

ECE M16/CS M51A Midterm

Parth Deshpande

TOTAL POINTS

84.5 / 105

QUESTION 1

1 Question 1-a 4 / 5

- **0 pts** Correct
- **1 pts** Missing 2x2 PI top/bottom right corner
- **1 pts** Missing bottom row PI
- **1 pts** Missing Upper Center PI
- ✓ - **1 pts** Missing Lower Center PI
- **1 pts** Missing Right Center PI
- **1 pts** Introducing additional wrong implicants (not PIs)
- **5 pts** Ambiguous circling.

QUESTION 2

2 Question 1-b 5 / 5

- ✓ - **0 pts** Correct
- **1 pts** Missed center EPI
- **1 pts** Missed right center EPI
- **1 pts** Missed 2x2 top/bottom-right corner EPI
- **1 pts** Missed bottom row EPI
- **0.5 pts** Missed bottom row EPI but consistent with previous answer
- **1 pts** Mistake made with EPI expression (circle is translated wrong)
- **1 pts** Inconsistent with previous answer (given previous answer the answer given here is wrong)
- **0.5 pts** Missed 2x2 top/bottom right EPI but consistent with previous answer
- **0.5 pts** * + notation error
- **5 pts** No answer
- **2 pts** A should be B, C should be D

QUESTION 3

3 Question 1-c 5 / 5

- ✓ - **0 pts** Correct
- **1 pts** Missed 2x2 bottom right corner

- **0.5 pts** Expression is wrong from the circle. (Wrong translation from circle).
- **1 pts** Expression inverses are wrong (dual form is given).
- **1 pts** Unnecessary PI included
- **2 pts** POS is given when SOP is asked.
- **1 pts** Top row PI is missed.
- **1 pts** Left center PI is missed
- **1 pts** Center PI is missed.
- **1 pts** Inverse at the beginning is wrong.
- **5 pts** Should have started with 0s.
- **2 pts** A should be B and C should be D
- **0.5 pts** Click here to replace this description.
- **1 pts** Simplifications are wrong.
- **4 pts** Formed correct POS but nothing else.

QUESTION 4

4 Question 2-a 5 / 5

- ✓ + **1 pts** Answered yes.
- ✓ + **1 pts** Wrote the correct form
- ✓ + **3 pts** Provided proof
- **1 pts** Proof incomplete
- + **5 pts** All correct

QUESTION 5

5 Question 2-b 5 / 6

- + **1 pts** Wrote XOR to Boolean
- + **2 pts** Expanded the Boolean expression
- + **1 pts** Simplified the expanded expression
- ✓ + **6 pts** Got the final DNF
- + **0 pts** Nothing
- ✓ - **1 pts** Did not simplify / not DNF / at least one term is wrong

QUESTION 6

6 Question 2-c 2 / 6

+ 6 pts Got the minimal terms

✓ + 2 pts Built on the previous answer

+ 1 pts Drew the k-map

+ 2 pts Verified using k-map

+ 0 pts Nothing

- 1 pts minor mistake

QUESTION 7

7 Question 2-d 7 / 8

+ 2 pts Used NOR for other logics

+ 2 pts Gate diagram for previous logic

✓ + 8 pts All correct

+ 0 pts Nothing

✓ - 1 pts Not minimum / minor issue

QUESTION 8

8 Question 3-a 4 / 4

✓ + 4 pts Correct

+ 0 pts Wrong

QUESTION 9

9 Question 3-b 3 / 3

✓ + 3 pts correct

+ 0 pts wrong

QUESTION 10

10 Question 3-c 0 / 4

+ 4 pts correct

✓ + 0 pts wrong

+ 2 pts partially correct from previous wrong answer

+ 1 pts taking 4's complement

QUESTION 11

11 Question 3-d 3 / 3

✓ + 3 pts Correct

+ 0 pts Wrong

QUESTION 12

Question 3-e 11 pts

12.1 3-e-1 4 / 4

✓ + 4 pts Correct

+ 0 pts Wrong

12.2 3-e-2 4 / 4

✓ + 4 pts Correct

+ 0 pts Wrong

+ 2 pts partially correct from previously wrong answer

+ 1 pts Taking 19's complement

12.3 3-e-3 0 / 3

+ 3 pts Correct

✓ + 0 pts Wrong

+ 1 pts partially correct

QUESTION 13

13 Question 4-a 8 / 8

✓ + 8 pts Correct

+ 4 pts Used blocks/non-gate components but correct output

+ 4 pts Carry wrong

+ 4 pts Difference output wrong

+ 0 pts No design

+ 0 pts Wrong output for carry and difference

QUESTION 14

14 Question 4-b 6 / 6

✓ + 6 pts Correct

+ 3 pts Didn't use half-subtractors but correct output

+ 3 pts carry out wrong

+ 3 pts difference wrong

+ 0 pts Wrong/No design

QUESTION 15

15 Question 4-c 6.5 / 7

✓ + 7 pts Correct

+ 6.5 pts Missing first cin but otherwise correct

✓ - 0.5 pts Missing labels for the bit order (order does matter)

+ 5 pts Close design but not correct

+ 5 pts Subtracted cin from all bits

+ 3 pts Incomplete/Significant design issues but on right track

+ 2 pts Implemented wrong device

- **0.5 pts** Implemented using half-sub instead of full-sub
- + **0 pts** Wrong/No design

QUESTION 16

16 Question 4-d 13 / 14

- ✓ + **2 pts** Correct addition function
- ✓ + **2 pts** Correct subtraction function
- ✓ + **3 pts** Correct negation function
- ✓ + **2 pts** Correct multiplication function
- + **3 pts** MUXs used properly for switching outputs
- ✓ + **2 pts** Completed design/put together properly/correct outputs
- **0.5 pts** Unclear which wires go where/splitting of wires
- **0.5 pts** Unclear which inputs are which for functions
- **0.5 pts** Wrong codes correspond to functions
- **0.5 pts** Wrong names for adder/subtractor
- + **0 pts** No Design
- + **2 Point adjustment**
 - pc for mux +2
- ① need a mux here too

QUESTION 17

17 Question 5 0 / 5

- + **5 pts** Correct
- + **4.5 pts** Correct design idea; minor mistake
- + **4 pts** Correct design idea; wrong codes match
- + **3 pts** Correct design idea; not complete/significant mistake
- + **3 pts** Gray-Binary converters wrong
- + **2 pts** Did not handle Gray codes properly/Wrong idea for codes
- + **1 pts** Gray-binary converter only/Fundamentally wrong design
- ✓ + **0 pts** No/Wrong design

Midterm Exam

Name (Last, First): Deshpande, Parth

Student Id #: 005129861

Student to Left: Robert

Student to Right: Daivik

Do not start working until instructed to do so.

1. You must answer in the **space provided** for answers after every question. We will ignore answers written anywhere else in the booklet. **All pages in this booklet must be accounted** for otherwise it will not be graded.
2. You are permitted 1 page of notes 8.5x11 (front and back).
3. You may not use any electronic device.

Following table to be filled by course staff only

	Maximum Score	Your Score
Question 1	15	
Question 2	25	
Question 3	25	
Question 4	35	
Question 5 (EC)	+5	
TOTAL	100	

Question #1 (15 pts)

Consider the following Karnaugh Map for the Boolean function, Y. A blank truth table is provided for your convenience.

		AB			
		"00"	"01"	"11"	"10"
CD	"00"	0	0	1	1
	"01"	0	1	0	1
	"11"	0	1	0	0
	"10"	1	1	X	X

100X
01X01
1XX0
XX10

A	B	C	D	Y
0	0	0	0	
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	
1	1	1	1	

(a) Circle the prime implicants on the map. (5 pts)

How many prime implicants are there? 4

(b) Write the Boolean (sum-of-product) expression of the essential prime implicants of (b) (if any). (5 pts)

EssentialPrimeImplicants = $(A \wedge \neg B \wedge \neg C) \vee (\neg A \wedge B \wedge \neg D) \vee (A \wedge \neg D) \vee (C \wedge \neg D)$

(c) Express as a minimal sum of product, $\neg Y$. (5 pts)

The K-map is provided for your convenience.

		AB			
		"00"	"01"	"11"	"10"
CD	"00"	0	0	1	1
	"01"	0	1	0	1
	"11"	0	1	0	0
	"10"	1	1	X	X

$$\neg Y = (A \wedge \neg C) \vee (\neg A \wedge \neg B \wedge D) \vee (A \wedge B \wedge D) \vee (A \wedge C)$$

Question #2 (25 pts)

(a) Is DeMorgan's theorem still true with more than two variables? If so, prove it in the case of three variables x, y and z. (5 pts)

$\neg(x \wedge y \wedge z) = \neg x \vee \neg y \vee \neg z$ ← Prove.

$= \neg((x \wedge y) \wedge z)$

$= \neg(x \wedge y) \vee \neg z$

$(\neg x \vee \neg y) \vee \neg z = \neg x \vee \neg y \vee \neg z$ Hence proved $\neg(x \wedge y \wedge z) = \neg x \vee \neg y \vee \neg z$

(b) Rewrite the following Boolean equation in (Disjunctive) Normal form. (6 pts)

$f = \overline{A \oplus B} + \overline{B \oplus C}$

where \oplus means XOR operation, i.e., $A \oplus B = \overline{A}B + A\overline{B}$

Answer:

$\neg((A \wedge \neg B) \vee (\neg A \wedge B)) \vee \neg((B \wedge \neg C) \vee (\neg B \wedge C))$

$\neg(A \wedge \neg B) \wedge \neg(\neg A \wedge B) \vee (\neg(B \wedge \neg C) \wedge \neg(\neg B \wedge C))$

$(\neg A \vee B) \wedge (A \vee \neg B) \vee (\neg B \vee C) \wedge (B \vee \neg C)$

$((A \wedge \neg A) \vee (\neg A \wedge B) \vee (B \wedge A) \vee (B \wedge \neg B)) \vee ((\neg B \wedge B) \vee (\neg B \wedge \neg C) \vee (C \wedge B) \vee (C \wedge \neg C))$

$f = \underline{(\neg A \wedge \neg B) \vee (B \wedge A) \vee (\neg B \wedge \neg C) \vee (C \wedge B) \vee (A \wedge \neg A) \vee (B \wedge \neg B) \vee (C \wedge \neg C)}$

(c) Simplify f from (b) to a minimum sum-of-products. List which Boolean properties you use at each step of the simplification. Hint: you may use K-map to **verify** your answer. (6 pts)

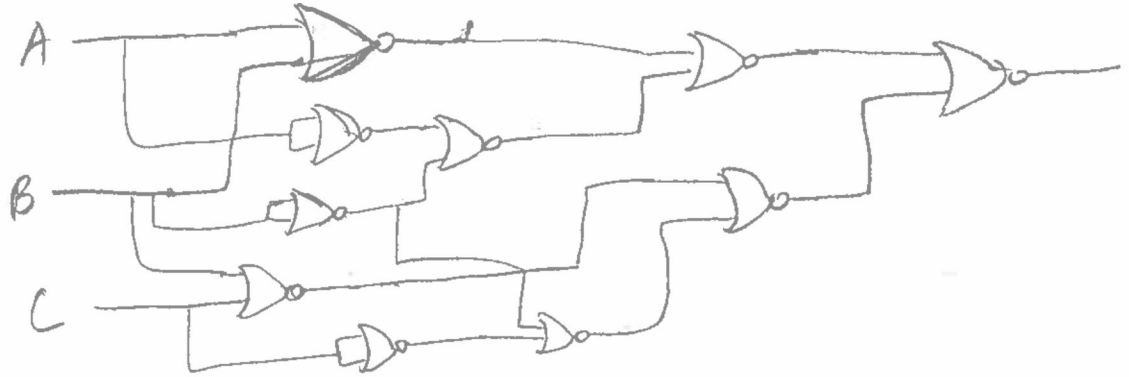
Answer:

$(\neg A \wedge \neg B) \vee (\neg B \wedge \neg C) \vee (B \wedge A) \vee (B \wedge C)$ \neq

$(\neg B \wedge (\neg A \vee \neg C)) \vee (B \wedge (A \vee C))$ Reverse Distributive

$f = \underline{\cancel{0(A \wedge \neg A)} (\neg A \wedge \neg B) \vee (\neg B \wedge \neg C) \vee (B \wedge A) \vee (B \wedge C)}$

(d) With only 2-input NOR gates, implement f with a minimal number of gates. Draw the gate diagram. (Note: no complemented inputs are given) (8 pts)



$$\begin{array}{r} 2048 \\ - 1645 \\ \hline -403 \end{array}$$

Question #3 (25 pts)

The following 12-bit word can be used to represent different numbers depending on the encoding

12b'1110_0110_1101

(a) If the word is 2's complement, what is the corresponding integer? (4 pts) -403

(b) If we convert the word (treated as unsigned) into base-4, what is the represented number? (3 pts)

321321

(c) If we take answer in (b), extending how we define 1's complement for base-2, write the 3's complement of the base-4 number. (4 pts)

012013

(d) What is this word in Hexadecimal? (3 pts) E6D

(e) In base-20 system, assume each digit is now 00, 01, 02, ..., 09, 10, 11, ..., 19 (each called a "vigint"). For example, 01,19 is 39 in decimal. Using 3 "vigints":

How would one represent a base-10 integer 1246? (4 pts) 03 02 06

What's the 20's complement representation of -1246 (i.e. the 20's complement of the 1246)? (4 pts) 16 17 14

Using the first vigint as the sign vigint, what is the most positive value in base-10 integer that can be represented? (3 pts) 7999

$$\begin{array}{r} 12 \\ 1024 \\ 512 \\ 64 \\ 32 \\ 8 \\ 4 \\ 1 \\ \hline 1645 \end{array}$$

$$\begin{array}{r} 1246 \\ - 2000 \\ \hline 246 \\ - 200 \\ \hline 46 \\ - 40 \\ \hline 6 \end{array}$$

$$\begin{array}{r} 319 \\ \times 4 \\ \hline 1276 \\ 1276 \\ \hline 1276 \\ \hline 1276 \end{array}$$

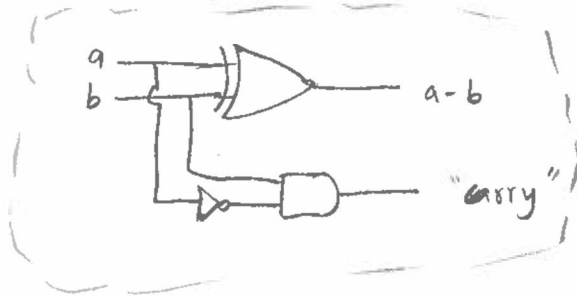
$$0 + 4^{11} + 4^{10} + 4^9 + 4^6 + 4^5 + 4^3 + 4^2 + 1$$

Question #4 (35 pts)

(a) Implement a one-bit "half-subtractor" from gates. The carry-out of this subtractor is 1 when the result is -1. The truth table for this is shown below: (8 pts)

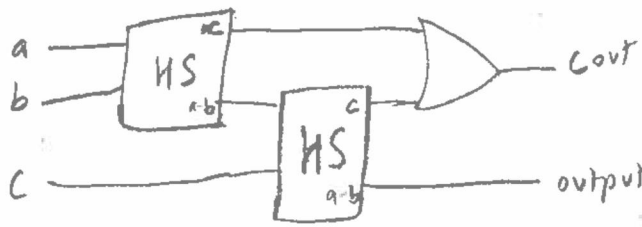
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a	b	a - b	"carry"
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0



HS → Half subtractor

(b) Implement a "full-subtractor" from "half-subtractor" blocks. (6 pts)

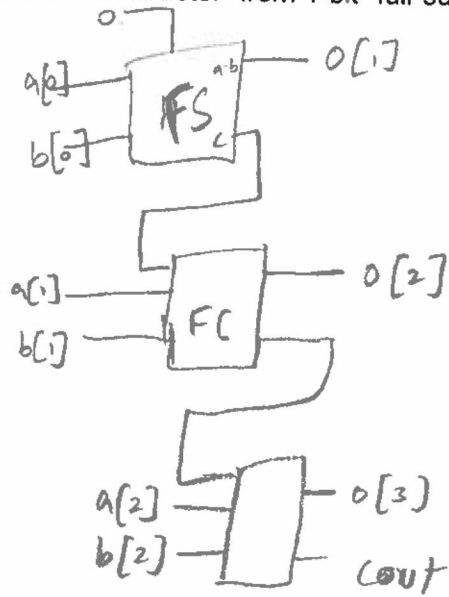


a	b	c	Carry
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0



a-b	c	s	c
0	0	0	0
1	1	1	1
0	1	1	1
1	0	0	0

(c) Implement a 3-bit "subtractor" from 1-bit "full-subtractor" blocks. (7 pts)



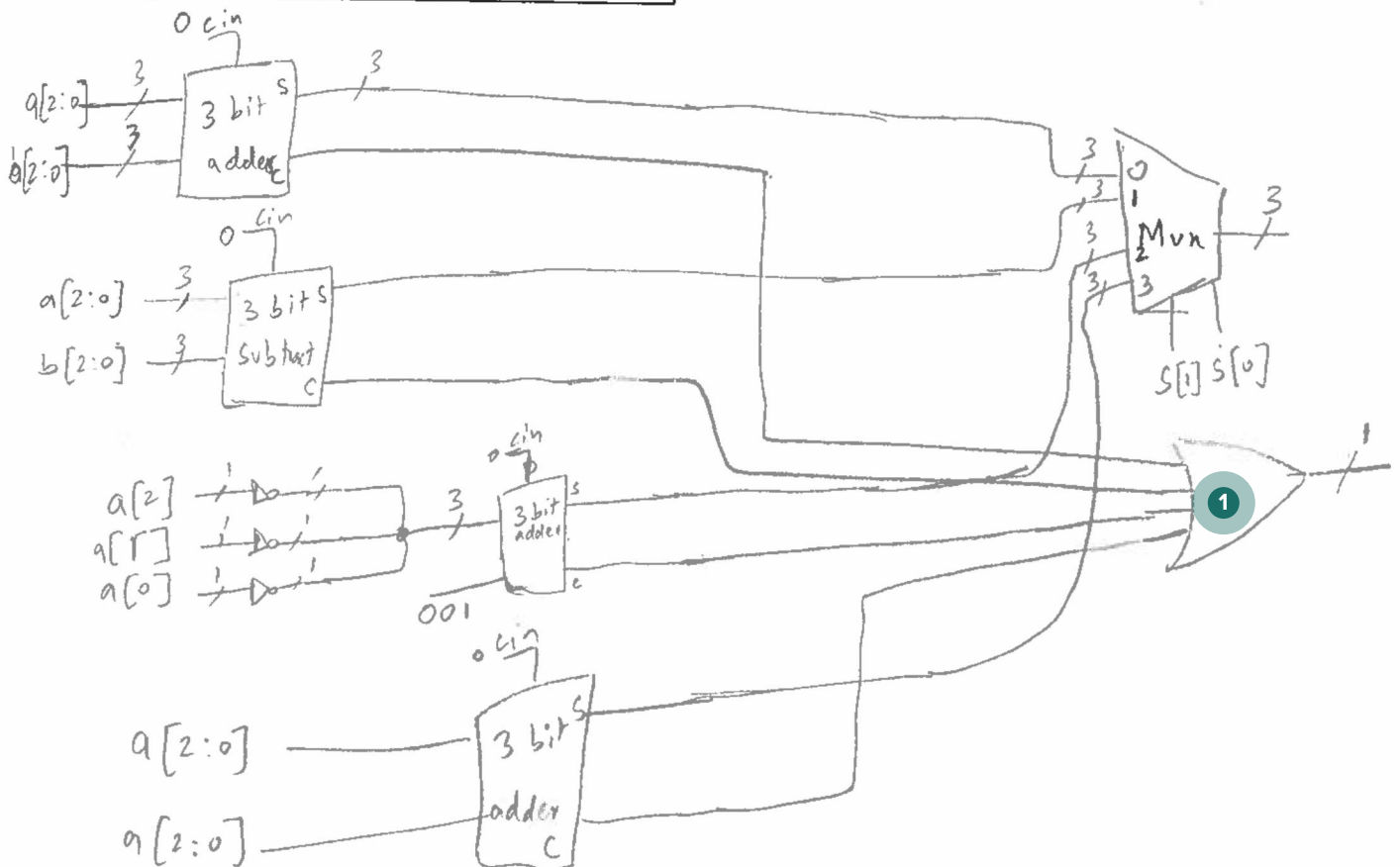
(d) Processors use a block called an ALU (Arithmetic Logic Unit) as part of their processing capability. Here we will implement a very basic ALU with a total of 4 functions, selected by a 2-bit code. Using the building blocks discussed in lecture and the 3-bit subtractor block, implement a 3-bit ALU that can add, subtract, negate one argument, and multiply by 2. The select codes are listed in the table below. Note that there are 3 inputs (3-bit a, 3-bit b, and the 2-bit select code) and 2 outputs (3-bit result and a 1-bit carry). (14 pts)

Hint: Multiplying a number is like shifting the bits to the left and using 0 as the lowest bit. An example: $a = 1 = 3'b001 \rightarrow 2a = 2 = 4'b0010$

Select Code	Result (3-bits)	Carry bit
00	$a + b$	carry out
01	$a - b$	carry out
10	$-a$	0
11	$2 \cdot a$	Product MSB

Select = S
a
b

000
001
010
011
100



Question #5 (Extra Credit - 5 pts)

Implement a 4-bit Gray code +1 incrementor using building blocks (no gates). The 4-bit Gray codes are shown below.

Decimal Number	Gray Code
0	0000
1	0001
2	0011
3	0010
4	0110
5	0111
6	0101
7	0100
8	1100
9	1101
10	1111
11	1110
12	1010
13	1011
14	1001
15	1000