1 Give a regular expression for each of the following languages:

(2 pts) a. binary strings of even length that contain at most one 0;

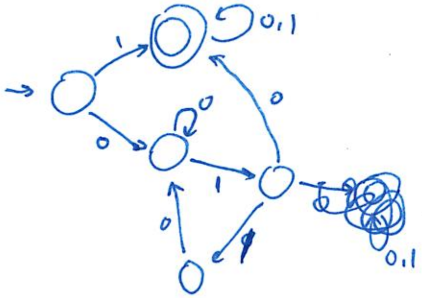
(2 pts) b. binary strings that do not contain a substring in $0^+1^+0^+1^+ \cup 1^+0^+1^+0^+$;

(2 pts) c. strings in $\{1,2\}^*$ such that the sum of the string's symbols is divisible by 3.

a). $(11)^* \cup 1(11)^* 0 (11)^* \cup (11)^* 0 1 (11)^*$

b). $\varepsilon \cup \Sigma \cup \Sigma^2 \cup \Sigma^3 \cup 0^+1^+0^+ \cup 1^+0^+1^+$

(3 pts) **2** Construct the smallest possible DFA for the language $1\Sigma^* \cup \Sigma^*010\Sigma^*$ over the binary alphabet Σ. You must prove that your DFA is the smallest possible.



Proof.

We want to show that, there are at least 5 equivalence classes.

ε, 1, 0, 01, ~~010~~, 011 belongs to different eq. classes.

See this matrix for suffix that differentiate them.

	ε	1	0	01	~~010~~ 011	~~011~~
ε						
1	ε					
0	1	0				
01	0	1	0			
~~010~~						
011	1	1	10	0		

There are at least 5 equivalence classes.

$\therefore$ # of states ≥ 5

$\therefore$ DFA with 5 states above is optimal.

3 Prove or disprove:

(2 pts) **a.** the concatenation of two languages is regular only if they are both regular;

(3 pts) **b.** for every language L, the indistinguishability relation $\equiv_L$ for L has as many equivalence classes as the indistinguishability relation for L^R.

a). ~~True~~. Disprove. See Counterexample below.

~~Proof. Let $L = L_1 L_2 = \{Q, \Sigma, \delta, q_0, F\}$.~~

~~Suppose L_2 is not regular.~~

Let $L_1 = \{w \mid w$ has more 1's than 0's$\}$

$L_2 = \{w \mid w$ has less 1's than 0's$\}$

$L_1 L_2$ concatenation is regular

because $L_1 L_2 = \Sigma^*$

However, as proven in class, both L_1 and L_2 are NOT regu

QED

b). True.

Proof. Let x_i, x_j be different eq. class in L

$x_i w_i \in L$, $x_j w_i \notin L$, $x_i w_j \notin L$, $x_j w_j \in L$.

$\Rightarrow$ $w_i^R x_i^R \in L^R$, $w_j^R x_j^R \in L^R$

$w_i^R x_j^R \notin L^R$ $w_j^R x_i^R \notin L^R$.

$\Rightarrow$ w_i^R, w_j^R belongs to different eq. classes.

(3 pts) 4 Let L be the language of binary strings obtained by concatenating two nonempty palindromes. Use the pumping lemma to prove that L is nonregular. You must not use the Myhill–Nerode theorem or closure properties.

Proof. $\forall p$, we construct string w' as below

$$w' = \overbrace{000\cdots0}^{2P\text{'s } 0\text{'s}}1\overbrace{000\cdots0}^{2P\text{'s } 0\text{'s}} = 0^{2P}10^{2P}$$

$$w'' = 1^{2P}01^{2P}$$

$$w = w'w''$$

$\forall x, y$, $|xy| \leq P$, $y = 0^k$ where k is a positive integer, $k \in [1, P]$

$$xy^2z = 0^{2P+k}10^{2P}1^{2P}01^{2P} \notin L$$

$$\cancel{xy^2z = 0^{P+k}10^{P}0^{P}10^{P} \notin L}$$

It ~~$0^{P+k}10^{P}0^{P}10^{P}$~~ cannot be expressed as concat of 2 palindrome.

① Suppose the first palindorme contains no 1's then the second palidrome begins with either 01, 00, or 10, but ends with 11, it's impossible

② Suppose the the first palindorme contains one or more 1's the 0^{2P+k} cannot find another ~~0 with~~ substring of 0's with length $\geq 2P+k$ as ending of the palindrome

$\therefore xy^2z \notin L$.

By pumping lamma, L failed in pumping Lemma.

(3 pts) 5 Your friend gives you two regular expressions over the binary alphabet. Describe a sequence of steps that would allow you to determine whether these regular expressions generate the same language. You must announce your answer in a finite number of steps.

First, generate DFA from these two Reg. Exp.

Let the DFA be D_1, D_2 with x_1, x_2 states respectively.

~~the~~ Let $x = \max(x_1, x_2)$. ~~we~~ .

$x \geq x_1, x_2 \geq$ number of equivalence classes in L_1, L_2.

~~Test every~~

~~Let k be the~~

Test every string with length $\leq x$, ~~and such~~

~~string must went~~ we ~~know that the base~~ ~~equival~~

Then, we check every loop within the DFA.

We test all strings with length $\leq x + k$,

where k is the max length of loop.

we have then tested every $w = x'y''z'$ with in these

two ~~a~~ DFA, ~~q~~ ~~B~~ we should know if they are equal.

6 For each of the following languages L, determine whether it is regular and prove your answer:

(2 pts) **a.** binary strings w such that some string of length 100 occurs as a substring in w at least twice;

(3 pts) **b.** the language $L = \{0, 1, 4, 9, 16, 25, 36, \ldots\}$ of strings that are the decimal representations (with no leading zeroes) of perfect squares, over the alphabet $\Sigma = \{0, 1, 2, 3, \ldots, 9\}$.

a). It's Regular. Let S= all string with length 100.
with S_i as i-th string in S.

~~Let S' be all string with prefix length =10 =suffix. and length < 2~~

$$\boxed{\bigcup_{i \in S} \Sigma^* S_i \Sigma^* S_i \Sigma^* \cup \bigcup_{i \in S'} \Sigma^* S_i \Sigma^*}$$

ex. 1 2

Let $S' = \boxed{1^n \cup 0^n \; 100 < n < 200}$

b). It's not regular. We want to find infinite # ~~Let i be prime no.~~ of equivalun classes.

$\forall i \in \mathbb{N}, \quad i^2 \in L. \quad i^2 00 = i^2 \times 10^2 = (10i)^2 \in L.$

However $(i^2+1)\,00 = (i^2+1) \times 100 = (10i)^2 + 100 \notin L$

$(10i+1)^2 = 100i^2 + 20i + 1.$ when $i > 5$.

It's not in L. ~~$a^2 - b^2 = (a+b)(a-b)$~~).

~~$(10i+1)^2 = 100i^2 + 20i + 1$~~

~~$200i + 100 = (a+b)(a-b)$~~

~~$= (a+10i)(a-10i)$~~