CS M51A / EE M16 MIDTERM

– closed books and notes –

– no calculators/laptops –

– 2 sheets of notes allowed –

THIS IS STRICTLY YOUR WORK:
ANY FORM OF COLLABORATION WILL BE PENALIZED

(6 problems, 110 minutes)

November 1, 2005

Name: Solutions

ID No.: ____________________

Problem	Points	Score
1	15	
2	10	
3	20	
4	20	
5	15	
6	20	
Total	100	

Problem 1. (15 points) Find a minimal sum of products and a minimal product of sums for the switching function specified below and compare their costs. Which one is less costly?

$$E(a,b,c,d) = (ad' \oplus b')(c+d) + (a' + bc)'cd'$$

$E = (ad'b + (ad')'b')(c+d) + a(bc)'cd'$

Show the transformation to a sum of products form

$= (abcd' + (ad')'(b'c + b'd)) + ab'cd'$

$= abcd' + a'b'c + a'b'd + b'cd + b'd + ab'cd'$

$= b'd + b'c(d + ad' + a') + a'b'd + abcd'$

$= b'd + b'c + abcd'$

$= b'd + c(b' + abd')$

$= b'd + c(b' + ad')$

$= b'd + b'c + acd'$

ab \ cd	00	01	11	10
00	0	1	1	1
01	0	0	0	0
11	0	0	0	1
10	0	1	1	1

$E_1(a,b,c,d) = b'd + b'c + acd'$

	0	1	1	1
	0	0	0	0
	0	0	0	1
	0	1	1	1

$E_2(a,b,c,d) = (c+d)(a+b')(b'+d')$

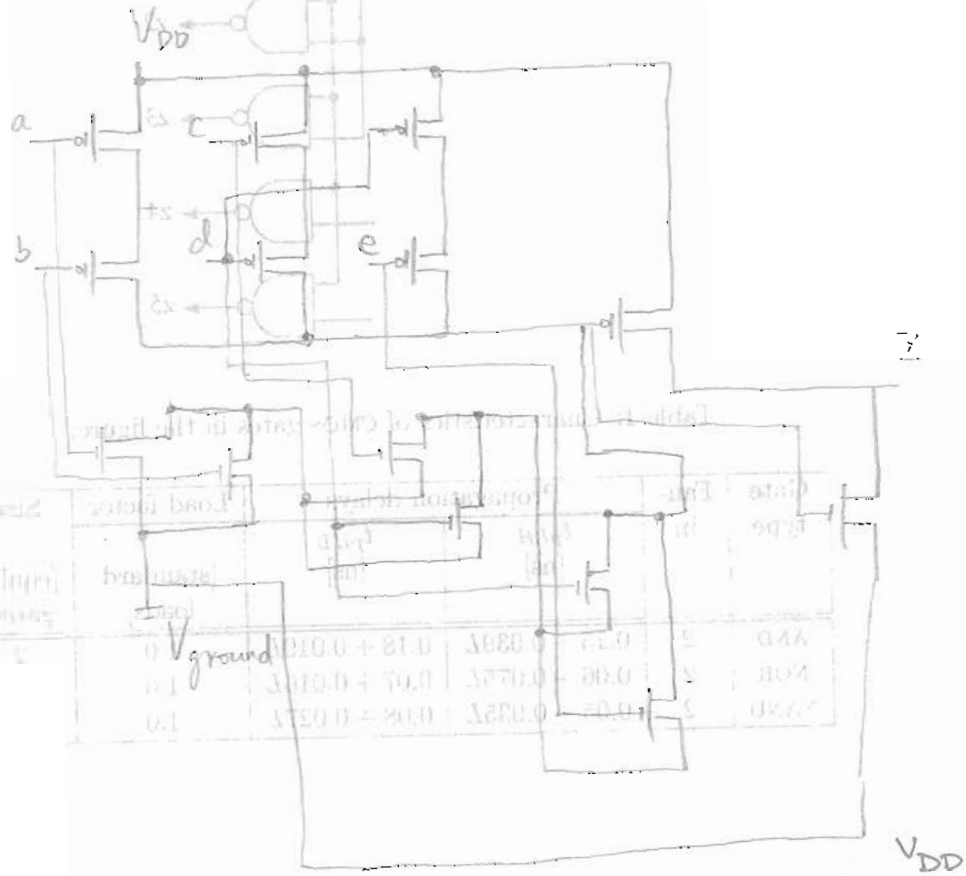
E_2 Less costly than E_1

Problem 2. (10 points)

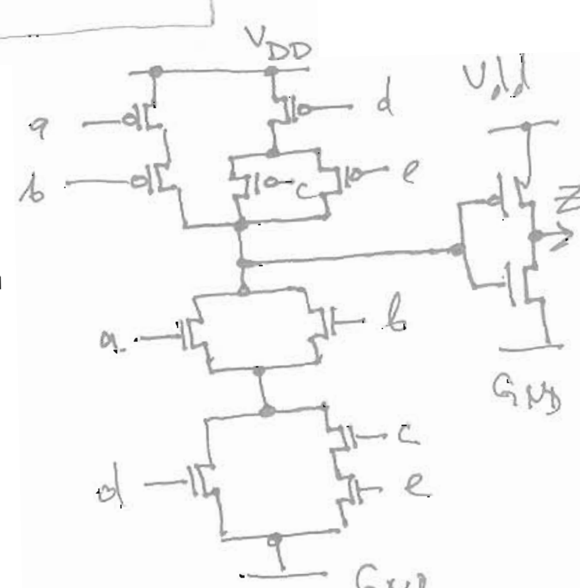
Design a CMOS circuit which implements the following switching expression

$$z = (a + b)(c + d)(d + e)$$

$$= (a'b' + c'd' + d'e')'$$



- show optimized version (12 transistors)



Problem 3. (20 points)

a) With the help of Table 1, determine the worst case propagation delay t_{pLH} for the output $z0$ of the network shown in Fig. 2. Assume that $L = 5$ for this output.

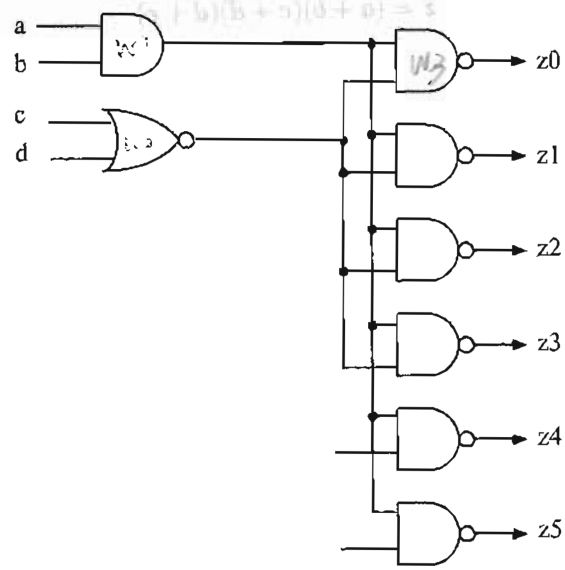


Table 1: Characteristics of CMOS gates in the figure.

Gate type	Fan-in	Propagation delays t_{pLH} [ns]	Propagation delays t_{pHL} [ns]	Load factor [standard loads]	Size [equiv. gates]
AND	2	$0.15 + 0.039L$	$0.18 + 0.019L$	1.0	2
NOR	2	$0.06 + 0.075L$	$0.07 + 0.016L$	1.0	1
NAND	2	$0.05 + 0.035L$	$0.08 + 0.027L$	1.0	1

If W1 → W3

$t_{pLH} = t_{pHL}(W1) + t_{pLH}(W3)$

$= 0.18 + 0.019 \times 6 + 0.05 + 0.035 \times 5$

$= 0.519$

If W2 → W3

$t_{pLH} = t_{pHL}(W2) + t_{pLH}(W3)$

$= 0.07 + 0.016 \times 4 + 0.05 + 0.35 \times 5$

$= 0.359$ ns

$t_{pLH\,worst} = 0.519$ ns

b) Determine the equivalent size of the network.

AND : 2

NOR : 1

NAND : 1 × 6

∴ equivalent size = 9

c) Determine the minimum sum of products expression for z0.

$$z_0 = (ab(c+d)')'$$

$$= a' + b' + c + d$$

Problem 4. (20 points)

Design a minimal NAND-NAND network to implement the following combinational system:

Input: $x, y \in \{0, 1, 2\}$

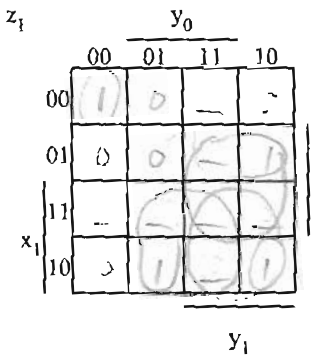
Output: $z \in \{0, 1, 2, 3\}$

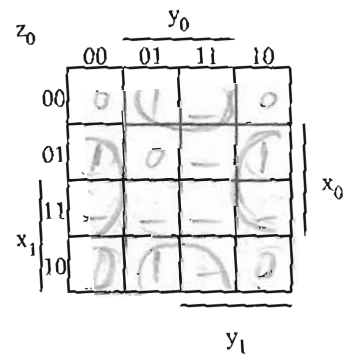
Function: $z = (3x - y + 2)\ mod\ 4$

a) Show a table for z at the binary level.

x_1x_0 \ y_1y_0 (z_1z_0)	00	01	10	11
00	10	01	00	–
01	01	00	11	–
10	00	11	10	–
11	–	–	–	–

b) Derive minimal sums of products expressions for the outputs. Use the binary code to represent the inputs and the output. Show all your work.

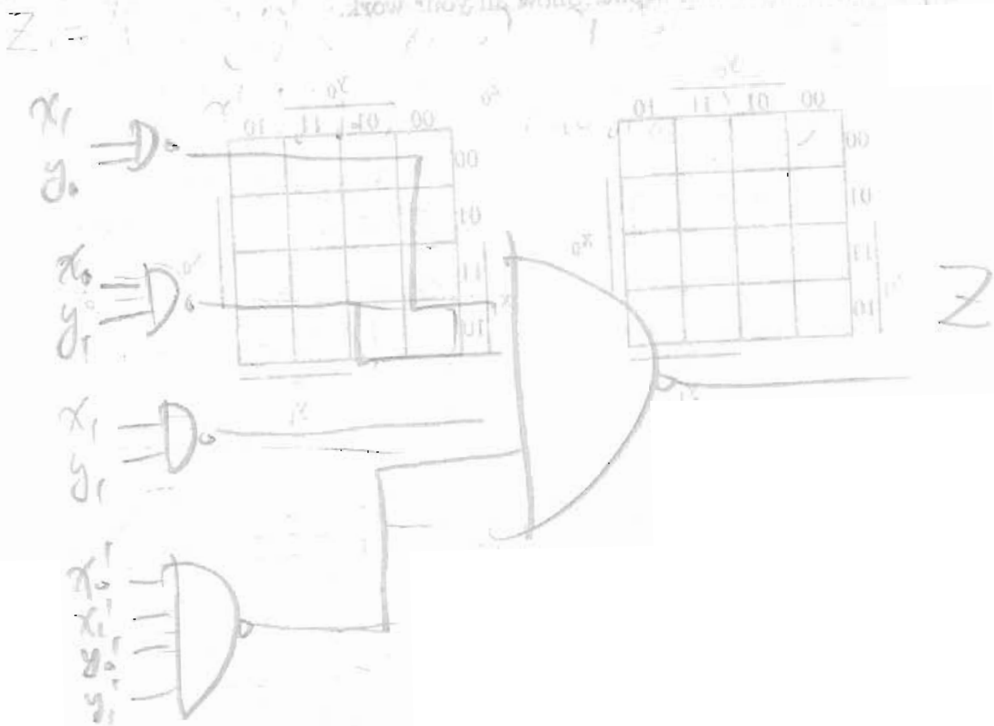


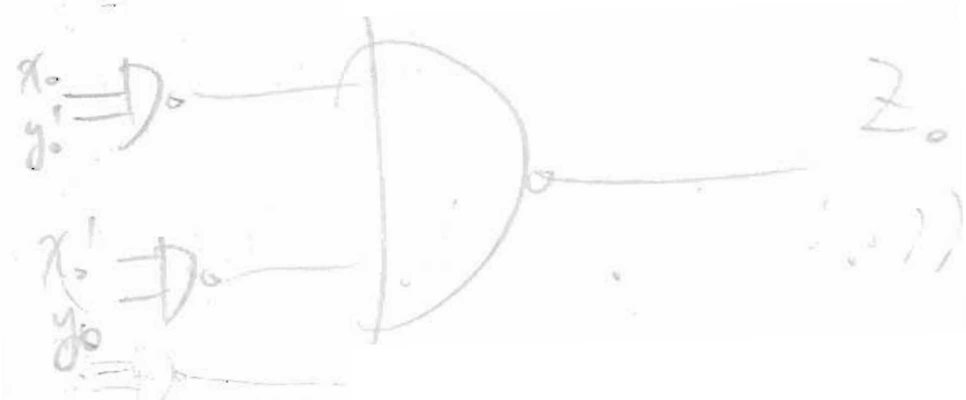


$Z_1 = x_1 y_0 + x_0 y_1 + x_1 y_1 + x_1' x_0' y_1' y_0'$

$Z_0 = x_0 y_0' + x_0' y_0$

c) Show the corresponding NAND-NAND network.

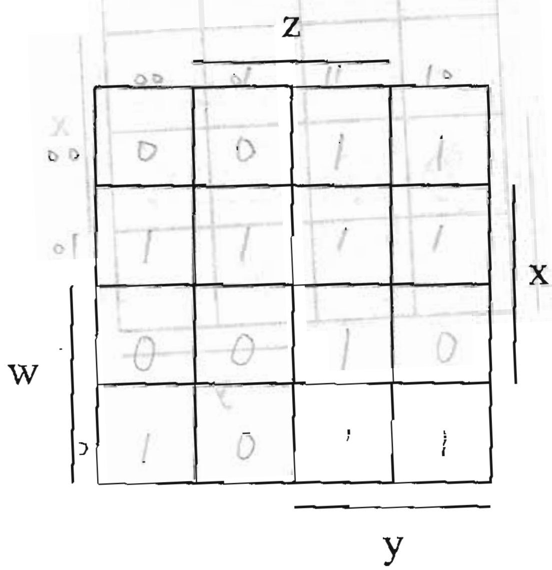




Problem 5. (15 points) Given

$$E(w,x,y,z) = wx'z' + w'x + w'y + yz$$

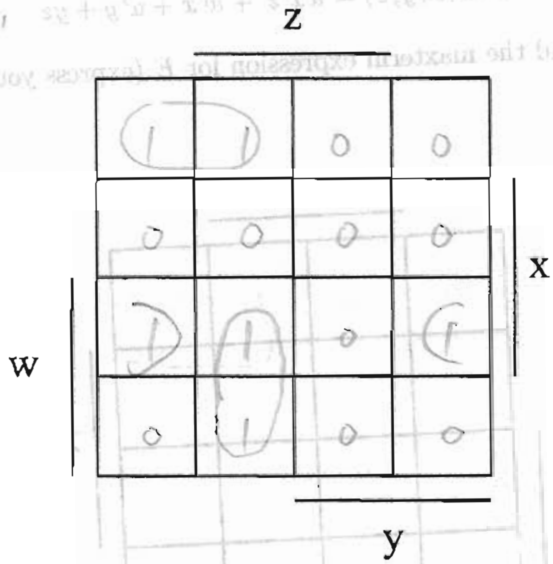
a) Use a K-map to find the maxterm expression for E (express your answer in maxterm notation).



$E(W,X,Y,Z) = (W+X+Y+Z)(W+X+Y+Z')$
$(W'+X'+Y+Z)(W'+X'+Y+Z')$
$(W'+X'+Y'+Z)(W'+X+Y+Z')$

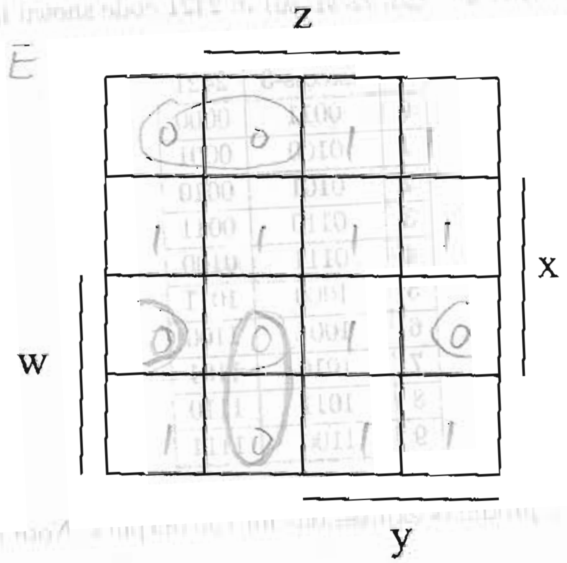
$= \Pi M(0,1,9,12,13,14)$

b) Use a K-map to find the minimum sum of products expressions for E' (complement of E)



$E' = w'x'y' + wy'z + wxz'$

c) Find the minimum product of sums for E.



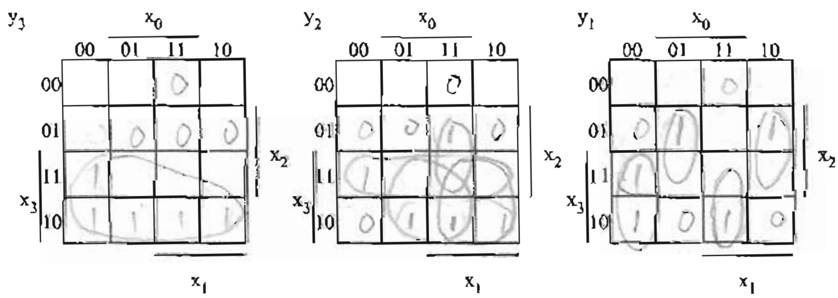
$E = (w + x + y)(w' + x' + z)(w' + y + z')$

Problem 6. (20 points)

Design a network to convert $x \in \{0, ..., 9\}$ represented in the Excess-3 code with a bit-vector $x = (x_3, x_2, x_1, x_0)$ to $y = (y_3, y_2, y_1, y_0)$ in 2421 code shown in the following table.

	Excess-3	2421
0	0011	0000
1	0100	0001
2	0101	0010
3	0110	0011
4	0111	0100
5	1000	1011
6	1001	1100
7	1010	1101
8	1011	1110
9	1100	1111

a) Derive minimal sum of products expressions for the outputs. Note that $y_0 = x_0'$. Show all your work



$y_3 = x_3$

$y_2 = x_3 x_2 + x_3 x_0 + x_3 x_1 + x_2 x_1 x_0$

$y_1 = x_2 x_1' x_0 + x_3 x_1' x_0' + x_3 x_1 x_0 + x_2 x_1 x_0'$

$y_0 = x_0'$

b) Implement your expressions from part a) by programming the PLA shown below.