[CS M51A Fall 15] Midterm exam

Date: 11/3/15

- The midterm is closed books and notes. Tablets and smartphone are not allowed.
- You can use calculators and have up to 2 sheets (= 4 pages) of summary notes.
- Please show all your work and write legibly, otherwise no partial credit will be given.
- **This should strictly be your own work; any form of collaboration will be penalized.**

Name : Mark Tai

Student ID : 504399847

2C

Problem	Points	Score
1	15	15
2	15	15
3	10	7
4	15	12
5	10	10
6	15	15
7	20	20
Total	100	94

Problem 1 (15 points)

1. (6 points) Given the following simplification of a boolean expression, identify all right and wrong steps and briefly explain what is wrong for each error.
   (For example, (10) → (11) wrong application of the Identity rule, (11) → (12) correct)

$$
\begin{aligned}
E_1(w,x,y,z) &= (((w+x+x'y')y+z)' + wx' + y')' && (1)\\
&= ((w+x+x'y')'y'z' + wx' + y')' && (2)\\
&= ((w+x+x'y')'y'z' + (w+y')(x'+y'))' && (3)\\
&= ((w+x'+y')'y'z' + (w+y')(x'+y'))' && (4)\\
&= (w'xyy'z' + (w+y')(x'+y'))' && (5)\\
&= (0 + (w+y')(x'+y'))' && (6)\\
&= wy + xy && (7)
\end{aligned}
$$

$(((w+x+x'y')y+z)' + wx' + y')'$

1→2 $((w+x+x'y')' + y'$

$w + y'$

$w'y +$

6→7 $((w+y')(x'+y'))'$

(1)→(2) should be $(w+y')' + (x'+y')'$

$(((w+x+x'y')' + y')z') + wx' + y')'$ $w'y + xy$

(3)→(4), should be $((w+x+y')'y'z' + (w+y')(x'+y'))'$

(6)→(7) should be $w'y + xy$

- (1)→(2) Wrong application of De Morgan's Law
- (2)→(3) Correct
- (3)→(4) Wrong application of $x + x'y = x + y$
- (4)→(5) Correct
- (5)→(6) Correct
- (6)→(7) Wrong application of De Morgan's law

2. (5 points) Obtain the minimal sum of products form for $E_2(w, x, y, z)$ using the identities of Boolean algebra. Show all the steps in your derivation.

$$E_2 = xy' + xzw + yw$$

$= x(y' + zw) + (x+1)yw$

$= x(y' + yw + zw) + yw$

$= x(y' + w + zw) + yw$

$= x(y' + w) + yw$

$= xy' + xw + yw$

$= xy' + xw(y + y') + yw$

$= xy' + xwy + xwy' + yw$

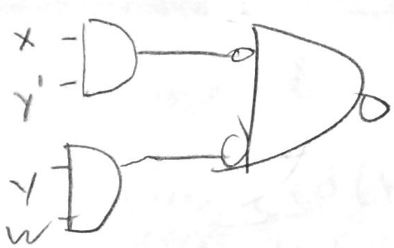
$= \boxed{xy' + yw}$

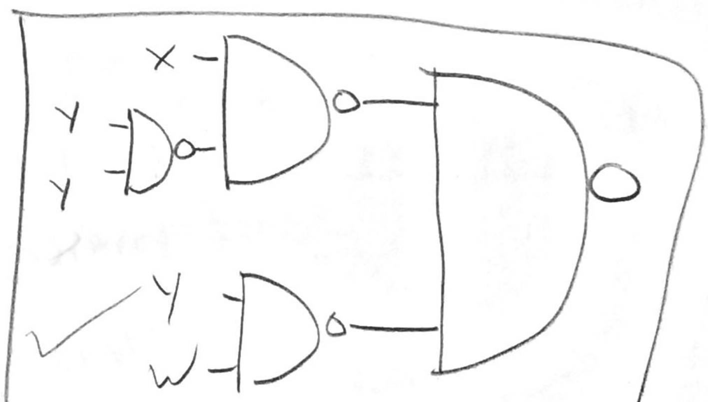
w x y z	E_2
0 0 0 0	0
0 0 0 1	0
0 0 1 0	0
0 0 1 1	0
0 1 0 0	1
0 1 0 1	1
0 1 1 0	0
0 1 1 1	0
1 0 0 0	0
1 0 0 1	0
1 0 1 0	1
1 0 1 1	1
1 1 0 0	1
1 1 0 1	1
1 1 1 0	1
1 1 1 1	1

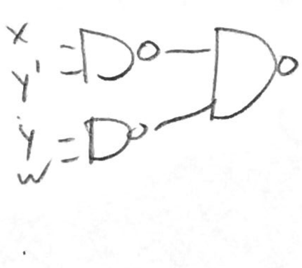
$xy' + yw$

3. (4 points) Using the expression obtained for E_2 from the previous step, obtain the NAND network that uses ONLY NAND gates. Inverted inputs are not available, and no constant inputs are allowed.

a NAND a = a'

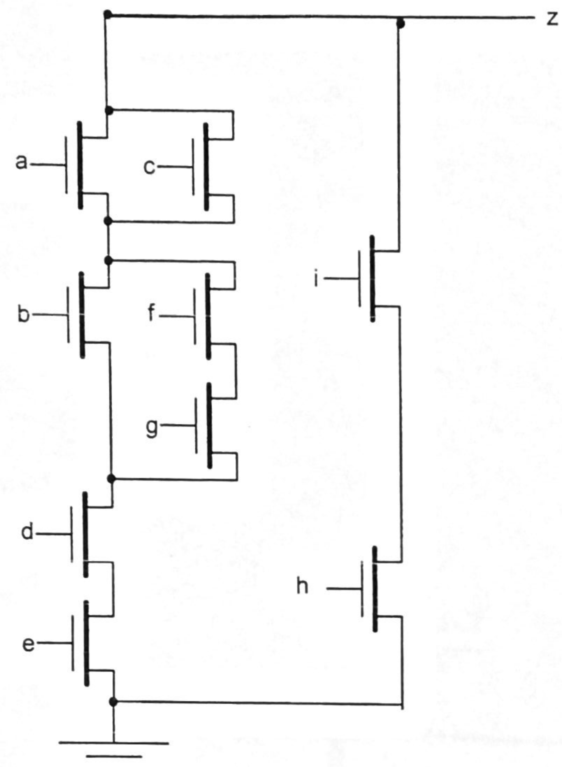






Problem 2 (15 points)

The following pull-down network is part of a complex CMOS gate that we want to implement.



1. **(8 points)** (a) Write the expression for the pull-down network.

   Pull-down network expression : z' = $hi + ed(b+gf)(a+c)$

   (b) Obtain the expression for the corresponding pull-up network.

   Pull-up network expression : z = $(h'+i')(e'+d' + b'(g'+f') + a'c')$

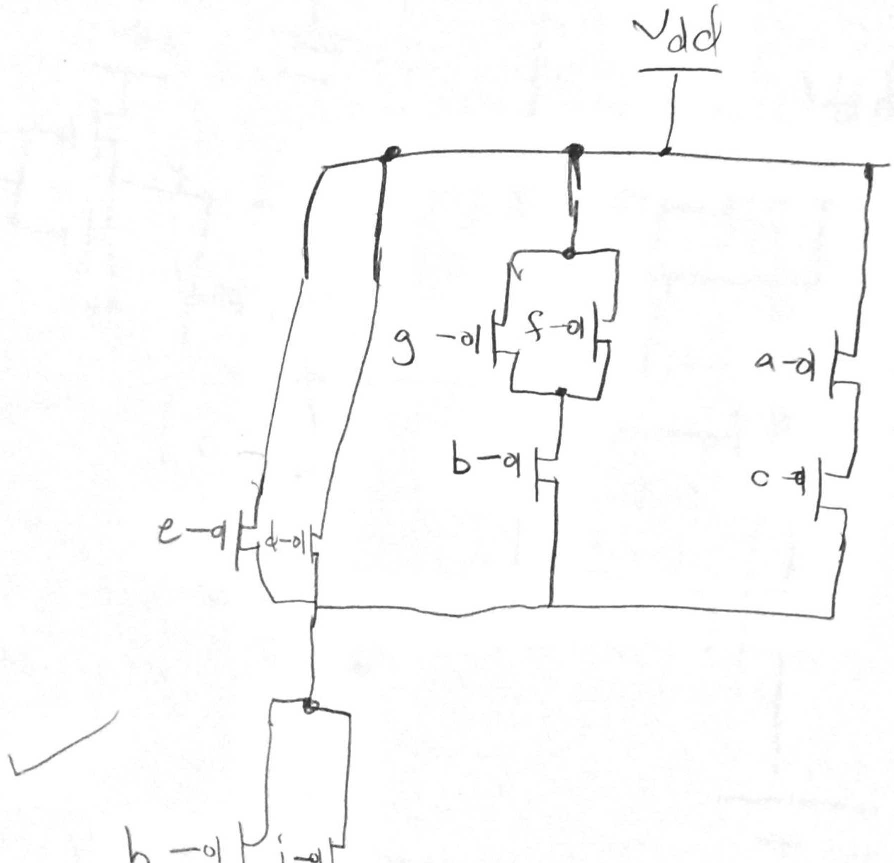
   $(hi + ed(b+gf)(a+c))'$

   $(hi)'(ed(b+gf)(a+c))'$

   $(h'+i')(e'+d' + (b+gf)' + (a+c)')$

   $(h'+i')(e'+d' + b'(g'+f') + (a'c'))$

(c) Draw the pull-up network.

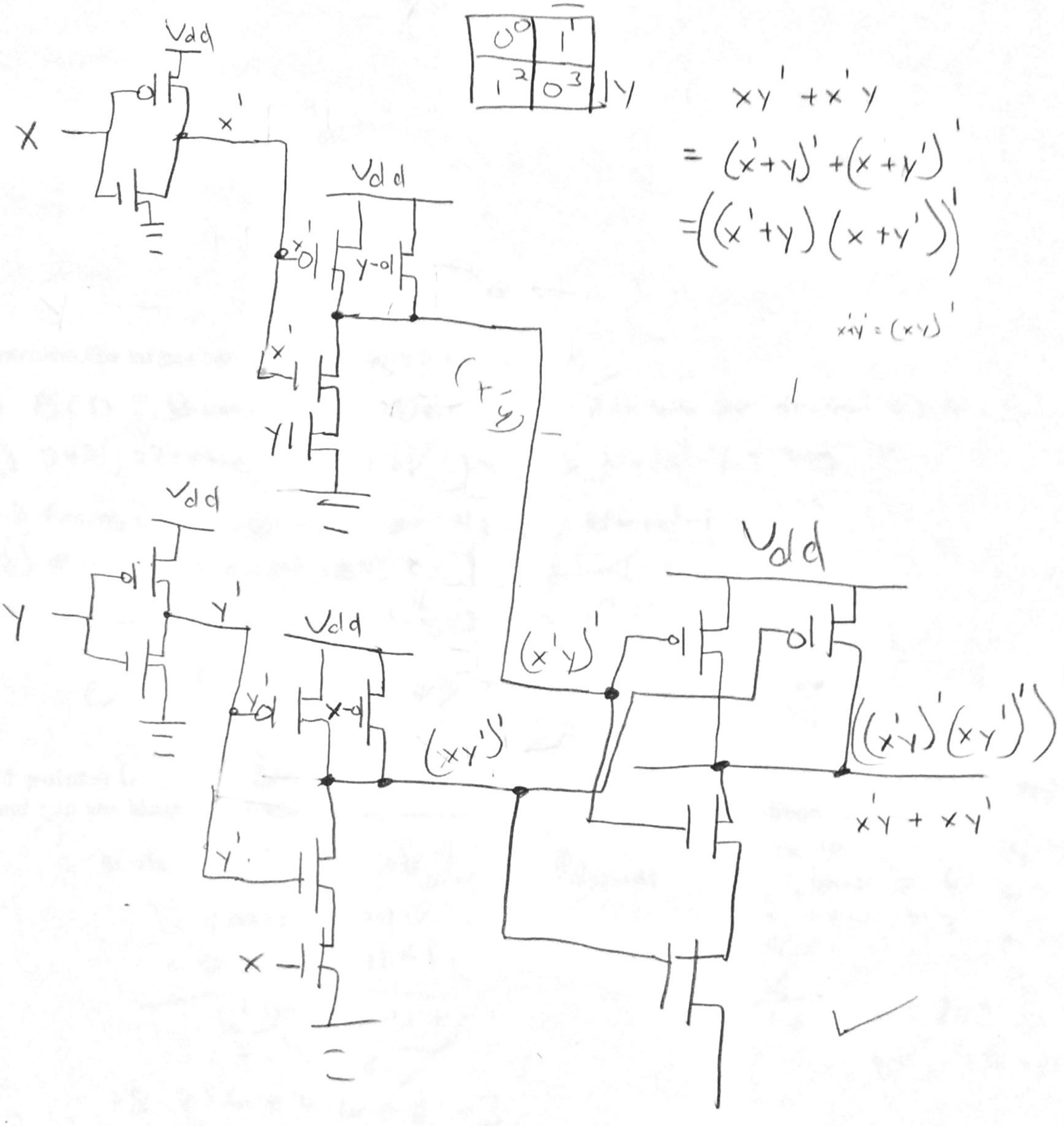


2. **(7 points)** Draw a CMOS network that implements f(x,y) = xy'+x'y (2-input XOR). x' and y' are not available as inputs. Do not use transmission gates. How many transistors does your solution have? OPTIONAL: If you find another solution with fewer transistors, you get 3 extra points. Show your reduced design and explain why it works.

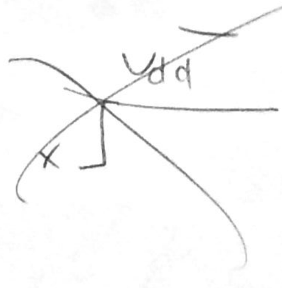
	x = 0	x = 1
y = 0	0	1
y = 1	1	0

$$xy' + x'y$$
$$= (x'+y)' + (x+y')'$$
$$= ((x'+y)(x+y'))'$$

$x'+y' = (xy)'$



16 transistors



Problem 3 (10 points) 7

1. **(5 points)** A 12-bit vector represents a set of positive integers {0,...,N}. Which of the following coding alternatives

   (a) BCD

   (b) 2421 code (a decimal code)

   (c) Excess-3 code (a decimal code)

   (d) Octal

   (e) Binary

   provides the largest range? Why? (Give N for each case).

a) BCD = Binary Coded Decimal => 4 bits per decimal => $N = 10^3 - 1 = 999$

b) 2421 => range of 10 per 4 bits => $N = 10^3 - 1 = 999$

c) Excess-3 => range of 10 per 4 bits => $N = 10^3 - 1 = 999$

d) Octal => 4 octal bits => $N = 8^4 - 1 = (2^3)^4 - 1 = 2^{12} - 1 = 4095$

e) Binary => 12 binary bids => $N = 2^{12} - 1 = 4095$ ✓

Octal and binary have the largest range. They don't waste any space for decimal

2. **(5 points)** Let $a = (101110010110)$ and $b = (001110110101)$. If a represents a number in the Excess-3 code and b in the binary code, what is the value in decimal of their sum $a + b$? Show all your work.

2

$a + b = a_{binary} + b_{binary} - 3_{decimal}$

```
  101110010110
+ 001110110101
--------------
  111101001011
   F   4   B
```

$15 \cdot 256 + 4 \cdot 16 + 11 - 3$

$3840 + 64 + 11 - 3$

$3904 + 11 - 3$

$\boxed{3912}$

$a_2 = 1011 = 8$ $\quad b_2 = 0011 = 3$

$a_1 = 1001 = 6$ $\quad b_1 = 1011 = B = 11$

$a_0 = 0110 = 3$ $\quad b_0 = 0101 = 5$

$256 \times 15 = 1280 + 2560 = 3840$

$a = 863$ ✓ +2

$b = 3 \cdot 256 + 4 \cdot 11 + 5$ ✗

$a + b = 863 + 3 \cdot 256 + 4 \cdot 11 + 5$

$= 863 + 768 + 44 + 5$

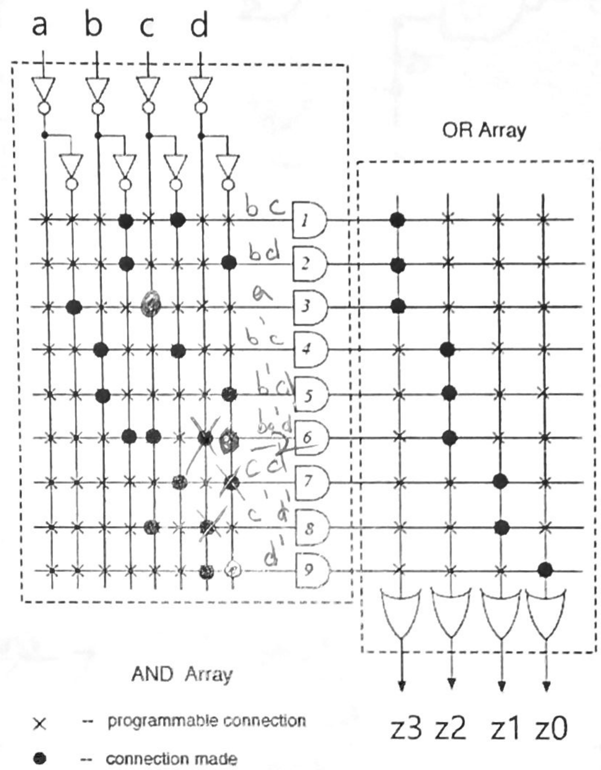
$= 1631 + 44 + 5$

$= \boxed{1680}$

Problem 4 (15 points) 12

We would like to verify that the PLA implementation shown here implements the following switching functions:

$$z3 = bc + bd + ac'$$
$$z2 = b'c + b'd + bc'd$$
$$z1 = 1$$
$$z0 = 0$$



correct→ac'

correct→x

correct→c'

correct→dd'

6

1. **(7 points)** Analyze the PLA shown above and show the output expressions.

$z_3 = bc + bd + a$

$z_2 = b'c + b'd + \underline{bc'd}$ −1

$z_1 = cd + c'd'$ / $(cd)' = c' + d'$

$z_1 = cd + c' + d'$

$z_0 = d'$ ✓

6

2. **(8 points)** Is the PLA implementation correct? If not, find errors and show the correct implementation (cross out wrong connections and insert correct ones)

c' on line 3 is connected

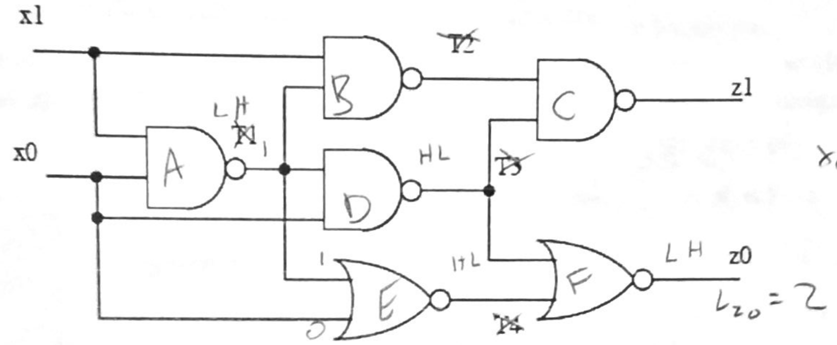
d on line 9 is connected

remove d on line 7

remove d' on line 8

Problem 5 (10 points) 10

Calculate the propagation delay $t_{pLH}(z0)$ when $x0$ changes. Assume that $z0$'s load value is 2. Fill in the blanks below with the appropriate values. You don't need to fill all the blanks.



$x_0 \to HL$

x_0 HL

Gate Type	Fan-in	Propagation Delays (ns) t_{pLH}	t_{pHL}	Load Factor I
NOT	1	$0.02 + 0.038L$	$0.05 + 0.017L$	1.0
AND	2	$0.15 + 0.037L$	$0.16 + 0.017L$	1.0
OR	2	$0.12 + 0.037L$	$0.20 + 0.019L$	1.0
NAND	2	$0.05 + 0.038L$	$0.08 + 0.027L$	1.0
NOR	2	$0.06 + 0.075L$	$0.07 + 0.016L$	1.0

	A	D	F			
Gate type & Fan-in	NAND →	NAND →	NOR →	→	→	
LH / HL	LH →	HL →	LH →	→	→	
Output load L	3 →	2 →	2 →	→	→	
Prop. Delay	$0.05+0.038\cdot3$ →	$0.08+0.027\cdot2$ →	$0.06+0.075\cdot2$ →	→	→	

$t_{pLH}(z_0)$ when x_0 changes is ✓

$0.05 + 0.038 \cdot 3 +$

$0.08 + 0.027 \cdot 2 +$

$0.06 + 0.075 \cdot 2$

Problem 6 (15 points)

A gate G is defined by the following expression

$$E(a,b,c,d) = cd + a'bc' + a'b'd' + bcd' + ab'c'd$$

15

Show that gate G forms a universal set assuming that constants 1 and 0 are available.

Specify a pre-established universal set you are using in the proof, and explicitly show the implementation for each element in the set using gate G with 1 and/or 0 as needed. For example, you can assign a=0 and b=1 in the expression E.

$E(a, b, 0, 1) = a'b' + ab'$

$= b' = \text{NOT}$

$E(0, b, 0, d) = b' + b'd'$

$= b'$

$E($

$E(a=0) = cd + b'c' + b'd + bcd'$

$a=1 = cd + bcd' + b'c'd$

$b=0 = cd + a'c' + a'd' + ac'd$

$b=1 = cd + cd' = c$

$c=0 = a'b' + a'b'd' + ab'd$

$\left.\begin{matrix} c=0 \\ d=0 \end{matrix}\right\} = a'b' + a'b' +$

$E(a, b, 0, 0) = a'b' = a \text{ NOR } b$

NOR is universal

$a'b' = (a+b)'$

20

Problem 7 (20 points)

For the switching function $f(x_3, x_2, x_1, x_0)$, we are given the information below for the dc-set and zero-set.

$$\text{dc-set} = (4, 15)$$

2 5

$$\text{zero-set} = \text{zero-set of function}$$

$$(x_3+x_2+x_1'+x_0)(x_3+x_2'+x_1+x_0')(x_3+x_2'+x_1'+x_0)(x_3'+x_2+x_1+x_0')(x_3'+x_2+x_1'+x_0)(x_3'+x_2'+x_1+x_0')$$

6 9 10 13

1. **(2 points)** Fill out the following K-map.

		x_0	x_0		
	0: 1	1: 1	3: 1	2: 0	
	4: –	5: 0	7: 1	6: 0	x_2
x_3	12: 1	13: 0	15: –	14: 1	x_2
x_3	8: 1	9: 0	11: 1	10: 0	
			x_1	x_1	

$x_3'x_2'x_0$ (g)
$x_3'x_2'x_1$ (f)
$x_1'x_0'$ (d)
$x_3x_2x_1$ (h)
$x_3x_2x_0'$ (i)
x_1x_0 (e)

2. **(4 points)** Which of the given expressions are prime implicants of the function given above? Circle **all** that apply. Do not circle implicants that are not prime.

   (a) x_3x_1
   (b) x_3x_2'
   (c) $x_3'x_1$
   (d) $x_1'x_0'$
   (e) x_1x_0
   (f) $x_3'x_2'x_1'$
   (g) $x_3'x_2'x_0$
   (h) ~~$x_3x_2x_1$~~
   (i) ~~$x_3x_2x_0'$~~
   (j) $x_3'x_2x_1'x_0$
   (k) $x_3x_2'x_1x_0$
   (l) $x_3'x_2x_1x_0'$

3. **(3 points)** Write down the complete set of **essential** prime implicants.

   $x_1'x_0'$ x_1x_0

4. **(2 points)** Write down the minimal sum of products expressions for f. If there are multiple forms of minimal sum of products expressions, you only need to write down one of them.

   $x_1'x_0' + x_1x_0 + x_3x_2x_0' + x_3'x_2'x_0$

5. **(4 points)** Which of the given expressions are prime implicates of the function given above? Circle all that apply. Do not circle implicates that are not prime.

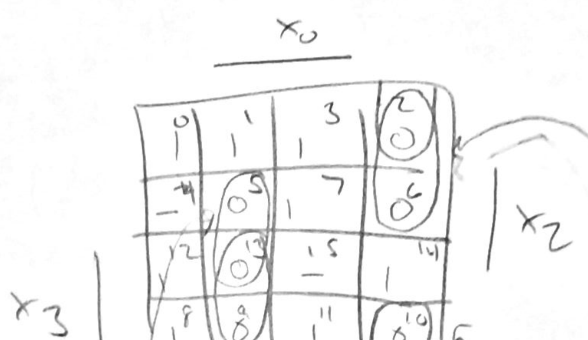
(a) $(x_3 + x_2' + x_1)$
(b) $(x_3' + x_2')$
(c) $(x_2' + x_1 + x_0')$
(d) $(x_3' + x_1 + x_0')$
(e) $(x_3 + x_2' + x_0)$
(f) $(x_3 + x_1' + x_0)$
(g) $(x_2 + x_1' + x_0)$
(h) $(x_3 + x_1 + x_0)$
(i) $(x_3' + x_2' + x_1)$
(j) $(x_3' + x_1')$
(k) $(x_3 + x_2 + x_1 + x_0')$
(l) $(x_3 + x_2' + x_1' + x_0)$

6. **(3 points)** Write down the complete set of **essential** prime implicates.

$x_2' + x_1 + x_0'$
$x_3' + x_1 + x_0'$
$x_2 + x_1' + x_0$
$x_3 + x_1' + x_0$

7. **(2 points)** Write down the minimal product of sums expressions for f. If there are multiple forms of minimal product of sums expressions, you only need to write down one of them.

$(x_2' + x_1 + x_0')(x_3' + x_1 + x_0')(x_2 + x_1' + x_0)(x_3 + x_1' + x_0)$



$x_2' + x_1 + x_0'$ (c)
$x_3' + x_1 + x_0'$ (d)
(g) $x_2 + x_1' + x_0$
(f) $x_3 + x_1' + x_0$