[CS M51A Fall 15] Midterm exam

Date: 11/3/15

- The midterm is closed books and notes. Tablets and smartphone are not allowed.
- You can use calculators and have up to 2 sheets (= 4 pages) of summary notes.
- Please show all your work and write legibly, otherwise no partial credit will be given.
- **This should strictly be your own work; any form of collaboration will be penalized.**

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Problem	Points	Score
1	15	9
2	15	8
3	10	4
4	15	15
5	10	10
6	15	15
7	20	17
Total	100	78

oblem 1 (15 points)

1. (6 points) Given the following simplification of a boolean expression, identify all right and wrong steps and briefly explain what is wrong for each error. (For example, (10) → (11) wrong application of the Identity rule, (11) → (12) correct)

$$
\begin{aligned}
E_1(w,x,y,z) &= (((w+x+x'y')y+z)'+wx'+y')' && (1)\\
&= ((w+x+x'y')'y'z'+wx'+y')' && (2)\\
&= ((w+x+x'y')'y'z'+(w+y')(x'+y'))' && (3)\\
&= ((w+x'+y')'y'z'+(w+y')(x'+y'))' && (4)\\
&= (w'xyy'z'+(w+y')(x'+y'))' && (5)\\
&= (0+(w+y')(x'+y'))' && (6)\\
&= wy+xy && (7)
\end{aligned}
$$

(1) → (2) $\boxed{\text{Wrong}}$ DeMorgan's expansion of $((w+x+x'y')y+z)'$.

Whereas $(a+b)'$ is $a'b'$ and is done correctly, $(ab)'$ is $(a'+b')$ and is NOT done correctly for $(w+x+x'y')y$ ← Two are multiplied instead of added

(2) → (3) Expansion of $wx'+y'$

working backwords

$(w+y')(x'+y') = wx' + wy' + x'y' + y'y'$

which simplifies to $w(x'+y') + (x'+1)y' = wx'+wy' + y' = wx'+$

So this is $\boxed{\text{fine}}$

(3) → (4) $\boxed{\text{Wrong}}$ simplification

$x+x'y'$ should be $x+y'$, not $x'+y'$

(4) → (5) $\boxed{\text{Correct}}$ DeMorgan's of $(w+x'+y')$ to $w'xy$

(5) → (6) $\boxed{\text{Correct}}$ reduction to 0 → y and y' will never happen. (complement)

(6) → (7) $((w+y')(x'+y'))' = (wx'+y')' = (wx')'y = (w'+x)y$

$= w'y + xy$

So this simplification was $\boxed{\text{WRONG}}$

(5 points) Obtain the minimal sum of products form for $E_2(w, x, y, z)$ using the identities of Boolean algebra. Show all the steps in your derivation.

$$E_2 = xy' + xzw + yw$$

$E_2 = xy' + xwz + wy$ $\qquad$ $wy(z+$

$= \cancel{xy'(w+w)(}$

$x(wz+y')$ $\qquad$ $xy' + xwz(y+y') + yw$

$x(y' + wzy') +$

correct

$\downarrow$

already ~~in minimal sop form~~?

$\downarrow$

$\boxed{\text{already in minimal SOP.}}$

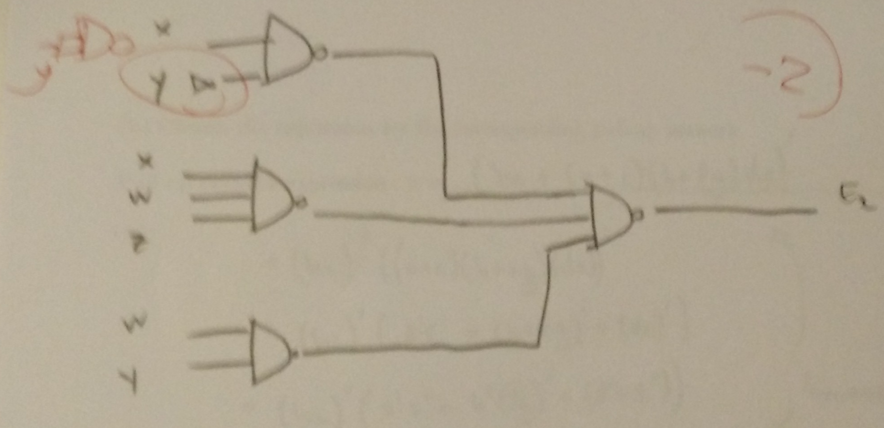
SOP

$xy' + yw$

-4

3. (4 points) Using the expression obtained for E_2 from the previous step, obtain the NAND network that uses ONLY NAND gates. Inverted inputs are not available, and no constant inputs are allowed.

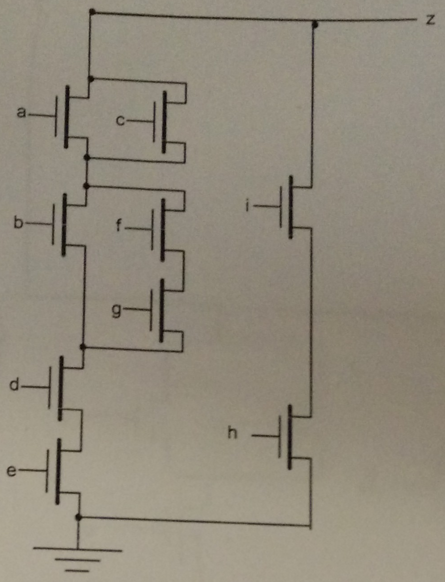
Recall NAND-NAND is same as AND-OR



-2

Problem 2 (15 points)

The following pull-down network is part of a complex CMOS gate that we want to implement.



1. **(8 points)** (a) Write the expression for the pull-down network.

Pull-down network expression : z' = $hi + (a+c)(b+fg)(d)(e)$

(b) Obtain the expression for the corresponding pull-up network.

Pull-up network expression : z = $(hi + (a+c)(b+fg)de)'$

$= (hi)'((a+c)(b+fg)(de))'$

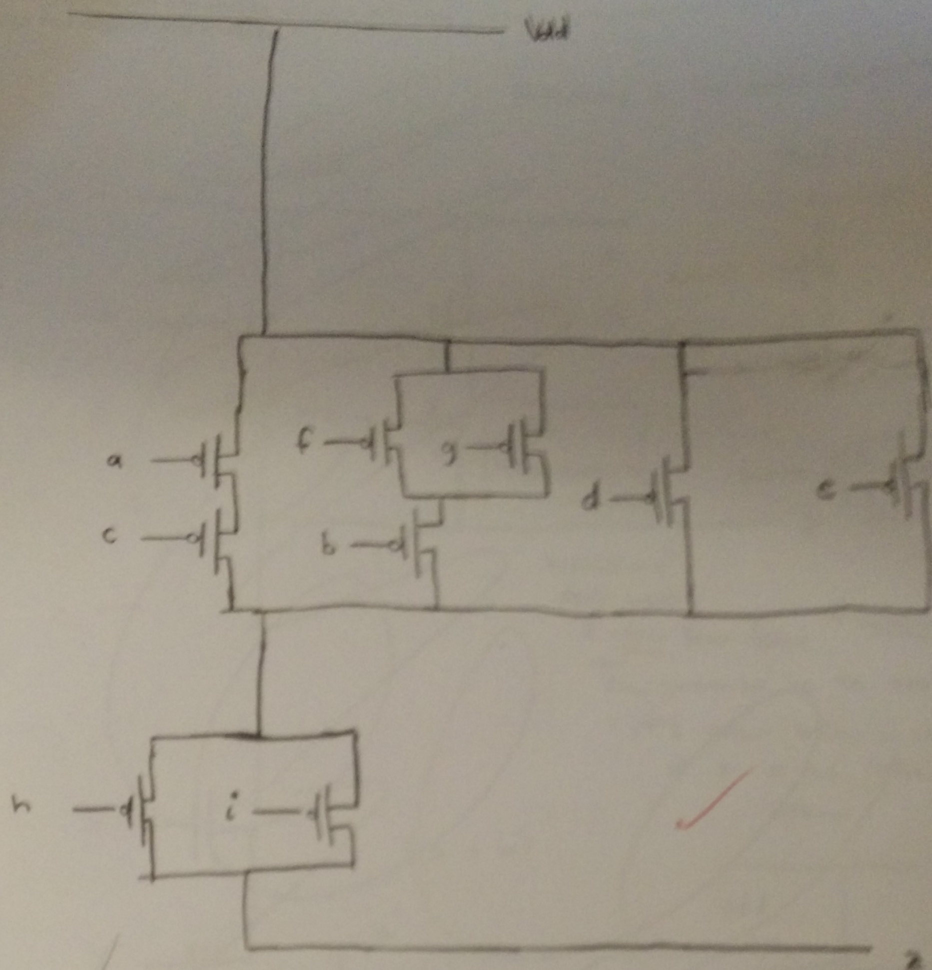
$= (hi)'(a'c' + (b+fg)' + (de)')$

$= (hi)'(a'c' + b'(fg)' + (d'+e'))$

$= (h'+i')(a'c' + b'(f'+g') + (d'+e'))$

however this should be equivalent

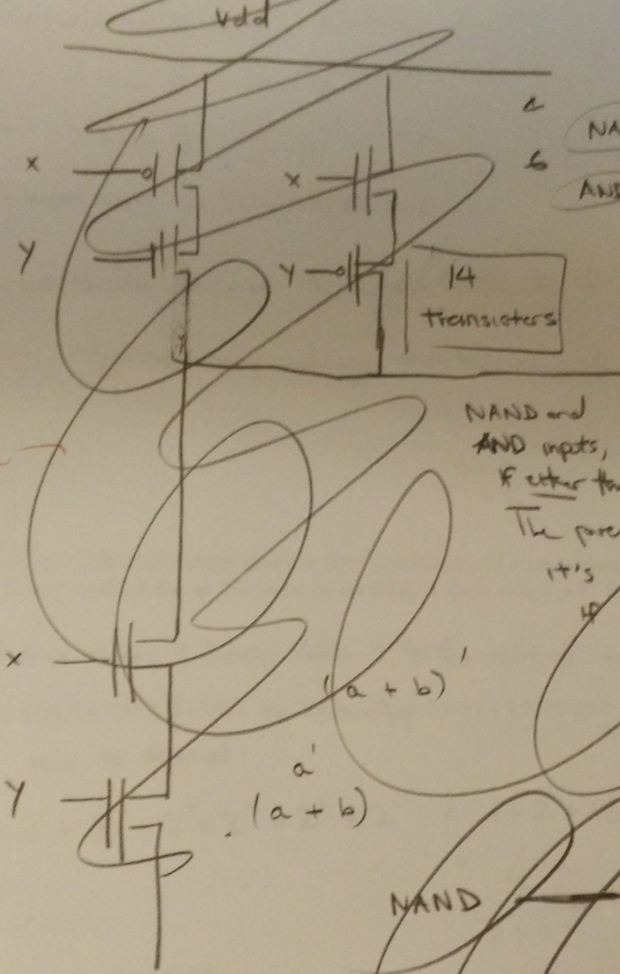
(c) Draw the pull-up network.

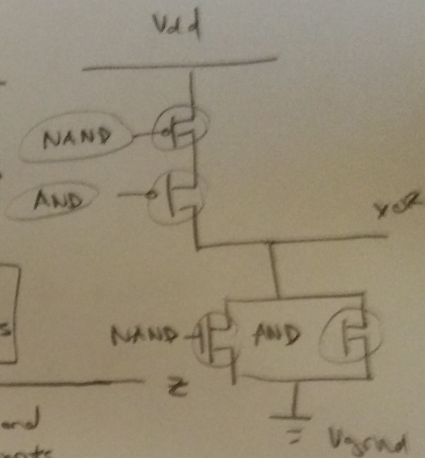


$(h'+i')(a'c' + b'(f'+g') + (d'+e'))$

2. **(7 points)** Draw a CMOS network that implements f(x,y) = xy'+x'y (2-input XOR). x' and y' are not available as inputs. Do not use transmission gates. How many transistors does your solution have? OPTIONAL: If you find another solution with fewer transistors, you get 3 extra points. Show your reduced design and explain why it works.

Probably in the book somewhere...

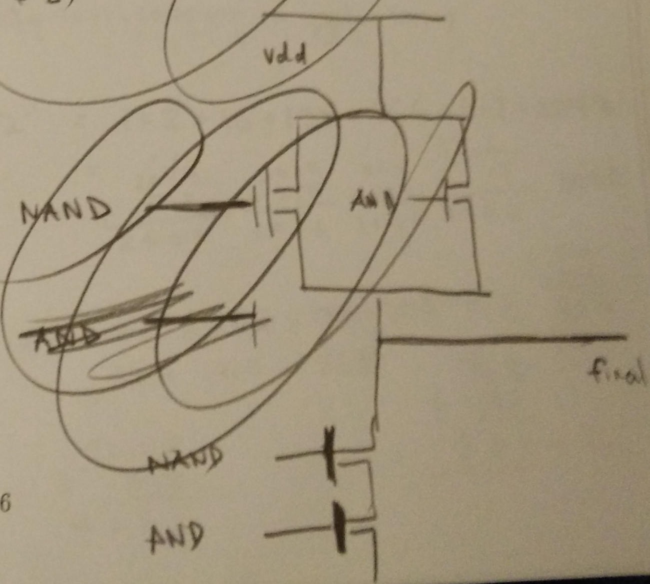




NAND and AND inputs,
If either then false
if neither then true
The premise is to see if
it's either both or neither
if it is the case for
either

$(a+b)'$

$a' \cdot (a+b)$



roblem 3 (10 points)

1. (5 points) A 12-bit vector represents a set of positive integers {0,...,N}. Which of the following coding alternatives

   (a) BCD $\Rightarrow$? X -1

   (b) 2421 code (a decimal code) $\rightarrow$ $2^{13}-1 \rightarrow 999$? — — — 3 dec digits

   (c) Excess-3 code (a decimal code) $\rightarrow$ ~~$2^{13}-1$~~ $\rightarrow$ 999 ?) +2

   (d) Octal $\rightarrow$ base 8 $\rightarrow$ $8^{12}-1$

   (e) Binary $\rightarrow$ base 2 $\rightarrow$ $2^{13}-1$) X

provides the largest range? Why? (Give N for each case).

Octal should because the largest value, 12 1's would be $8^{13}-1$

Whereas regular binary codings and the transf X

6-3 = 3   110 - 011

2. (5 points) Let a = (101110010110) and b = (001110110101). If a represents a number in the Excess-3 code and b in the binary code, what is the value in decimal of their sum $a+b$? Show all your work.

recall that excess 3 simply adds 3 to the value of the item.

So a: 101110010110 is actually 101110010011 in regular binary

convert this to decimal

$2^0+2^1+2^4+2^7+2^8+2^9+2^{11} = 1+2+16+128+256+512+2048$

$= 2048 + 512 = 2560$, $2560 + 256 = 2816$, $2816 + 128 = 3044$, $3044 + 19 = 3063$ 3063

b is 001110110101

convert this to decimal.

$2^0+2^2+2^4+2^5+2^7+2^8+2^9 = 1+4+16+32+128+256+512$ 512 + 256 = 768

21 + 160 + 768   768 + 181 = 949 +2

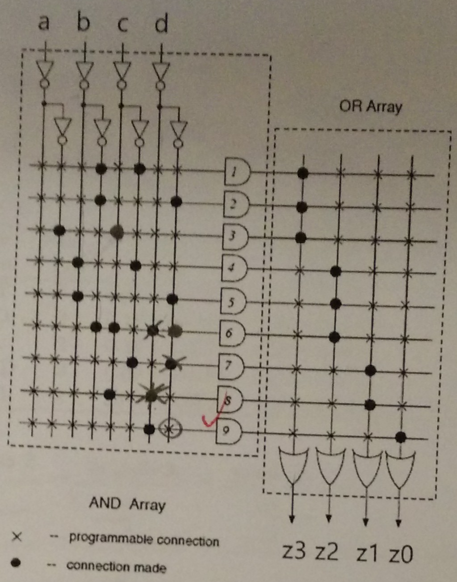
$a+b = 949 + 3063$ X

$= \boxed{4012}$ 3063 + 949 = 4012

Problem 4 (15 points) 15

We would like to verify that the PLA implementation shown here implements the following switching functions:

$$z3 = bc + bd + ac'$$
$$z2 = b'c + b'd + bc'd$$
$$z1 = 1$$
$$z0 = 0$$



1. **(7 points)** Analyze the PLA shown above and show the output expressions.

$z_3 \Rightarrow a + bd + bc$

$z_2 \Rightarrow b'c + b'd + bc'd'$

$z_1 \Rightarrow cd + c'd'$

$z_0 \Rightarrow d'$

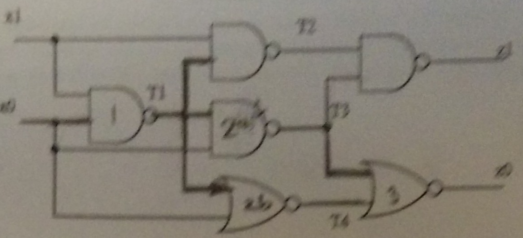
2. **(8 points)** Is the PLA implementation correct? If not, find errors and show the correct implementation (cross out wrong connections and insert correct ones)

No. Change z_0 to $dd' = 0$ z_2-Change $bc'd'$ to $bc'd$

Change z_1 to $c' + c = 1$ 8 z_3: change a to ac'

Problem 5 (10 points) 10

Calculate the propagation delay $t_{pLH}(z0)$ when $x0$ changes. Assume that $z0$'s load value is 2. Fill in the blanks below with the appropriate values. You don't need to fill all the blanks.



1
0.38
× 2
0.76

Gate Type	Fan-in	Propagation Delays (ns)		Load Factor
		t_{pLH}	t_{pHL}	f
NOT	1	$0.02 + 0.038L$	$0.05 + 0.017L$	1.0
AND	2	$0.15 + 0.037L$	$0.16 + 0.017L$	1.0
OR	2	$0.12 + 0.037L$	$0.20 + 0.019L$	1.0
NAND	2	$0.05 + 0.038L$	$0.08 + 0.027L$	1.0
NOR	2	$0.06 + 0.075L$	$0.07 + 0.016L$	1.0

2 NAND 2's is 0.10 + 0.76L which is worse than 1 NOR 2

Gate type & Fan-in	NAND 2 →	NAND 2 →	NOR 2 →	→	→	
LH / HL	LH →	HL →	LH →	→	→	
Output load L	3 →	2 →	2 →	→	→	
Prop. Delay	0.05+0.038(3),	0.08+0.027(2),	0.06+0.075(2),	✓ →	→	

worst path only considered

0.05 + 0.76
0.06 + 0.75

It turns out that taking either 2a or 2b will both result in 0.81, for LH, but for HL, path 2a will produce more load

Problem 6 (15 points)

$c + cd' +$ $\quad$ $c + a'c' +$

$c + a'c'$ $\quad$ $c(d$ $\quad$ 15

A gate G is defined by the following expression

$$E(a,b,c,d) = cd + a'b'c' + a'b'd' + bcd' + ab'c'd$$

$d = 1$

$b = 0$

Show that gate G forms a universal set assuming that constants 1 and 0 are available.

Specify a pre-established universal set you are using in the proof, and explicitly show the implementation for each element in the set using gate G with 1 and/or 0 as needed. For example, you can assign a=0 and b=1 in the expression E.

{AND, NOT}

{OR, NOT}

$a'd'$ $\quad$ $c'd + cd$ $\quad$ d $\quad$ $\cancel{cd} + cd' + c'd$ $\quad$ $\cancel{c + c'd}$

$a = 1$ $\quad$ $b = b$ $\quad$ $c = 0$ $\quad$ $d = 1$

then this becomes...

$E(1, b, 0, 1) = b'$, which is a NOT gate.

Now we find an and or an or

$\cancel{cd} + \cancel{a'b'c'} + \cancel{a'b'd'} + bcd' + \cancel{ab'c'd}$

b $\quad$ ab', b, $a'b'$ a', b'

~~probably done with~~ an or

~~or with~~

~~a NAND.~~

$E(1, b, c, 0)$

produces AND

$c(0) +$

So we have

17

Problem 7 (20 points)

For the switching function $f(x_3, x_2, x_1, x_0)$, we are given the information below for the dc-set and zero-set.

dc-set = (4, 15) 8 4 1

zero-set = zero-set of function

2 5

$(x_3+x_2+x_1'+x_0)(x_3+x_2'+x_1+x_0')(x_3+x_2'+x_1'+x_0)(x_3'+x_2+x_1+x_0')(x_3'+x_2+x_1'+x_0)(x_3'+x_2'+x_1+x_0')$

6 9 10 13

1. **(2 points)** Fill out the following K-map.

x_0 (columns 2–3), x_1 (columns 3–4), x_2 (rows 2–3), x_3 (rows 3–4)

1	1	1	0
–	0	1	0
1	0	–	1
1	0	1	0

$x_3'x_2'x_0$

$x_3'x_2'x_1'$

$x_3x_2x_1$

2. **(4 points)** Which of the given expressions are prime implicants of the function given above? Circle **all** that apply. Do not circle implicants that are not prime.

~~(a)~~ x_3x_1 ~~(b)~~ x_3x_2' ~~(c)~~ $x_3'x_1$

$\boxed{\text{(d) } x_1'x_0'}$ $\boxed{\text{(e) } x_1x_0}$ $\boxed{\text{(f) } x_3'x_2'x_1'}$

$\boxed{\text{(g) } x_3'x_2'x_0}$ $\boxed{\text{(h) } x_3x_2x_1}$ ~~(i)~~ $x_3x_2x_0'$

~~(j)~~ $x_3'x_2x_1'x_0$ ~~(k)~~ $x_3x_2'x_1x_0$ ~~(l)~~ $x_3'x_2x_1x_0'$

−1

3. **(3 points)** Write down the complete set of **essential** prime implicants.

$x_3x_2x_1$, $x_1'x_0'$, x_1x_0

−1

4. **(2 points)** Write down the minimal sum of products expressions for f. If there are multiple forms of minimal sum of products expressions, you only need to write down one of them.

$x_3x_2x_1 + x_1'x_0' + x_1x_0 + x_3'x_2'x_0$ ✓

5. **(4 points)** Which of the given expressions are prime implicates of the function given above? Circle **all** that apply. Do not circle implicates that are not prime.

(a) $(x_3 + x_2' + x_1)$
(b) $(x_3' + x_2')$
(c) $\boxed{(x_2' + x_1 + x_0')}$
(d) $\boxed{(x_3' + x_1 + x_0')}$
(e) $(x_3 + x_2' + x_0)$
(f) $\boxed{(x_3 + x_1' + x_0)}$
(g) $(x_2 + x_1' + x_0)$
(h) $(x_3 + x_1 + x_0)$
(i) $(x_3' + x_2' + x_1)$
(j) $(x_3' + x_1')$
(k) $(x_3 + x_2 + x_1 + x_0')$
(l) $(x_3 + x_2' + x_1' + x_0)$

6. **(3 points)** Write down the complete set of **essential** prime implicates.

There is an epi that was not in the above.

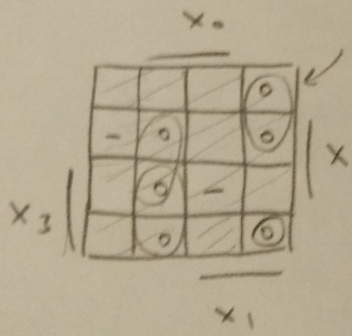
$x_2' + x_1 + x_0' \quad x_3' + x_1 + x_0' \quad x_3' + x_2 + x_1' + x_0 \quad x_3 + x_1' + x_0$

7. **(2 points)** Write down the minimal product of sums expressions for f. If there are multiple forms of minimal product of sums expressions, you only need to write down one of them.

The minimal POS is all the EPI's

$(x_2' + x_1 + x_0')(x_3' + x_1 + x_0')(x_3' + x_2 + x_1' + x_0)(x_3 + x_1' + x_0)$

$x_3' + x_2 + x_1' + x_0$



$x_3 + x_1' + x_0$

$x_1 + x_2' + x_0'$

$x_1 + x_3' + x_0'$