[CS M51A Fall 14] Midterm exam

Date: 11/04/14

- Please show all your work and write legibly, otherwise no partial credit will be given.
- **This should strictly be your own work; any form of collaboration will be penalized.**

Name : [redacted]

Student ID : [redacted]

Problem	Points	Score
1	10	2
2	10	4
3	20	20
4	20	20
5	10	10
6	30	30
Total	100	86

Problem 1 (10 points) 2

X = (x, y, z) is a 3-digit weighted mixed-radix number system: x is a radix-16 digit, y a is radix-3 digit, and z is a radix-12 digit.

0

1. **(5 points)** Convert X=(6, 1, 11) to a decimal number.

$16 \cdot 6 + 3 \cdot 1 + 12^0 \cdot 11$

6 1 11

$6 \cdot 16^2 + 1 \cdot 3^1 + 11 \cdot 12^0$

$1536_{10} + 3_{10} + 11_{10} = 1550$ X

$$\begin{array}{r} 16 \\ 16 \\ \hline 96 \\ 16 \\ \hline 256 \\ 6 \\ \hline 1536 \end{array}$$

2

2. **(5 points)** What is the largest number of X in decimal?

X = (15, 2, 11) +2

$15 \cdot 16^2 + 2 \cdot 3^1 + 11 \cdot 12^0$

$3840 + 6 + 11 = 3857$

$$\begin{array}{r} 256 \\ 15 \\ \hline 1280 \\ 2560 \\ \hline 3840 \end{array}$$

Problem 2 (10 points)

Simplify the following boolean expression by using postulates of Boolean Algebra.

$(a+b)' = a'b'$
$(ab)' = a'+b'$

$$(a'b' + c)(a + b)(b' + a'c')'$$

$(a'b'+c)(a+b)((b)(a'c')')'$

$(a'b'+c)(a+b)(b(a+c))'$

$((a+b)'+c)(a+b)(b(a+c))$

$(ac+cb)(b(a+c))$

$ac+cb+b'+a'c'$

$ac+c+b'+a'c'$

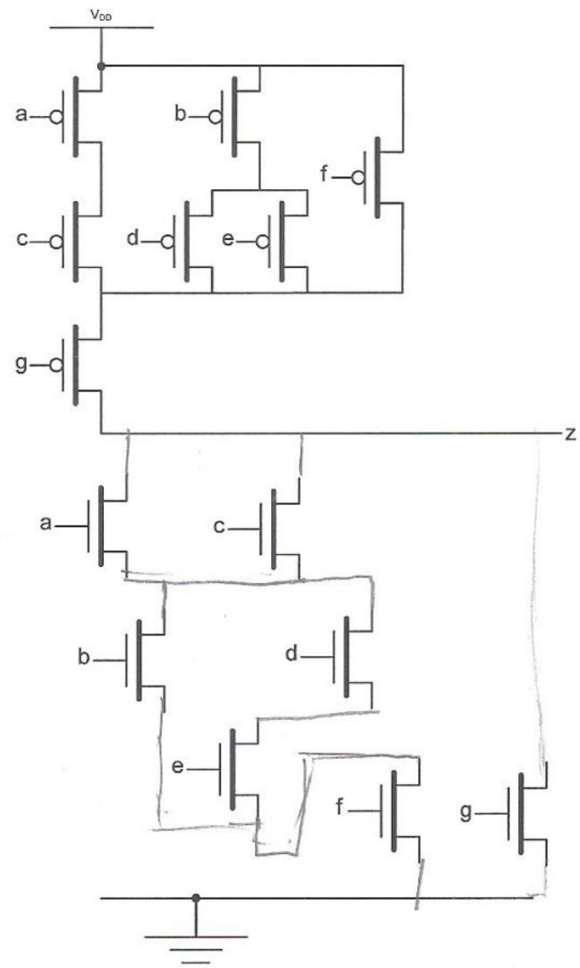
$c+b'+a'c'$

$c+a'+b'$

4

Problem 3 (20 points)

We are given the following partial CMOS network.



1. **(10 points)** Write the expression for the pull-up network. From this, derive the expression for the pull-down network using switching algebra.

pull-up

$$z = ((a'c') + f' + (d'+e')b')g'$$

$$Z' = [(a'c') + f' + (d'+e')b')g']'$$

$$((a'c') + f' + (d'+e')b')' + g$$

$$(a'c')'f((d'+e')b')' + g$$

$$(a+c)f((d'+e')' + b) + g$$

pull-down $(a+c)f(de+b) + g$

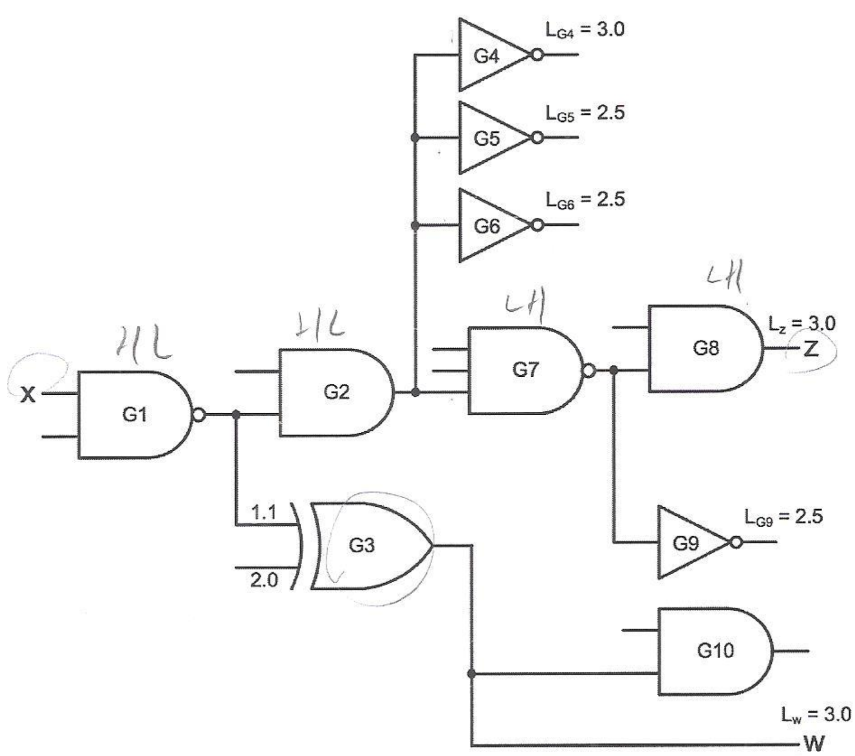
2. **(10 points)** Connect the NMOS transistors in page 4 to complete the pull-down network so that it corresponds to the expression obtained from part 1 and drives the output z to a valid output - i. e. either V_{DD} or ground - for any combination of inputs.

   You can directly draw on the CMOS implementation on page 4.

Problem 4 (20 points)

We would like to determine the propagation delay of the gate network shown here. The output is z with a input x. The necessary gate characteristics are given in the table below.

Gate Type	Fan-in	Propagation Delays (ns) t_{pLH}	t_{pHL}	Load Factor I
NOT	1	$0.02 + 0.038L$	$0.05 + 0.017L$	1.0
AND	2	$0.15 + 0.037L$	$0.16 + 0.017L$	1.0
NAND	2	$0.05 + 0.038L$	$0.08 + 0.027L$	1.0
NAND	3	$0.07 + 0.038L$	$0.09 + 0.039L$	1.0
XOR	2	$0.30 + 0.036L$	$0.30 + 0.021L$	1.1
		$0.16 + 0.036L$	$0.15 + 0.020L$	2.0



1. **(10 points)** (a) Determine the output load of gate G3. 10

(b) If the fanout factor of gate G3 is 8, then how many additional gate inputs with load factor of 2 can be connected to the output of gate G3?

a) $1+3 = \boxed{4}$

b) $8-4 = 4/2 = \boxed{2}$

10

2. **(10 points)** Find the worst case value of $t_{pLH}(x \to z)$. Fill in the blanks below with the appropriate values.

Gate type and fan-in	G1: NAND, 2	→ G2: AND, 2	→ G7: NAND, 3	→ G8: AND, 2
LH / HL	G1: HL	→ G2: HL	→ G7: LH	→ G8: LH
Output load L	G1: 2.1	→ G2: 4	→ G7: 2	→ G8: 3
Propagational delay	G1: .08 + .027(2.1)	→ G2: .16 + .017(4)	→ G7: .07 + .038(2)	→ G8: .15 + .037(3)
$t_{pLH}(x \to z)$	________			

Problem 5 (10 points)

Decide whether the function E defined in (a) and (b) are universal functions.. If Yes, show your proof. You can use constant 0 or 1 as your input.

10

(a) **(5 points)**

x	y	$E(x,y)$
0	0	1
0	1	1
1	0	0
1	1	1

NOR
"OR
XOR
OR

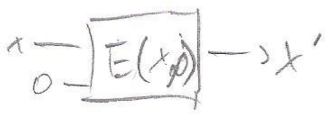
00 → NAND
01 → NAND
10 → AND
11 → AND

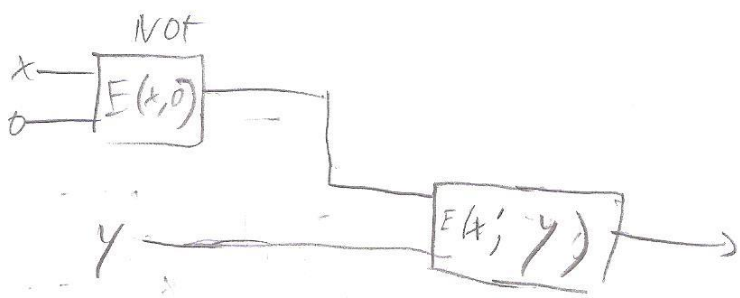
NOT ✓

y = 0

x	E(x,0)
0	1
1	0

← NOT gate





x y	output
0 0	0
0 1	1
1 0	1
1 1	

OR ✓

universal

(b) **(5 points)**

x	y	z	E(x,y,z)
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

$E(1,0,z) \rightarrow$ NOT

z	E(1,0,z)
0	1
1	0

= NOT ✓

$E(0,y,z) \rightarrow$ AND

y z	E(0,y,z)
0 0	0
0 1	0
1 0	0
1 1	1

AND ✓

{AND, NOT}

Universal set

Problem 6 (30 points)

3 -> dc

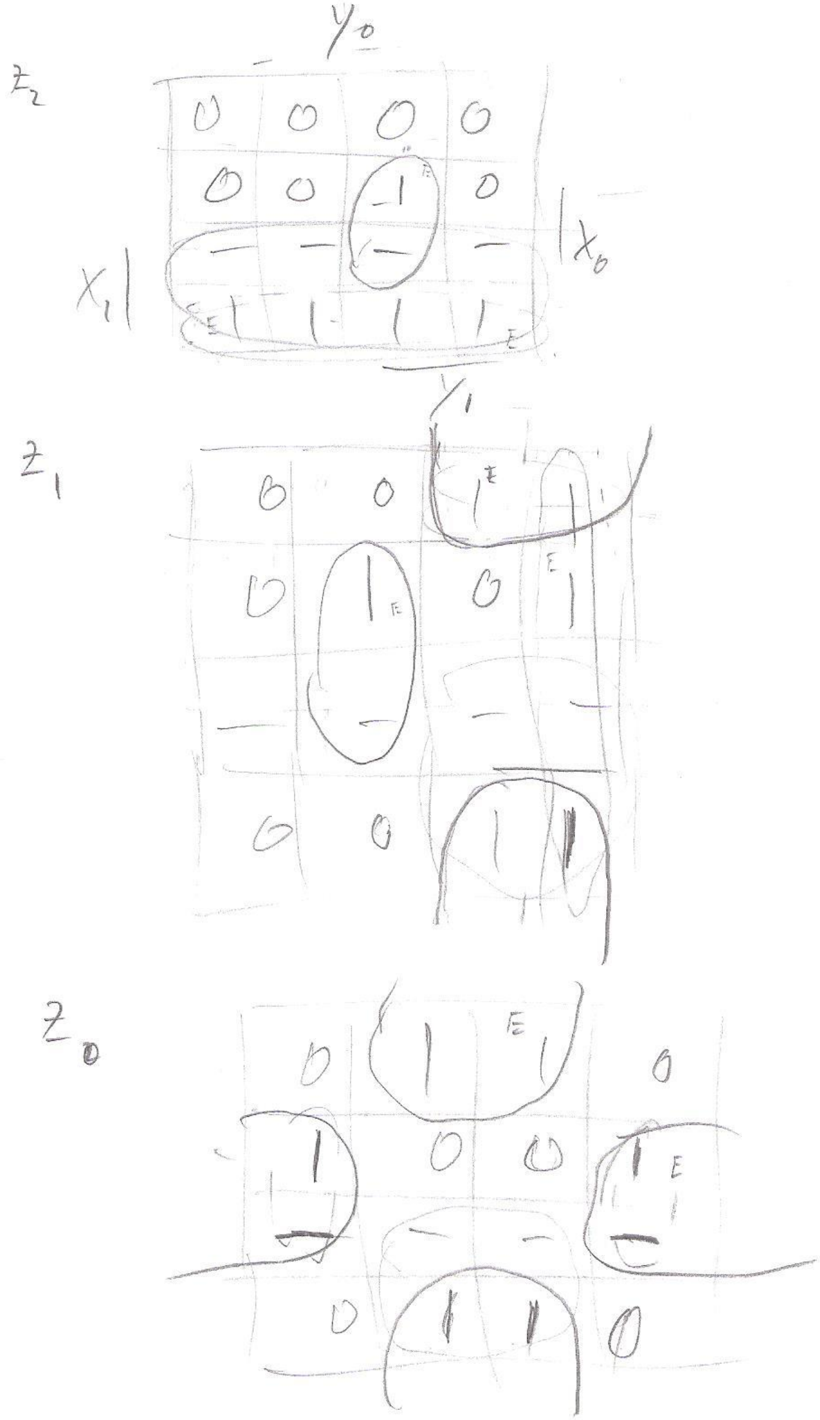
30

F is a function that accepts inputs $x \in \{0, 1, 2\}$, $y \in \{0, 1, 2, 3\}$, and outputs $z = x^2 + y$. Suppose you use binary code to encode x, y, and z. x is encoded as x_1x_0, y is encoded as y_1y_0, z is encoded as $z_2z_1z_0$.

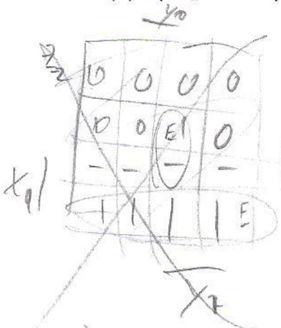
(a) **(5 points)** Fill in the following truth table.

	x_1	x_0	y_1	y_0	z_2	z_1	z_0	
0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	1	1
2	0	0	1	0	0	1	0	2
3	0	0	1	1	0	1	1	3
4	0	1	0	0	0	0	1	1
5	0	1	0	1	0	1	0	2
6	0	1	1	0	0	1	1	3
7	0	1	1	1	1	0	0	4
8	1	0	0	0	1	0	0	4
9	1	0	0	1	1	0	1	5
10	1	0	1	0	1	1	0	6
11	1	0	1	1	1	1	1	7
12	1	1	0	0	–	–	–	
13	1	1	0	1	–	–	–	
14	1	1	1	0	–	–	–	
15	1	1	1	1	–	–	–	

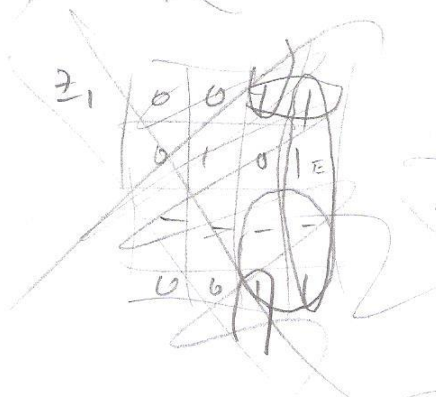
(b) **(5 points)**Based on the truth table in (a), draw the K-map for z_2, z_1, and z_0.



(c) **(5 points)** Use the K-maps in (b) to find all the prime implicants for z_2, z_1, and z_0 respectively.



z_2 - $x_0 y_1' y_0$, x_1



z_1 - $x_1 y_1$, $y_1 y_0'$, $x_0' y_1$, $x_0 y_1' y_0$

z_0 - $x_0 y_0'$, $x_0' y_0$, $x_1 y_0$

(d) **(5 points)** Use the K-maps in (b) to find all the essential prime implicants for z_2, z_1, and z_0 respectively.

z_2 - $x_0 y_1' y_0$, x_1

z_1 - $x_0 y_1' y_0$, $y_1 y_0'$, $x_0' y_1$

z_0 - $x_0 y_0'$, $x_0' y_0$

(e) **(5 points)** Give minimal SOP expression for z_2, z_1, and z_0. State if the expressions are unique.

$z_2 - (x_0 y_1 y_0) + (x_1)$

unique

$z_1 - (x_0 y_1' y_0) + (y_1 y_0') + (x_0' y_1)$

unique

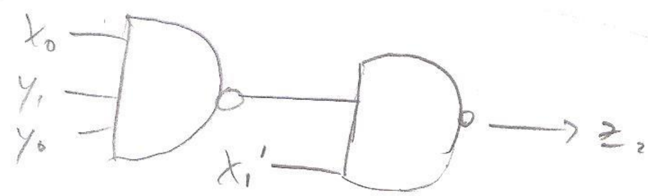
$z_0 - (x_0 y_0') + (x_0' y_0)$

unique

(f) **(5 points)** Implement z_2, z_1, and z_0 using minimal NAND-NAND networks. Note that each output has a separate gate network. You can directly use x_1', x_0', y_1', y_0' as inputs.

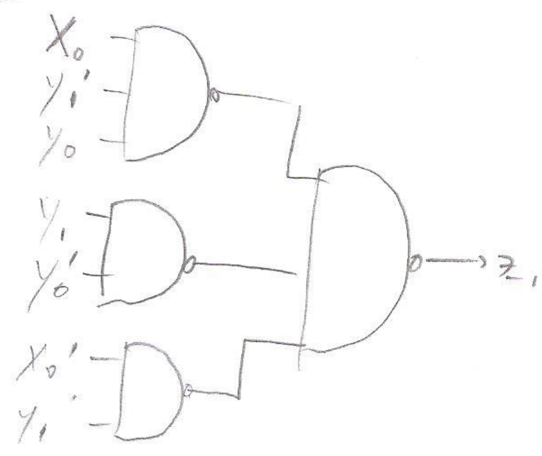
$x_0 y_1 y_0 + x_1$

$\left((x_0 y_1 y_0)' (x_1)' \right)'$



$x_0 y_1' y_0 + y_1 y_0' + x_0' y_1$

$\left((x_0 y_1' y_0)' (y_1 y_0')' (x_0' y_1)' \right)'$



$x_0 y_0' + x_0' y_0$

$\left((x_0 y_0')' (x_0' y_0)' \right)'$