Name: ______________________________

Student ID: ______________________________

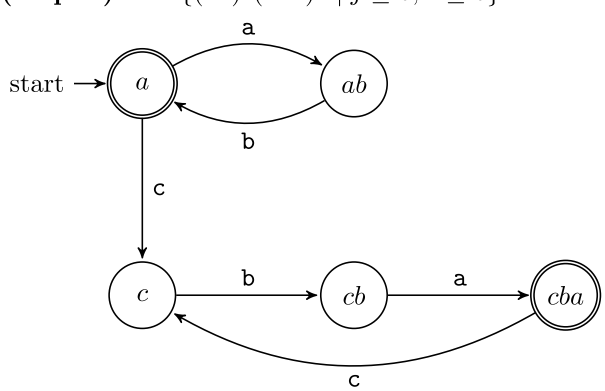
CS181 Spring 2012 - Quiz 1

Friday, April 27, 2010

You will have 45 minutes to complete this quiz. Read all problem statements carefully! You must show and explain all your work to get full credit. You may use any results given in class as long as you state them clearly.

1. **(40 pts. total) Regular Languages.** Indicate whether each of the following languages is regular or not. When a language is regular construct a NFSA without ε moves that accepts it. Otherwise, provide a brief proof as to why the language is not regular. Let $\Sigma = \{\mathtt{a,b,c}\}$.

   (a) **(10 pts.)** $L = \{(\mathtt{ab})^j(\mathtt{cba})^k \mid j \geq 0, k \geq 0\}$



   (b) **(10 pts.)** $L = \{(\mathtt{ab})^j\mathtt{c}^k(\mathtt{ba})^k \mid j \geq 0, k \geq 0\}$ **Expected Solution:** Here we use pumping lemma. Let p be the pumping length, set $\alpha = (ab)^0c^p(ba)^p$. Let $x = \epsilon, y = c^p, z = (ba)^p$. Now regardless of what u, v, w are, for $i = 2$ we have that the number of c's and the number of $(ba)'$s are different in the string, resulting in a contradiction.

   (c) **(20 pts.)** $L = \{\mathtt{a}^j\mathtt{b}^k \mid j \geq k \geq 0\}$ **Expected Solution:** Here the pumping lemma is a bit tricky. Let $\alpha = a^pb^p$. Then either $x = \epsilon, y = a^p, z = b^p$ would work when $i = 0$ or $x = a^p, y = b^p, z = \epsilon$ would work for any $i \geq 2$.

2. **(40 pts. total) Closure Properties.**

Let $\Sigma = \{\texttt{a, b}\}$ be the alphabet and let w_1, w_2 be two words over Σ. We say that w_1 is a PREFIX of w_2 if w_2 can be constructed by adding *0 or more* letters from the alphabet to w_1. For example, `abab` is a PREFIX of `ababaa`, `a` is a PREFIX of `a` and ϵ is a PREFIX of any word. Let L be any language. Define PREFIX(L) to be the language formed by the set of all PREFIXs of all the words in L. Formally,

$$\text{PREFIX}(L) = \{w \in \Sigma^* \mid \text{ there exists } y \in L, w = \text{PREFIX}(y)\}$$

Show that regular languages are closed under the PREFIX operation. i.e if L is any regular language, show that PREFIX(L) is a regular language.

Expected Solution: Let $M = (Q, \Sigma, \delta, q_0, F)$ be any DFSA for the language L. Then we claim that the DFSA where you change every live state of M into an accepting state accepts PREFIX(L). i.e $M' = (Q, \Sigma, \delta, q_0, F')$, where $F' \subseteq Q$ is the set of all live states accepts PREFIX(L).

This requires a proof and the proof proceeds in 2 parts. For part 1, we say that suppose $w \in \text{PREFIX}(L)$ then $w \in L(M')$. This is proved as follows.

Since $w \in \text{PREFIX}(L)$, there exists a string s such that $w \cdot s \in L$.

Thus when the automata consumes w and reaches a state q, q is a live state because there exists a string namely s which takes q to a final state. Thus $q \in F'$ and so $w \in L(M')$.

For part 2, we say that suppose $w \in L(M')$ then $w \in \text{PREFIX}(L)$. This is proved as follows.

Since $w \in L(M')$ upon consuming the w, the state q reached is in F' and is thus a live state.

By definition, there exists some string s which allows the automata to transition from q to a final state $\in F$. Thus, $w \cdot s \in L$ and thus $w \in \text{PREFIX}(L)$.

3. **(30 pts total) True or False.**

For each of the following below state true or false. Supplement your answer with a brief proof.

(a) **(20 pts.)** Define $\text{NFSA}_n = \{M \mid M \text{ is a NFSA with } \epsilon \text{ moves and } M \text{ has atmost } n \text{ states.}\}$. There is a finite state language not accepted by any machine in the set $\text{NFSA}_{4272012}$.

Expected Solution: True. The number of FSLs is infinite, but the number of automata with less than 4272012 states is finite. Hence it follows that there is a FSL not accepted by any automata in $\text{NFSA}_{4272012}$

(b) **(10 pts.)** Let L be any language. If $(L^*)^* = L$ then L has to be $\{\epsilon\}$. **Expected Solution:** False. $L = \Sigma^*$ also works.