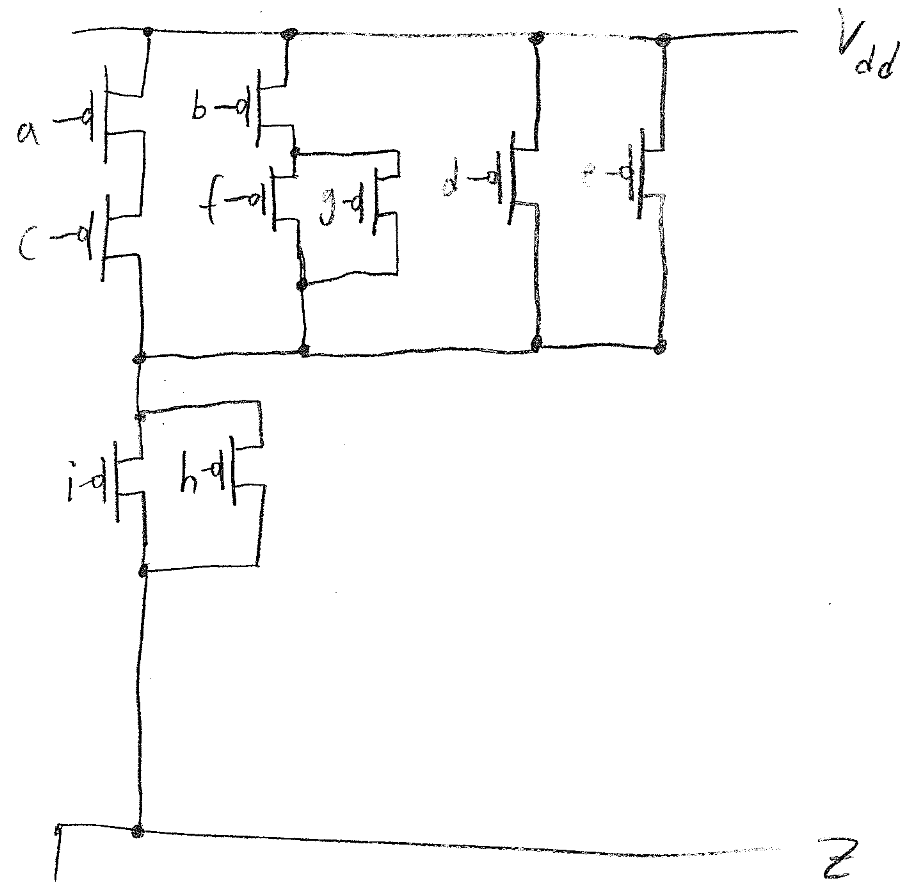
   Pull-down network expression : z' = $((a+c)(b+fg)de)+ih$

   (b) Obtain the expression for the corresponding pull-up network.

   Pull-up network expression : z = $(a'c'+b'(f'+g')+d'+e')(i'+h')$

$$(((a+c)(b+fg)de)+ih)'$$
$$= ((a+c)(b+fg)de)'(i'+h')$$
$$= ((a+c)'+(b+fg)'+d'+e')(i'+h')$$
$$= (a'c'+b'(f'+g')+d'+e')(i'+h')$$

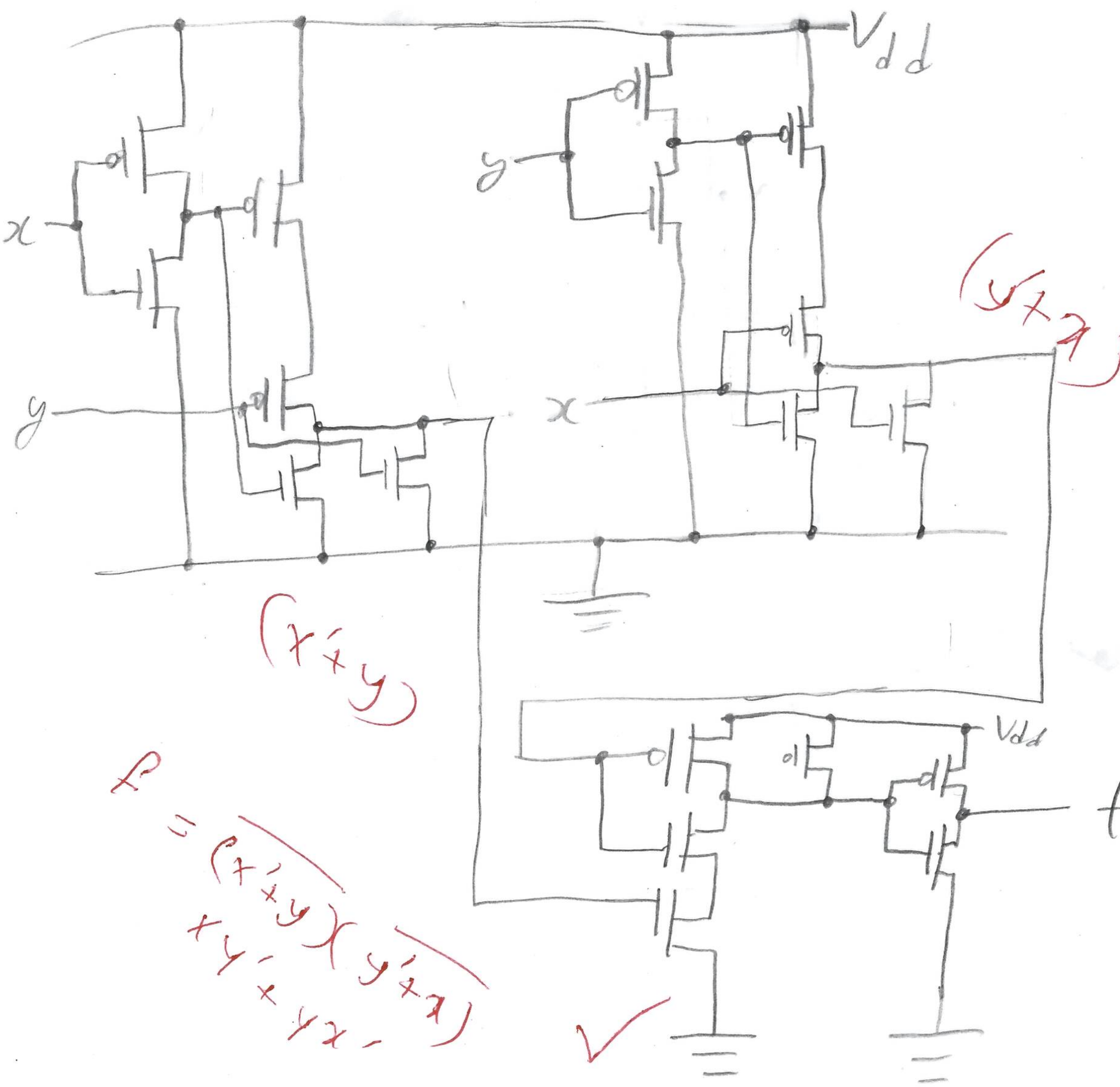
(c) Draw the pull-up network.



2. **(7 points)** Draw a CMOS network that implements f(x,y) = xy'+x'y (2-input XOR). x' and y' are not available as inputs. Do not use transmission gates. How many transistors does your solution have? OPTIONAL: If you find another solution with fewer transistors, you get 3 extra points. Show your reduced design and explain why it works.

$f' = (xy'+x'y)' = xy+x'y'$

$f(x,y) =$



this uses 18 transistors

You could also get a 14 transistor solution:

xy takes 6

$x'y'$ takes 4

$(xy+x'y')'$ takes 4

×2

3

Problem 3 (10 points)

1. **(5 points)** 3 A 12-bit vector represents a set of positive integers {0,...,N}. Which of the following coding alternatives

   (a) BCD $2^{12}-1$ -1

   (b) 2421 code (a decimal code) 10^3-1

   (c) Excess-3 code (a decimal code) 10^3-1

   (d) Octal $8^{12}-1$ -1

   (e) Binary $2^{12}-1$

   provides the largest range? Why? (Give N for each case).

   Octal in this case provides the largest range.

2. **(5 points)** 0 Let $a = (101110010110)$ and $b = (001110110101)$. If a represents a number in the Excess-3 code and b in the binary code, what is the value in decimal of their sum $a+b$? Show all your work.

a in binary = 101110010011
b = 001110110101
111101001000

11 10 9 8 7 6 5 4 3 2 1 0

$2^{11}+2^{10}+2^{9}+2^{8}+2^{6}+2^{3}$

2048 1024 512 256 64 8

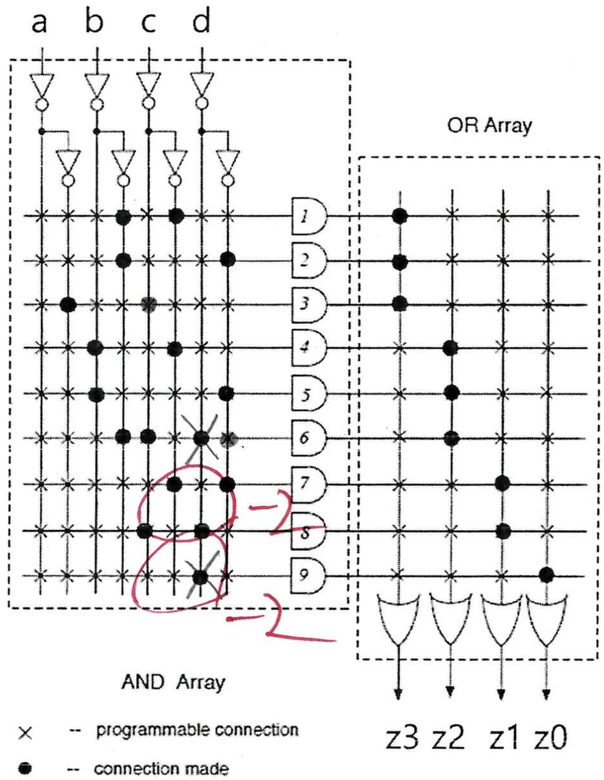
2048
1024
512
256
64
8
3912

3912

Problem 4 (15 points)

We would like to verify that the PLA implementation shown here implements the following switching functions:

$$z3 = bc + bd + ac'$$
$$z2 = b'c + b'd + bc'd$$
$$z1 = 1$$
$$z0 = 0$$



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1. **(7 points)** Analyze the PLA shown above and show the output expressions.

term 1: bc
term 2: bd
term 3: a
term 4: b'c
term 5: b'd
term 6: bc'd'
term 7: cd
term 8: c'd'
term 9: d'

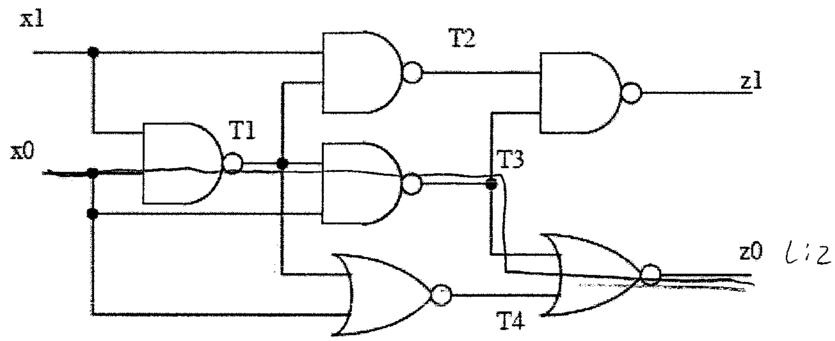
$Z_3 = bc + bd + a$
$Z_2 = b'c + b'd + bc'd'$
$Z_1 = cd + c'd' \neq 1$
$Z_0 = d'$ ✓

X

2. **(8 points)** Is the PLA implementation correct? If not, find errors and show the correct implementation (cross out wrong connections and insert correct ones)

Problem 5 (10 points) 10

Calculate the propagation delay $t_{pLH}(z0)$ when $x0$ changes. Assume that $z0$'s load value is 2. Fill in the blanks below with the appropriate values. You don't need to fill all the blanks.



Gate Type	Fan-in	Propagation Delays (ns) t_{pLH}	t_{pHL}	Load Factor I
NOT	1	$0.02 + 0.038L$	$0.05 + 0.017L$	1.0
AND	2	$0.15 + 0.037L$	$0.16 + 0.017L$	1.0
OR	2	$0.12 + 0.037L$	$0.20 + 0.019L$	1.0
NAND	2	$0.05 + 0.038L$	$0.08 + 0.027L$	1.0
NOR	2	$0.06 + 0.075L$	$0.07 + 0.016L$	1.0

Gate type & Fan-in	NAND2 →	NAND2 →	NOR 2 →	→	→	
LH / HL	LH →	HL →	LH →	→	→	
Output load L	3 →	2 →	2 →	→	→	
Prop. Delay	$0.05+0.038\cdot3$ →	$0.08+0.027\cdot2$ →	$0.06+0.075\cdot2$ →	✓	→	→

Problem 6 (15 points)

15

A gate G is defined by the following expression

$$E(a,b,c,d) = cd + a'b'c' + a'b'd' + bcd' + ab'c'd$$

Show that gate G forms a universal set assuming that constants 1 and 0 are available.

Specify a pre-established universal set you are using in the proof, and explicitly show the implementation for each element in the set using gate G with 1 and/or 0 as needed. For example, you can assign a=0 and b=1 in the expression E.

I will be using the NOR universal set

I use k-maps to simplify E:

ab \ cd	00	01	11	10
00	1	1	1	1
01			1	1
11			1	1
10		1	1	

$$E = a'b' + b'd + bc$$

$$E(a,b,0,0) = a'b' + b' \cdot 0 + b \cdot 0$$
$$= a'b' = (a+b)' = a \text{ NOR } b$$

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Problem 7 (20 points)

For the switching function $f(x_3, x_2, x_1, x_0)$, we are given the information below for the dc-set and zero-set.

$$\text{dc-set} = (4, 15)$$
$$\text{zero-set} = \text{zero-set of function}$$
$$(x_3+x_2+x_1'+x_0)(x_3+x_2'+x_1+x_0')(x_3+x_2'+x_1'+x_0)(x_3'+x_2+x_1+x_0')(x_3'+x_2+x_1'+x_0)(x_3'+x_2'+x_1+x_0')$$

2 5 6 9 10 13

1. **(2 points)** Fill out the following K-map.

$x_3x_2 \backslash x_1x_0$	00	01	11	10
00	1	1	1	0
01	d	0	1	0
11	1	0	d	1
10	1	0	1	0

PI: $x_1'x_0'$

2. **(4 points)** Which of the given expressions are prime implicants of the function given above? Circle **all** that apply. Do not circle implicants that are not prime.

(a) x_3x_1
(b) x_3x_2'
(c) $x_3'x_1$
(d) $x_1'x_0'$
(e) x_1x_0
(f) $x_3'x_2'x_1'$
(g) $x_3'x_2'x_0$
(h) $x_3x_2x_1$
(i) $x_3x_2x_0'$
(j) $x_3'x_2x_1'x_0$
(k) $x_3x_2'x_1x_0$
(l) $x_3'x_2x_1x_0'$

3. **(3 points)** Write down the complete set of **essential** prime implicants.

EPI: $x_1'x_0'$, x_1x_0, $x_3x_2x_0'$

4. **(2 points)** Write down the minimal sum of products expressions for f. If there are multiple forms of minimal sum of products expressions, you only need to write down one of them.

$f = x_1'x_0' + x_1x_0 + x_3x_2x_0' + x_3'x_2'x_0$

5. **(4 points)** Which of the given expressions are prime implicates of the function given above? Circle **all** that apply. Do not circle implicates that are not prime.

(a) $(x_3 + x_2' + x_1)$	(d) $(x_3' + x_1 + x_0')$	(g) $(x_2 + x_1' + x_0)$	(j) $(x_3' + x_1')$
(b) $(x_3' + x_2')$	(e) $(x_3 + x_2' + x_0)$	(h) $(x_3 + x_1 + x_0)$	(k) $(x_3 + x_2 + x_1 + x_0')$
(c) $(x_2' + x_1 + x_0')$	(f) $(x_3 + x_1' + x_0)$	(i) $(x_3' + x_2' + x_1)$	(l) $(x_3 + x_2' + x_1' + x_0)$

6. **(3 points)** Write down the complete set of **essential** prime implicates.

EPI: $(x_3'+x_1+x_0')$, $(x_2'+x_1+x_0')$, $(x_3+x_1'+x_0)$, $(x_2+x_1'+x_0)$

7. **(2 points)** Write down the minimal product of sums expressions for f. If there are multiple forms of minimal product of sums expressions, you only need to write down one of them.

$f = (x_3'+x_1+x_0')(x_2'+x_1+x_0')(x_3+x_1'+x_0)(x_2+x_1'+x_0)$

x_0

1	1	1	0
d	0	1	0
1	0	d	1
1	0	1	0