[CS M51A Fall 15] Midterm exam

Date: 11/3/15

- The midterm is closed books and notes. Tablets and smartphone are not allowed.
- You can use calculators and have up to 2 sheets (= 4 pages) of summary notes.
- Please show all your work and write legibly, otherwise no partial credit will be given.
- **This should strictly be your own work; any form of collaboration will be penalized.**

Name : Max Chern Dis 2B

Student ID : ______

Problem	Points	Score
1	15	14
2	15	8
3	10	3
4	15	15
5	10	10
6	15	15
7	20	18
Total	100	83

Problem 1 (15 points)

1. (6 points) Given the following simplification of a boolean expression, identify all right and wrong steps and briefly explain what is wrong for each error.
(For example, (10) → (11) wrong application of the Identity rule, (11) → (12) correct)

$$
\begin{aligned}
E_1(w,x,y,z) &= (((w+x+x'y')y+z)'+wx'+y')' && (1)\\
&= ((w+x+x'y')'y'z'+wx'+y')' && (2)\\
&= ((w+x+x'y')'y'z'+(w+y')(x'+y'))' && (3)\\
&= ((w+x'+y')'y'z'+(w+y')(x'+y'))' && (4)\\
&= (w'xyy'z'+(w+y')(x'+y'))' && (5)\\
&= (0+(w+y')(x'+y'))' && (6)\\
&= wy+xy && (7)
\end{aligned}
$$

(1) → (2) wrong application of DeMorgan's Law

X (2) → (3) wrong application of Distributivity

(3) → (4) wrong application of Simplification

(4) → (5) correct

(5) → (6) correct

(6) → (7) wrong application of DeMorgan's Law

−1

2. (5 points) Obtain the minimal sum of products form for $E_2(w, x, y, z)$ using the identities of Boolean algebra. Show all the steps in your derivation.

$$E_2 = xy' + xzw + yw$$

$= xy' + xzw(y+y') + yw$ ($= 1$)

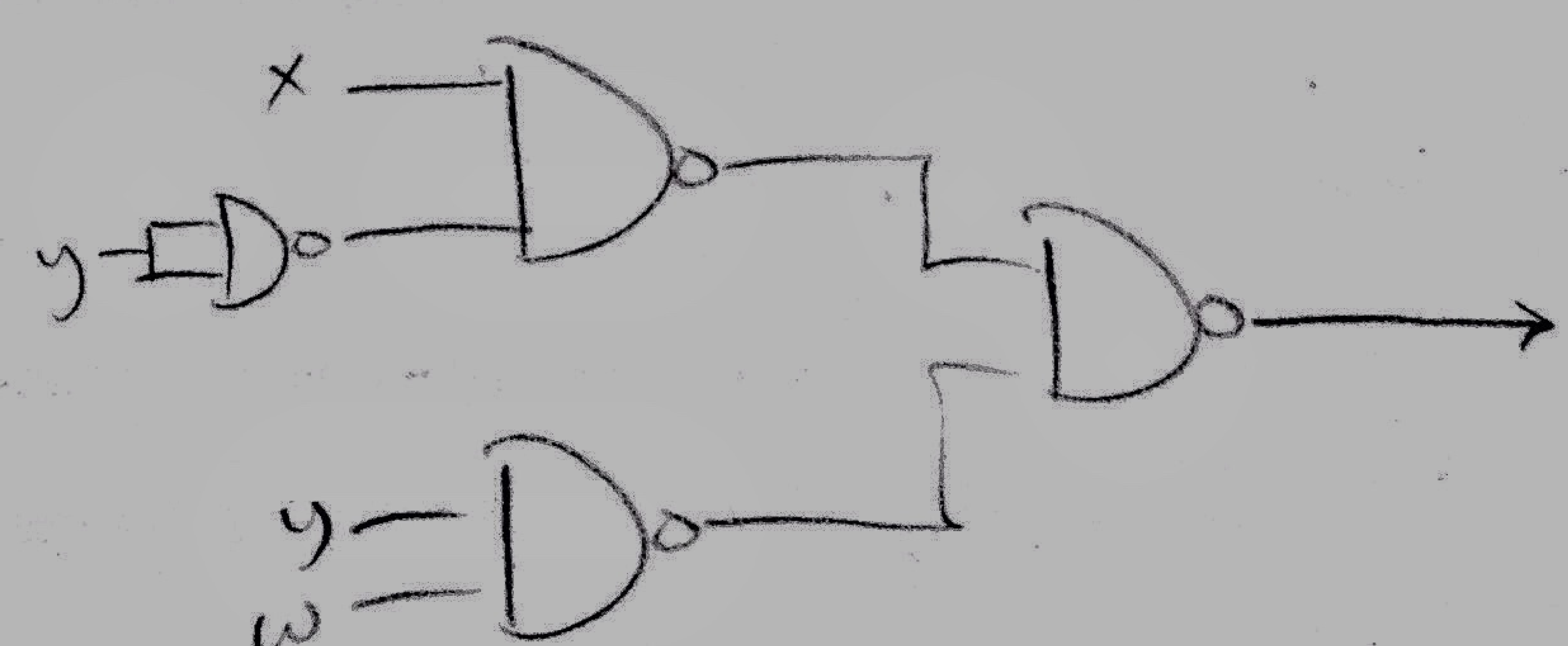
$= xy' + xzwy + xzwy' + yw$

$= [xy' + xy'wz] + [yw + ywxz]$ $a + ab = a$

$= xy' + yw$

✓

3. (4 points) Using the expression obtained for E_2 from the previous step, obtain the NAND network that uses ONLY NAND gates. Inverted inputs are not available, and no constant inputs are allowed.



✓

$(x_1 x_2)'$

11 →	0
00 →	1
10 →	1
01 →	1

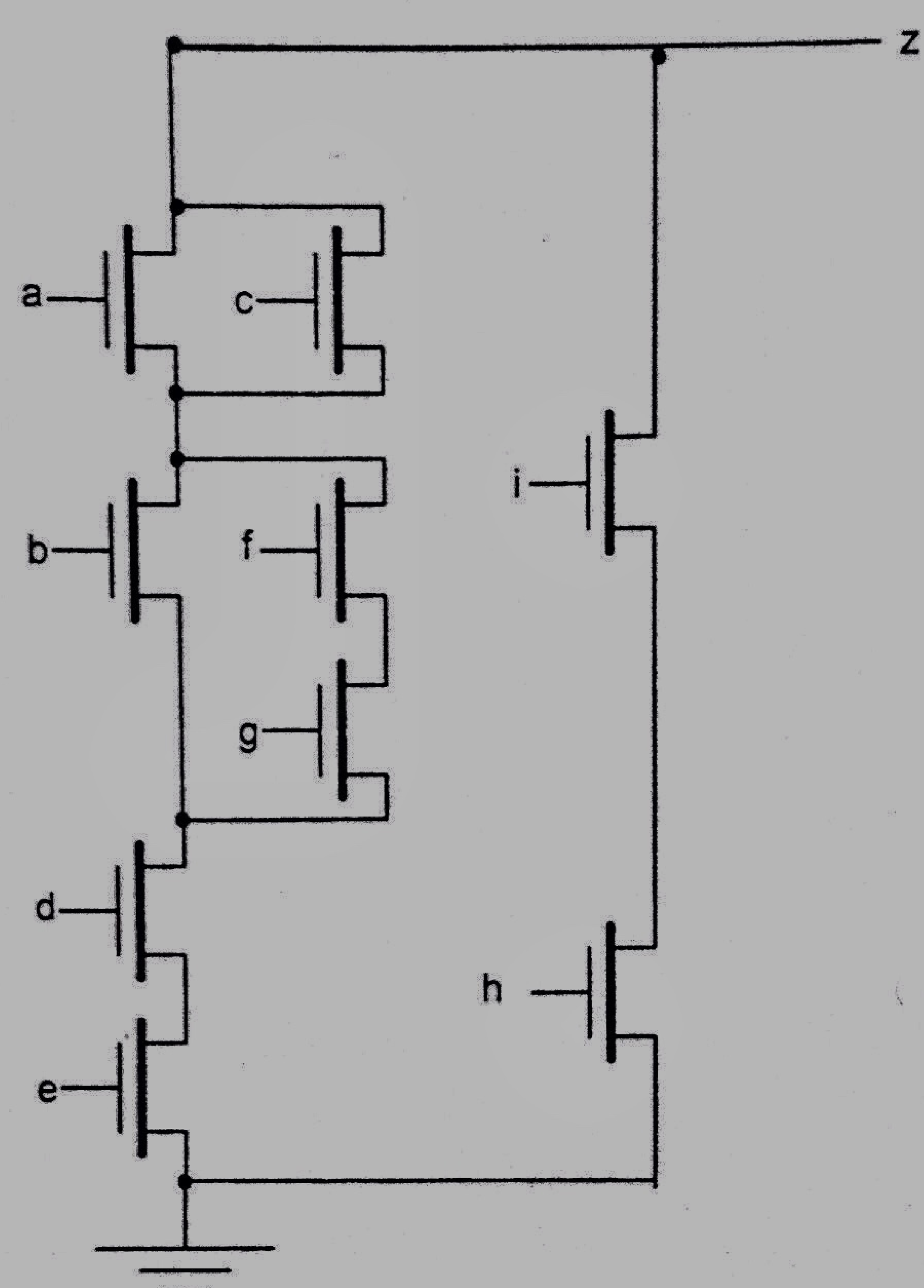
y	y	NAND
1	1	0
0	0	1

$((xy')'(yw)')'$

$= (xy') + (yw)$

Problem 2 (15 points)

The following pull-down network is part of a complex CMOS gate that we want to implement.



1. **(8 points)** (a) Write the expression for the pull-down network.

Pull-down network expression : z' = $[(a+c)(b+fg)de] + hi$

✓

(b) Obtain the expression for the corresponding pull-up network.

Pull-up network expression : z = $[a'c' + b'(f'+g') + d'+e'][h'+i']$

$z = \left([(a+c)(b+fg)\,de] + hi \right)'$ ✓

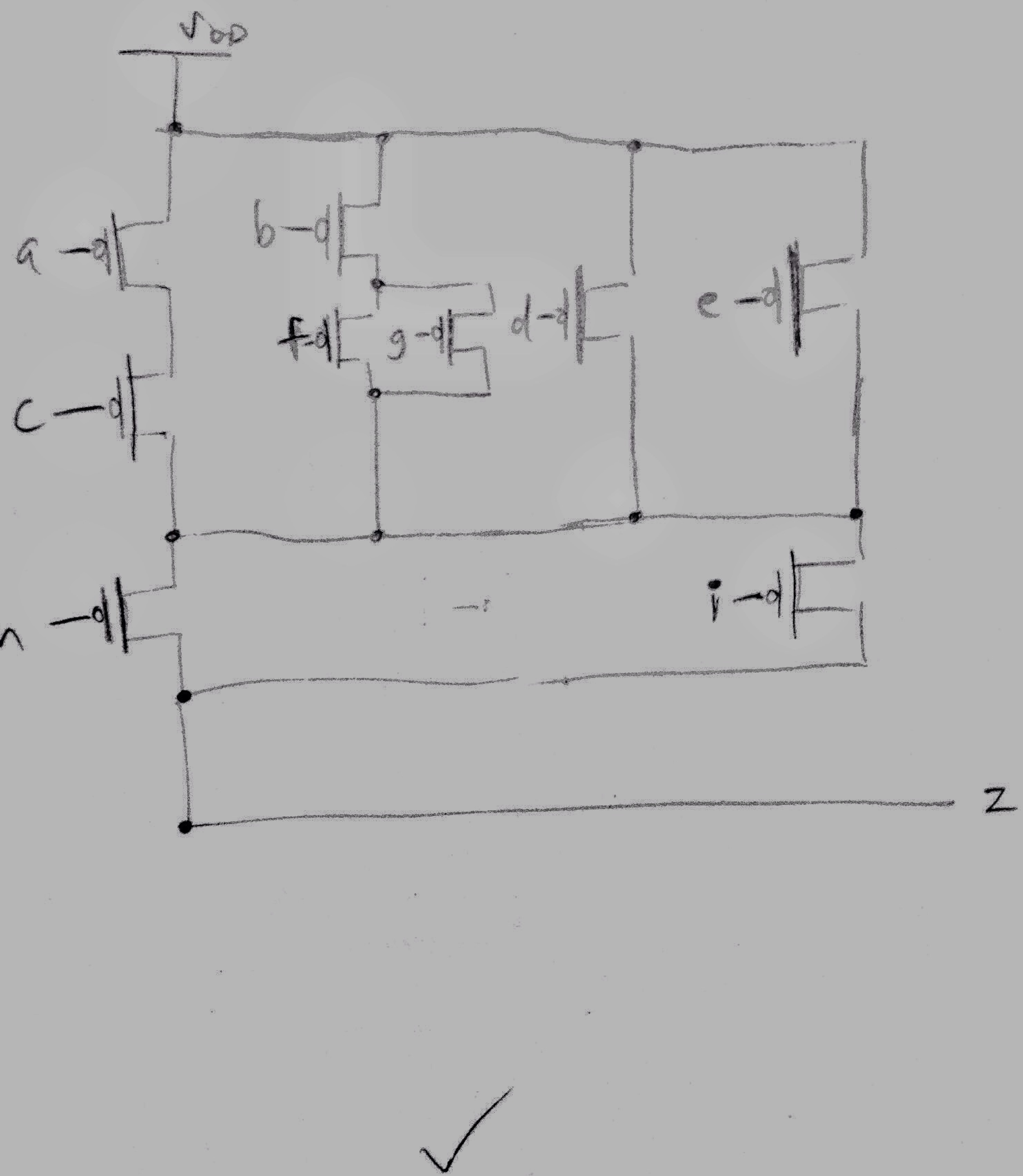
$= [(a+c)(b+fg)de]'(hi)'$

$= [(a+c)' + (b+fg)' + d' + e'][h' + i']$

$= [a'c' + b'(fg)' + d' + e'][h' + i']$

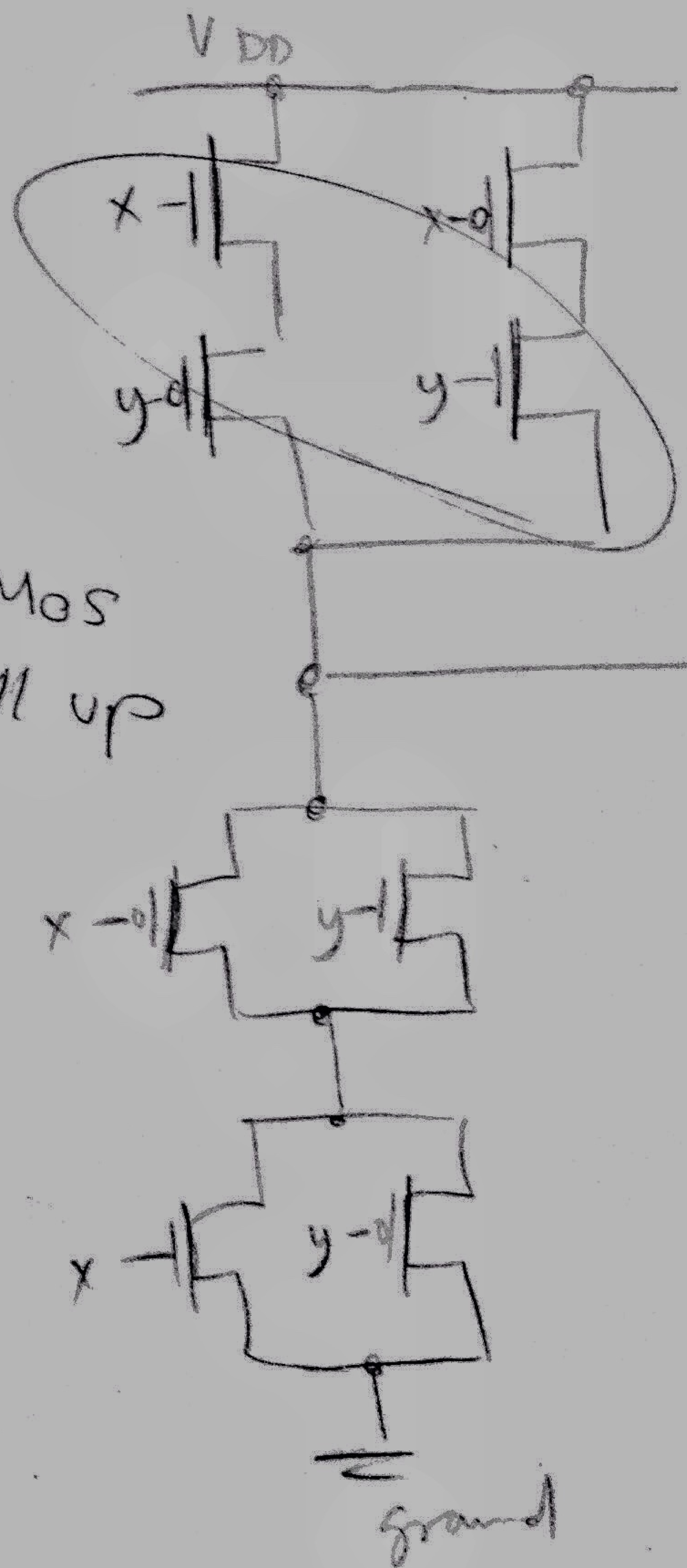
$= (a'c' + b'(f'+g') + d' + e')(h' + i')$

(c) Draw the pull-up network.

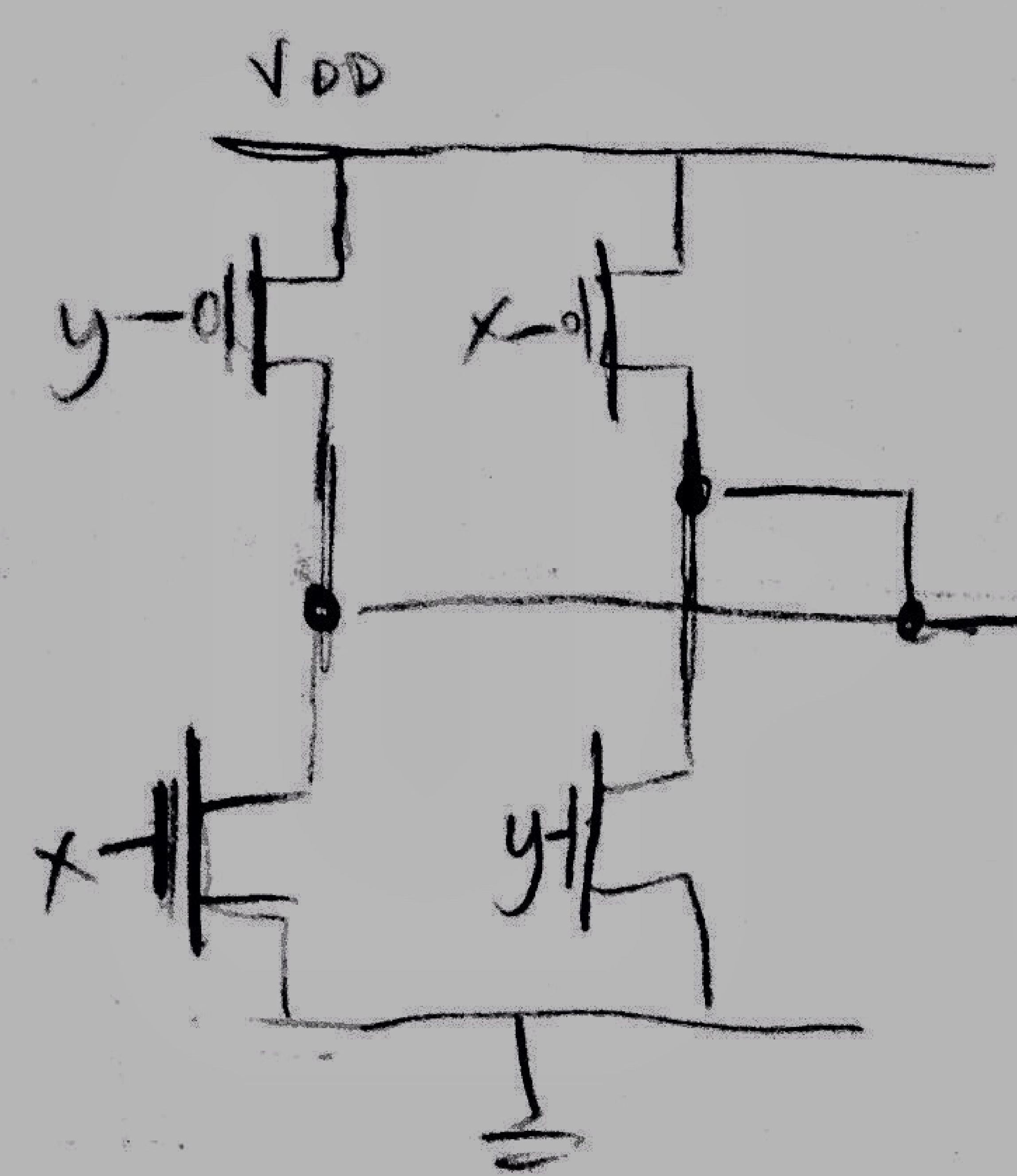


2. **(7 points)** Draw a CMOS network that implements f(x,y) = xy'+x'y (2-input XOR). x' and y' are not available as inputs. Do not use transmission gates. How many transistors does your solution have? OPTIONAL: If you find another solution with fewer transistors, you get 3 extra points. Show your reduced design and explain why it works.

Wrong
No NMOS in Pull up



I don't know if this is right :(



Fewer Transistors

Bad wiring, sorry!

Problem 3 (10 points) 3

1. (5 points) A 12-bit vector represents a set of positive integers {0,...,N}. Which of the following coding alternatives

(a) BCD

(b) 2421 code (a decimal code)

(c) Excess-3 code (a decimal code)

(d) Octal

(e) Binary

provides the largest range? Why? (Give N for each case).

(a) BCD
each 4 bits has range 16, 0-15
so 12 bit range is $16^3 = 4096$, N=4095

(b) 2421 code
each 4 bits has range 9
so 12 bit range is $9^3 = 729$, N = 728

(c) Excess-3 code
each 4 bits has range 16 still, N=4095 X

(d) Octal
each 4 bits has range 4096, 0-4095
so 12 bit range is 4096^3, $N = 4096^3 - 1$ X

(e) binary
same as BCD X
N = 4095 ✓ +1

Octal has largest range because each bit has a range of 0-7

2. (5 points) Let $a = (101110010110)$ and $b = (001110110101)$. If a represents a number in the Excess-3 code and b in the binary code, what is the value in decimal of their sum $a + b$? Show all your work.

b = 949 +2 5+11(16) + 3(256)

$a = 3 + 6(16) + 8(256)$

= 2147 X

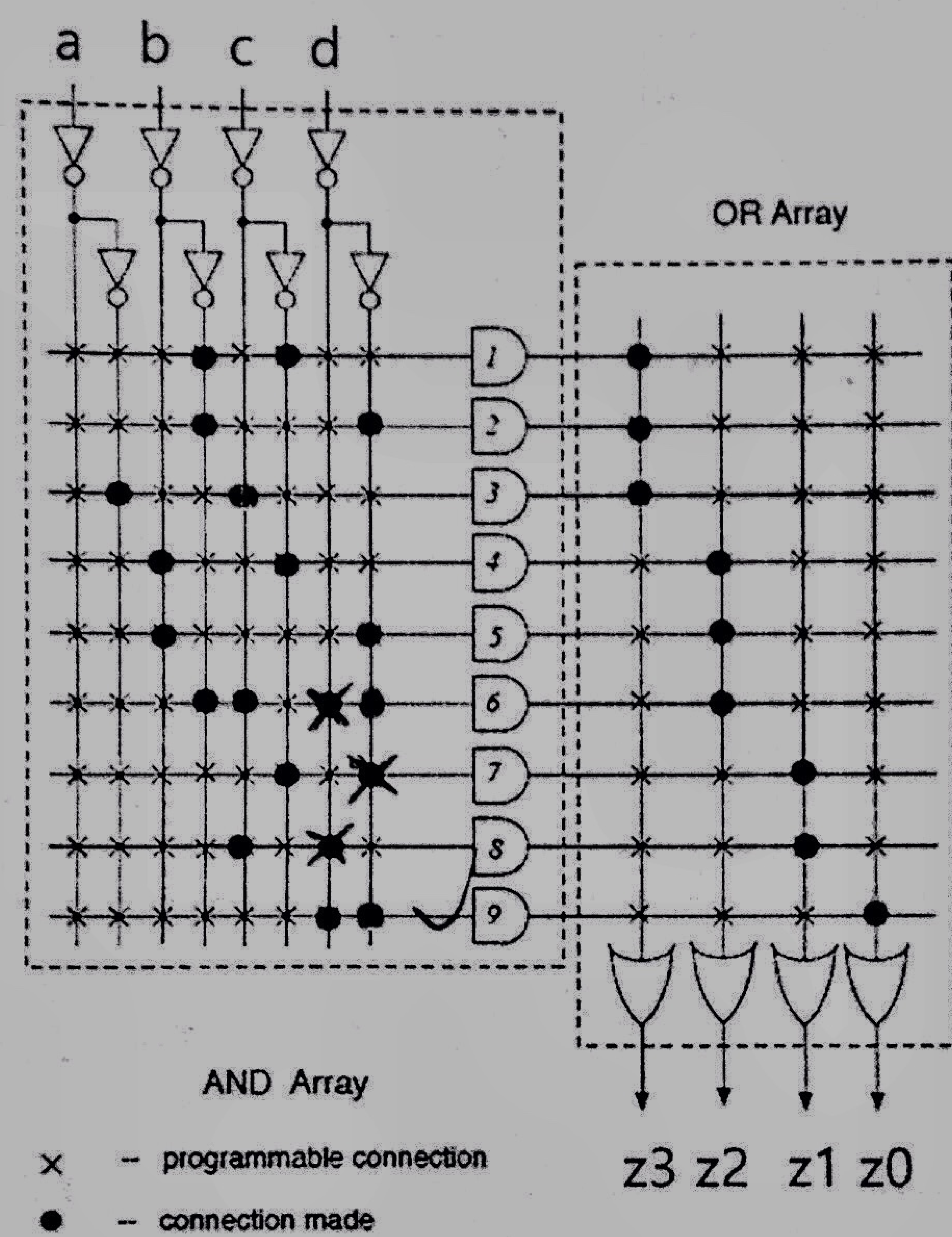
$a + b = 949 + 2147 = 3096$

Problem 4 (15 points)

15

We would like to verify that the PLA implementation shown here implements the following switching functions:

$$z3 = bc + bd + ac'$$
$$z2 = b'c + b'd + bc'd$$
$$z1 = 1$$
$$z0 = 0$$



1. **(7 points)** Analyze the PLA shown above and show the output expressions.

$z_3 = bc + bd + a$

$z_2 = b'c + b'd + bc'd'$

$z_1 = cd + c'd'$ ✓

$z_0 = d$

2. **(8 points)** Is the PLA implementation correct? If not, find errors and show the correct implementation (cross out wrong connections and insert correct ones)

Not Correct!

Problem 5 (10 points)

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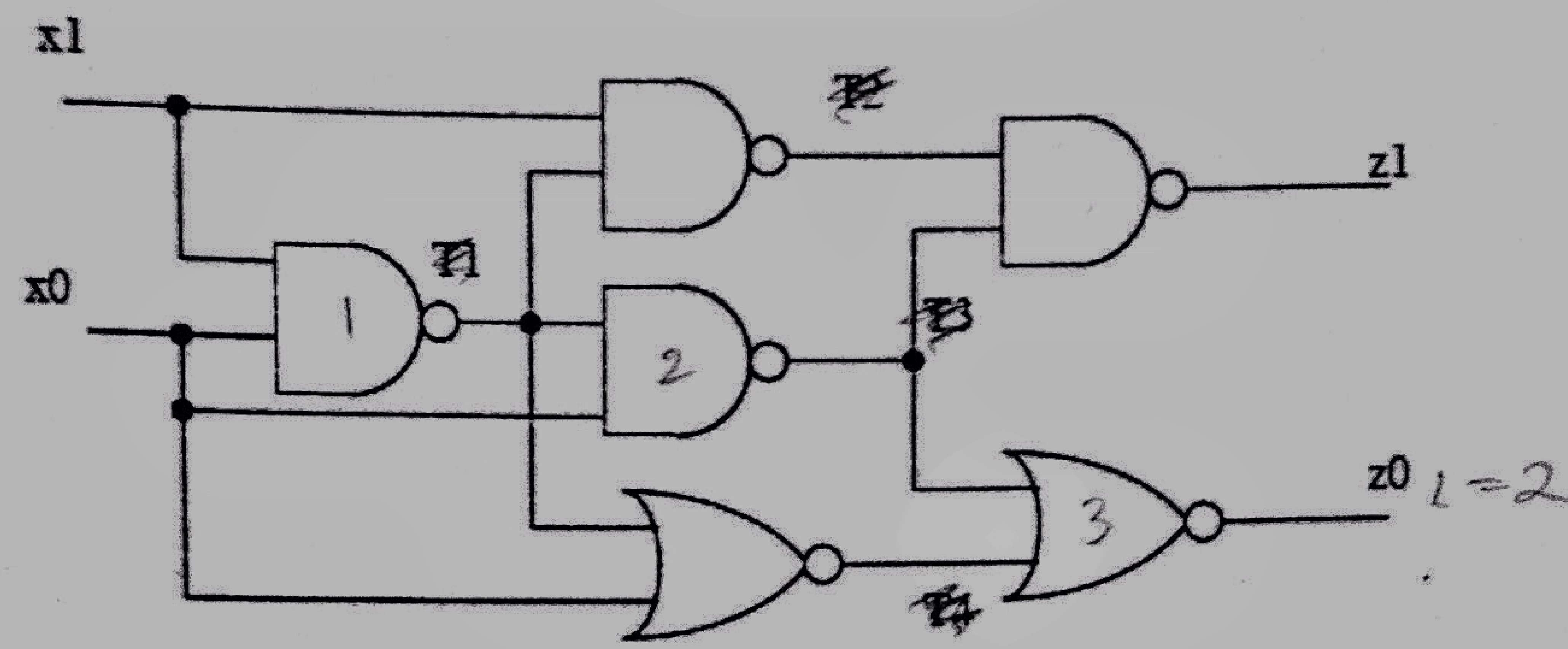
Calculate the propagation delay $t_{pLH}(z0)$ when $x0$ changes. Assume that $z0$'s load value is 2. Fill in the blanks below with the appropriate values. You don't need to fill all the blanks.



Gate Type	Fan-in	Propagation Delays (ns) t_{pLH}	t_{pHL}	Load Factor I
NOT	1	$0.02 + 0.038L$	$0.05 + 0.017L$	1.0
AND	2	$0.15 + 0.037L$	$0.16 + 0.017L$	1.0
OR	2	$0.12 + 0.037L$	$0.20 + 0.019L$	1.0
NAND	2	$0.05 + 0.038L$	$0.08 + 0.027L$	1.0
NOR	2	$0.06 + 0.075L$	$0.07 + 0.016L$	1.0

Gate type & Fan-in	NAND 2 →	NAND 2 →	NOR 2 →	→	→	
LH / HL	LH →	HL →	LH →	→	→	
Output load L	3.0 →	2.0 →	2.0 →	→	→	
Prop. Delay	0.05 + 0.038 (3.0) ns →	0.08 + 0.027 (2.0) ns →	0.06 + 0.075 (2.0) → ✓	→	→	

Problem 6 (15 points)

A gate G is defined by the following expression

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$$E(a,b,c,d) = cd + a'b'c' + a'b'd' + bcd' + ab'c'd$$

Show that gate G forms a universal set assuming that constants 1 and 0 are available.

Specify a pre-established universal set you are using in the proof, and explicitly show the implementation for each element in the set using gate G with 1 and/or 0 as needed. For example, you can assign a=0 and b=1 in the expression E.

c = 0, d = 0

E(a,b) = a'b' + a'b' = a'b' = (a+b)'

NOR Gate ! → E(a,b,0,0) = NOR(a,b)

{NOR} is a universal set

Problem 7 (20 points)

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For the switching function $f(x_3, x_2, x_1, x_0)$, we are given the information below for the dc-set and zero-set.

$$\text{dc-set} = (4, 15)$$

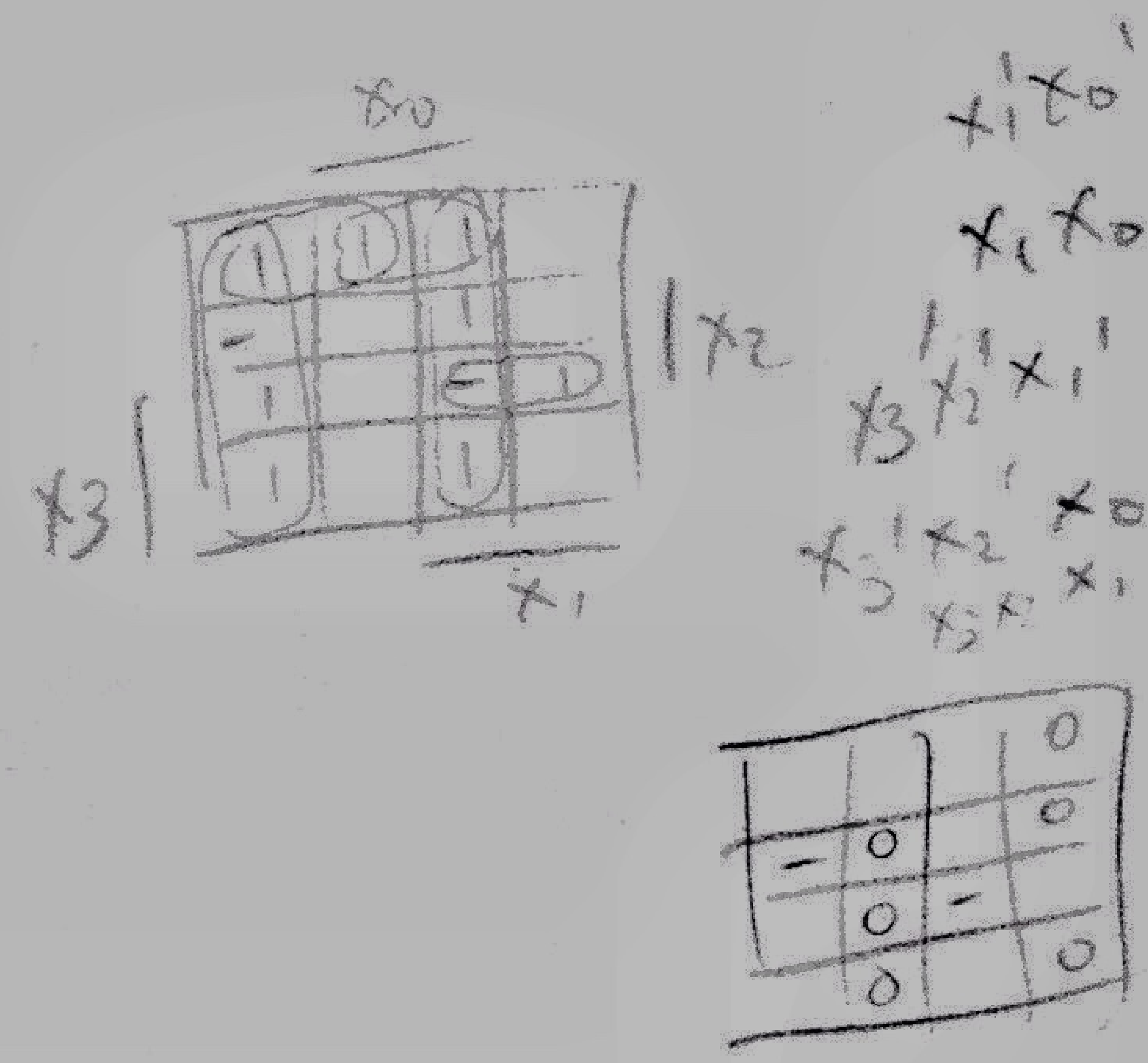
$$\text{zero-set} = \text{zero-set of function}$$

$$(x_3+x_2+x_1'+x_0)(x_3+x_2'+x_1+x_0')(x_3+x_2'+x_1'+x_0)(x_3'+x_2+x_1+x_0')(x_3'+x_2+x_1'+x_0)(x_3'+x_2'+x_1+x_0')$$

1. **(2 points)** Fill out the following K-map.

K-map: x_0 marks the 2nd and 3rd columns, x_1 marks the 3rd and 4th columns, x_2 marks the 2nd and 3rd rows, and x_3 marks the 3rd and 4th rows.

1	1	1	0
–	0	1	0
1	0	–	1
1	0	1	0



2. **(4 points)** Which of the given expressions are prime implicants of the function given above? Circle **all** that apply. Do not circle implicants that are not prime.

(a) x_3x_1
(b) x_3x_2'
(c) $x_3'x_1$
(d) $x_1'x_0'$
(e) x_1x_0
(f) $x_3'x_2'x_1'$
(g) $x_3'x_2'x_0$
(h) $x_3x_2x_1$
(i) $x_3x_2x_0'$
(j) $x_3'x_2x_1'x_0$
(k) $x_3x_2'x_1x_0$
(l) $x_3'x_2x_1x_0'$

–1

3. **(3 points)** Write down the complete set of **essential** prime implicants.

$x_1'x_0'$, x_1x_0, $x_3x_2x_1$

–1

4. **(2 points)** Write down the minimal sum of products expressions for f. If there are multiple forms of minimal sum of products expressions, you only need to write down one of them.

$x_1'x_0' + x_1x_0 + x_3x_2x_1 + x_3'x_2'x_1'$

5. **(4 points)** Which of the given expressions are prime implicates of the function given above? Circle **all** that apply. Do not circle implicates that are not prime.

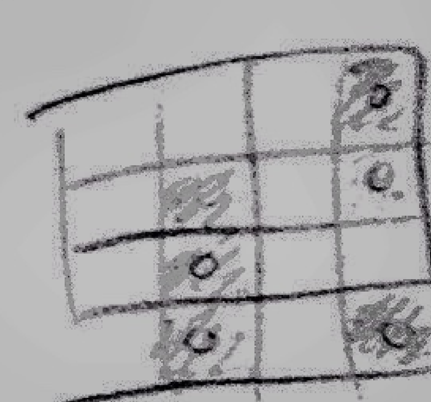
(a) $(x_3 + x_2' + x_1)$
(b) $(x_3' + x_2')$
(c) $(x_2' + x_1 + x_0')$
(d) $(x_3' + x_1 + x_0')$
(e) $(x_3 + x_2' + x_0)$
(f) $(x_3 + x_1' + x_0)$
(g) $(x_2 + x_1' + x_0)$
(h) $(x_3 + x_1 + x_0)$
(i) $(x_3' + x_2' + x_1)$
(j) $(x_3' + x_1')$
(k) $(x_3 + x_2 + x_1 + x_0')$
(l) $(x_3 + x_2' + x_1' + x_0)$

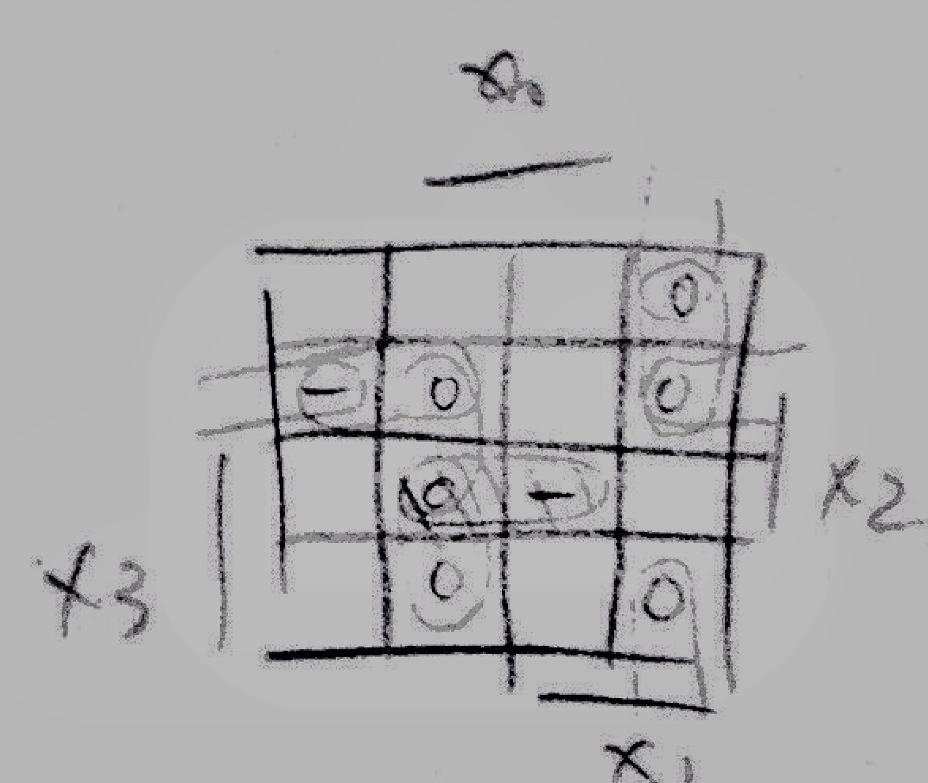
6. **(3 points)** Write down the complete set of **essential** prime implicates.

$X_2 + X_1' + X_0$, $X_3' + X_1 + X_0'$ ✓

7. **(2 points)** Write down the minimal product of sums expressions for f. If there are multiple forms of minimal product of sums expressions, you only need to write down one of them.

$(X_2 + X_1' + X_0)(X_3' + X_1 + X_0')(X_2' + X_1 + X_0')(X_3 + X_2' + X_0)$ ✓





$X_3 + X_1' + X_0$

$X_2 + X_1' + X_0$

$X_3 + X_2' + X_0$

$X_3 + X_2' + X_1$

$X_2' + X_1 + X_0'$

~~$X_3' + X_2' + X_1'$~~

$X_3' + X_1 + X_0'$