Name: ______________________________

Student ID: ______________________________

CS181 Winter 2017 - Midterm

You will have 110 minutes to take this exam. This exam is open-book and open-notes, but any material not used in this course is prohibited. There are two questions and a total of 125 points and the first question has one part with 25 extra credit points. You will **not** get 20% points for just writing "I don't know". However, if you describe a non-trivial approach that you tried using to solve the problem but realized it doesn't work and explain correctly why it doesn't work and then write "I don't know" you will get 20% points for that problem. It is solely the grader's discretion as to whether your stated approach was indeed non-trivial. Place your name and UID on every page of your solutions. **Please use separate pages for each question.**

Honor Code Agreement: I understand this exam is open-book and open-notes, but any material not used in this course is strictly prohibited. I also understand that this exam is to be taken individually without any outside help (except possibly from the professor or the TAs) within the time limits set forth. I agree to adhere to the course honor code and if I am unsure of any rules of the honor code, I will ask for clarification from the professor or the TAs.

Signature: ______________________________

Question	Points
1	
2	
EC	
Total	

Note: While solving the problems, you can always reference theorems and claims from your homeworks, lectures, and anything in the book without having to prove those again.

Problem 1. Let a *Convergent DFA (ConDFA)* be a DFA with the following changes:

- A ConDFA only runs on input of even length and instead of being processed from one end to the other, the input is processed from both ends simultaneously. More specifically, if the ConDFA M is run on input $x = v_1 v_2 \dots v_n u_1 u_2 \dots u_n$ then M makes its first transition based on (v_1, u_n), its second transition based on (v_2, u_{n-1}), and its nth (and last) transition based on (v_n, u_1).
- Just as in the case of a DFA, a ConDFA is a five tuple $(Q, \Sigma, q_0, \delta, F)$. However, the transition function $\delta \colon \Sigma \times \Sigma \times Q \to Q$ now sends two symbols of Σ and a state (the pair of symbols that it gets to see and its current state) to another state. A ConDFA accepts its input if after processing its input it ends up in one of the states of F.

We say that a language is *convergent* if there exists a ConDFA recognizing it.
Example: A ConDFA can recognize the language $\{0^n 1^n \mid n \geq 0\}$ as follows. The ConDFA simply checks that every pair of symbols that it reads is $(0, 1)$. If this is the case then it accepts. Otherwise it rejects.

a) **(15 points).** Construct a ConDFA recognizing the language

$$L = \{0^i 1^j 2^k \mid j > 0 \text{ and } i \neq k \text{ and } i + j + k \text{ is even}\}.$$

Answer guide: In this problem it is sufficient to provide a construction and a short informal explanation about why your construction recognizes L.

b) **(30 points).** Prove that convergent languages satisfy the following pumping lemma.
Let L be a convergent language. Then there exists a $p \in \mathbb{N}$ such that for every string $s \in L$ with $|s| \geq 2p$ there exists a partition $s = xyzuv$ such that the following is satisfied

(i) $|yu| > 0$.
(ii) For every $i \geq 0$, $xy^i zu^i v \in L$.
(iii) $|y| = |u| > 0$.
(iv) $p \geq |xy|$ and $|xy| = |uv|$.

Note: You must attempt to prove (i) and (ii) to get partial credit. For a full solution all conditions must be proven.

c) **Extra Credit (25 points).** Using the pumping lemma, prove that the set of convergent languages is **not** closed under concatenation.

Problem 2. Let a *Crossing DFA (CroDFA)* be a DFA with the following changes:

- A CroDFA is related to a ConDFA. It only runs on input of even length that it process from both ends simultaneously. However, instead of stopping at the middle, a CroDFA completes a complete pass in each direction. I.e. if the CroDFA M is run on input $x = x_1x_2 \dots x_n$ then M makes its first transition based on (x_1, x_n), its second transition based on (x_2, x_{n-2}), and its nth (and last) transition based on (x_n, x_1).

We say that a language is *crossing* if there exists a CroDFA recognizing it.

a) **(25 points).** Let M_1 and M_2 be CroDFAs with input alphabet $\Sigma = \{0, 1\}$ recognizing the languages L_1 and L_2, respectively. Construct a CroDFA M recognizing the language

$$L_1 \bowtie L_2 = \{u\$v\$w \mid |u| = |w|, uw \in L_1 \text{ and } v \in L_2\}.$$

Answer guide: In this problem it is sufficient to provide a construction and a short informal explanation about why your construction recognizes L.

b) **(15 points).** Let $\Sigma = \{0, 1, 2, 3, 4, 5\}$ be the input alphabet of a CroDFA M. All we know about M is that

(i) M accepts the input 012345.

(ii) The computation path for M when processing 012345 is

$$q_0 \xrightarrow{(0,5)} q_1 \xrightarrow{(1,4)} q_1 \xrightarrow{(2,3)} q_2 \xrightarrow{(3,2)} q_3 \xrightarrow{(4,1)} q_3 \xrightarrow{(5,0)} q_4.$$

Find a word different from 012345 that is also accepted by M.

c) **(40 points).** Prove that crossing languages satisfy the same pumping lemma as convergent languages.
I.e. prove that for every crossing language L there exists a $p > 0$ such that for every string $s \in L$ with $|s| \geq 2p$ there exists a partition $s = xyzuv$ such that the following is satisfied

(i) $|yu| > 0$.

(ii) For every $i \geq 0$, $xy^izu^iv \in L$.

(iii) $|y| = |u| > 0$.

(iv) $p \geq |xy| = |uv|$.

Note: You must attempt to prove (i) and (ii) to get partial credit. For a full solution all conditions must be proven.
Hint: Think about part b!

Name: ______________________________

Student ID: ______________________________

Name: ______________________________

Student ID: ______________________________

Name: ______________________________

Student ID: ______________________________

Name: ______________________________

Student ID: ______________________________

Name: ______________________________

Student ID: ______________________________

Name: ____

Student ID: ____

Name: ______________________________

Student ID: ______________________________