[CS M51A Fall 14] Midterm exam

Date: 11/04/14

- Please show all your work and write legibly, otherwise no partial credit will be given.
- **This should strictly be your own work; any form of collaboration will be penalized.**

Name : Mario Tinoco

Student ID : 404-500-045 (2C)

Problem	Points	Score
1	10	10
2	10	10
3	20	20
4	20	[illegible] 17
5	10	10
6	30	28
Total	100	~~93~~ 95

Problem 1 (10 points) 10

X = (x, y, z) is a 3-digit weighted mixed-radix number system: x is a radix-16 digit, y a is radix-3 digit, and z is a radix-12 digit.

1. **(5 points)** Convert X=(6, 1, 11) to a decimal number.

$= 6(3)(12) + 1(12) + 11$

$= 6(36) + 12 + 11$

$= 216 + 23$

$= \boxed{239_{10}}$ ✓

$36 \times 6 = 216$

$12 + 11 = 23$

$216 + 23 = 239$

1 1 1

$1(2)(2 \quad + 1(2) + 1$

2. **(5 points)** What is the largest number of X in decimal?

largest = (15, 2, 11)

$= 15(3)(12) + 2(12) + 11$

$= 15(36) + 24 + 11$

$= \boxed{15(36) + 35}$ ✓

Problem 2 (10 points)

Simplify the following boolean expression by using postulates of Boolean Algebra.

$$(a'b' + c)(a + b)(b' + a'c')'$$

$= (a'b'a + ac + bc)(b' + a'c')'$ distributive

$= (0 + ac + bc)((b)(a + c))$ de Morgans, complement

$= (ac + bc)(ab + bc)$ distributive

$= abc + abc + abc + bc$ distributive

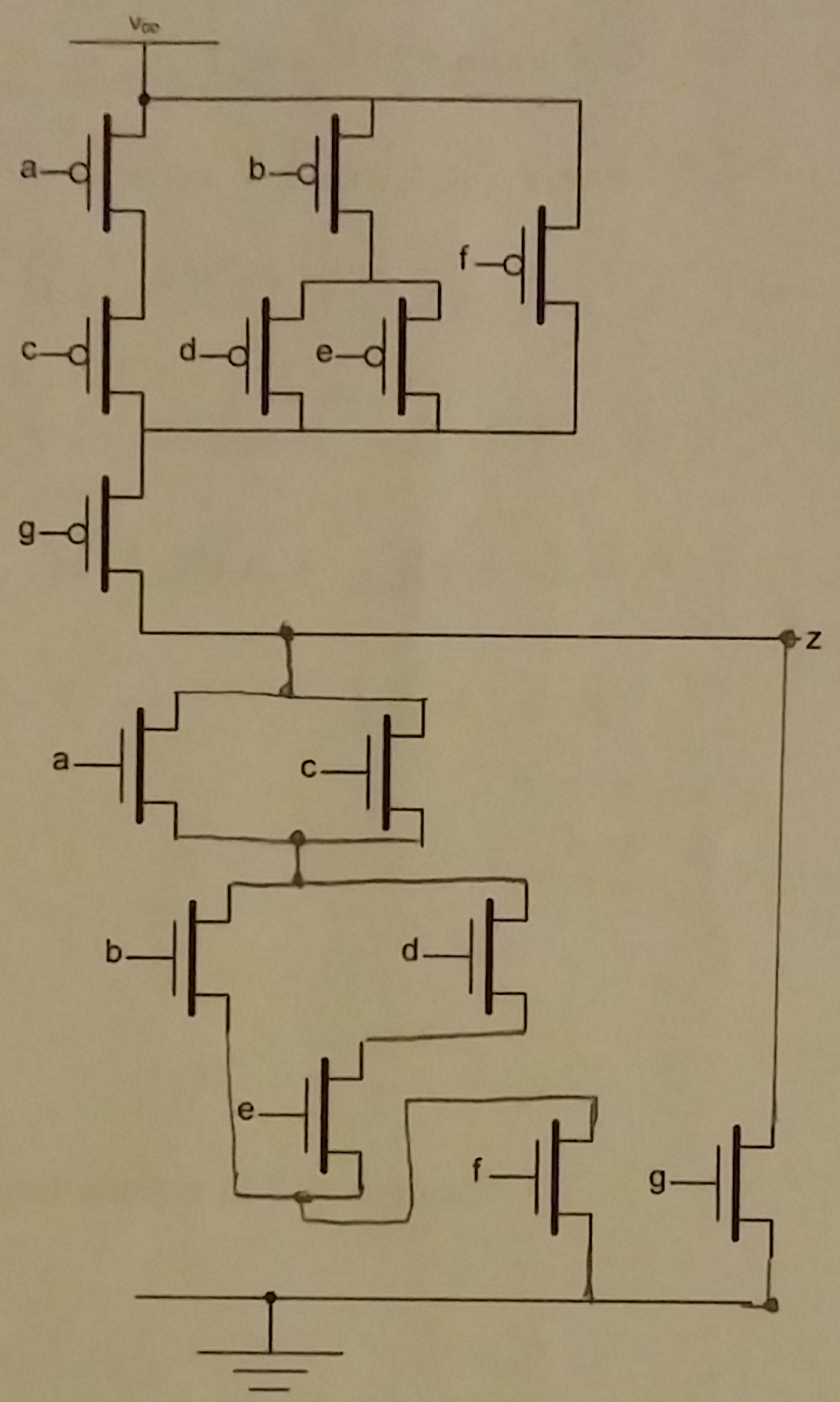
$= abc + bc$ idempotent

$\boxed{= bc}$ absorption

10

Problem 3 (20 points)

We are given the following partial CMOS network.



1. **(10 points)** Write the expression for the pull-up network. From this, derive the expression for the pull-down network using switching algebra.

$$z = (g')((a'c') + (b'(d'+e')) + (f'))$$

$[b'(d'+e')]'$

Pull Down $= z'$

$b + de$

$$Z = g'(a'c' + b'(d'+e') + f')$$

$$z' = (g) + (a'c' + b'(d'+e') + (f'))'$$ deMorgans

$$= g + (a+c)(b+de)(f)$$

Pull-Down Network Expression

b

2. **(10 points)** Connect the NMOS transistors in page 4 to complete the pull-down network so that it corresponds to the expression obtained from part 1 and drives the output z to a valid output - i. e. either V_{DD} or ground - for any combination of inputs.

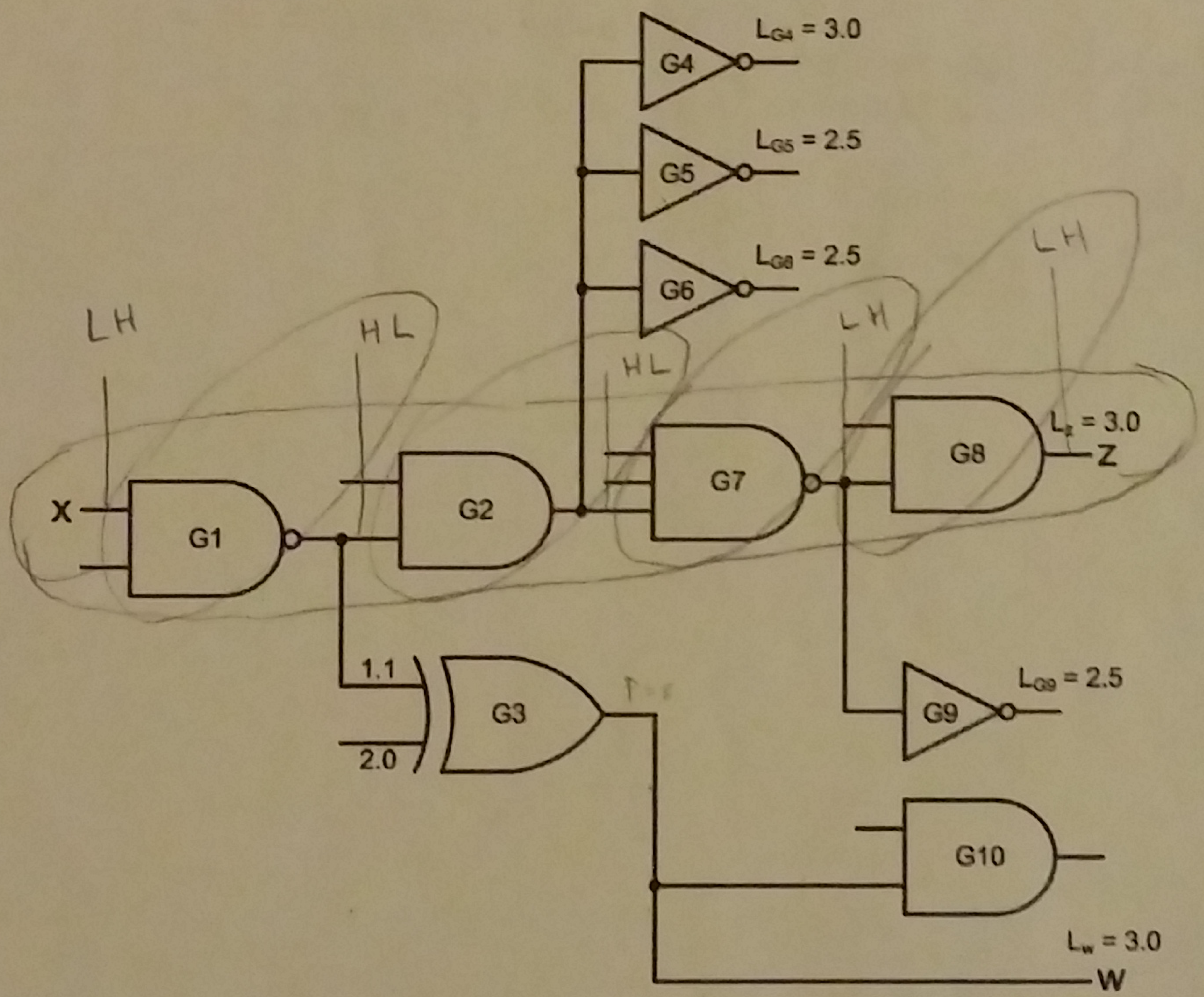
   You can directly draw on the CMOS implementation on page 4.

drawn on pg. 4

Problem 4 (20 points) #5 14

We would like to determine the propagation delay of the gate network shown here. The output is z with a input x. The necessary gate characteristics are given in the table below.

Gate Type	Fan-in	Propagation Delays (ns) t_{pLH}	Propagation Delays (ns) t_{pHL}	Load Factor l
NOT	1	$0.02 + 0.038L$	$0.05 + 0.017L$	1.0
AND	2	$0.15 + 0.037L$	$0.16 + 0.017L$	1.0
NAND	2	$0.05 + 0.038L$	$0.08 + 0.027L$	1.0
NAND	3	$0.07 + 0.038L$	$0.09 + 0.039L$	1.0
XOR	2	$0.30 + 0.036L$	$0.30 + 0.021L$	1.1
		$0.16 + 0.036L$	$0.15 + 0.020L$	2.0



2.1 4 2 3

1. **(10 points)** (a) Determine the output load of gate G3.

7/10

(b) If the fanout factor of gate G3 is 8, then how many additional gate inputs with load factor of 2 can be connected to the output of gate G3?

$= 8.0 - L(G_{10}) - L_w$

$= 8.0 - 1 - 3$

(a)?

$= 8.0 - 4$

$= \underline{4.0}$ +2

additional inputs of I=2

$\frac{4.0}{2.0} = \boxed{2}$

10

2. **(10 points)** Find the worst case value of $t_{pLH}(x \to z)$. Fill in the blanks below with the appropriate values.

Gate type and fan-in	G1: NAND 2 → G2: AND 2 → G7: NAND 3 → G8: AND 2
LH / HL	G1: HL → G2: HL → G7: LH → G8: LH
Output load L	G1: 2.1 → G2: 4 → G7: 2 → G8: 3
	0.08 + 0.16 + 0.07 + 0.15 +
Propagational delay	G1: 0.027(2.1) → G2: 0.017(4) → G7: 0.038(2) → G8: 0.037(3)
$t_{pLH}(x \to z)$	↓

$$= 0.08 + 0.16 + 0.07 + 0.15 +$$
$$0.027(2.1) + 0.017(4) + 0.038(2) + 0.037(3)$$

10

Problem 5 (10 points)

Decide whether the function E defined in (a) and (b) are universal functions.. If Yes, show your proof. You can use constant 0 or 1 as your input.

(a) (5 points)

x	y	$E(x,y)$
0	0	1
0	1	1
1	0	0
1	1	1

$E(x,y) = (x'y)$

$E(x,y) = x'y' +$
$x'y +$
xy
$= x' + y$

* NOT-CASE

$E(x,0) \cdot x'(1) + x'(0) + x(0) = x' = NOT(x)$ ✓

* OR-CASE

$E(E(x,0),y) = ((x'+0)' + y) = ((x')' + y)$
$= (x+y)$
$= OR(x,Y)$

OR-NOT is a known universal set, and I have shown both can be constructed throug E

Therefor E $\boxed{\text{is also universal.}}$

(b) **(5 points)**

$x'yz + xy'z' + xyz' + xyz$

$x'yz + x(y'z' + yz' + yz)$

$x'yz + xz' + xy$

x	y	z	E(x,y,z)
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

$E(x,y,z) =$
$x'yz +$
$xy'z' +$
$xyz' +$
xyz

~~$\boxed{\text{Not Universal}}$ because there is no input that can generate an inversion with this setup.~~

~~For any two zeros the output will not invert the third ones~~

~~Therefore Not, NAND, NOR are Not Possible~~

$E(x, 1, y) = x'y + xy' + xy = x + y = OR(x, Y)$

$E(x, y, 1) = x'y + x + xy = xy = AND(x, Y)$

Therefore E is Universal
because it can form OR and AND
which are known to be universal

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Problem 6 (30 points)

F is a function that accepts inputs $x \in \{0, 1, 2\}$, $y \in \{0, 1, 2, 3\}$, and outputs $z = x^2 + y$. Suppose you use binary code to encode x, y, and z. x is encoded as x_1x_0, y is encoded as y_1y_0, z is encoded as $z_2z_1z_0$.

(a) **(5 points)** Fill in the following truth table.

x	y	x²+y
0	0	0
0	1	1
0	2	2
0	3	3
1	0	1
1	1	2
1	2	3
1	3	4
2	0	4
2	1	5
2	2	6
2	3	7

	x_1	x_0	y_1	y_0	z_2	z_1	z_0
6	0	0	0	0	0	0	0
	0	0	0	1	0	0	1
	0	0	1	0	0	1	0
3	0	0	1	1	0	1	1
4	0	1	0	0	0	0	1
	0	1	0	1	0	1	0
	0	1	1	0	0	1	1
7	0	1	1	1	1	0	0
8	1	0	0	0	1	0	0
	1	0	0	1	1	0	1
	1	0	1	0	1	1	0
11	1	0	1	1	1	1	1
12	1	1	0	0	–	–	–
	1	1	0	1	–	–	–
	1	1	1	0	–	–	–
16	1	1	1	1	–	–	–

$x = 3 \rightarrow \{-\}$

(b) **(5 points)** Based on the truth table in (a), draw the K-map for z_2, z_1, and z_0.

essential

Z_2:

$x_1x_0 \backslash y_1y_0$	00	01	11	10
00	0	0	0	0
01	0	0	1	0
11	–	–	–	–
10	1	1	1	1

$P_1 = x_1 x_0'$ ✓

$P_2 = y_1 y_0 x_0$ ✓

Z_1:

0	0	1	1
0	1	0	1
–	–	–	–
0	0	1	1

$P_1 = y_0' y_1$ ✓

$P_2 = y_1 x_0'$ ✓

$P_3 = y_0 y_1' x_0$ ✓

Z_0:

0	1	1	0
1	0	0	1
–	–	–	–
0	1	1	0

$P_1 : x_0' y_0$ ✓

$P_2 : y_0' x_0$ ✓

(c) **(5 points)** Use the K-maps in (b) to find all the prime implicants for z_2, z_1, and z_0 respectively

z_2: $p_1 = x_1 x_0'$

$p_2 = y_1 y_0 x_0$

z_1: $p_1 = y_0' y_1$

$p_2 = y_1 x_0'$

$p_3 = y_0 y_1' x_0$

z_0: $p_1 = x_0' y_0$

$p_2 = y_0' x_0$

(d) **(5 points)** Use the K-maps in (b) to find all the essential prime implicants for z_2, z_1, and z_0 respectively.

z_2: $e_1 = x_1 x_0'$

$e_2 = y_1 y_0 x_0$

z_1: $e_1 = y_0' y_1$

$e_2 = y_1 x_0'$

$e_3 = y_0 y_1' x_0$

z_0: $e_1 = x_0' y_0$

$e_2 = y_0' x_0$

(e) **(5 points)** Give minimal SOP expression for z_2, z_1, and z_0. State if the expressions are unique.

$z_2 = x_1 x_0' + y_1 y_0 x_0$

$z_1 = y_0' y_1 + y_1 x_0' + y_0 y_1' x_0$

$z_0 = x_0' y_0 + y_0' x_0$

All 3 are unique

(f) **(5 points)**Implement z_2, z_1, and z_0 using minimal NAND-NAND networks. Note that each output has a separate gate network. You can directly use x_1', x_0', y_1', y_0' as inputs.