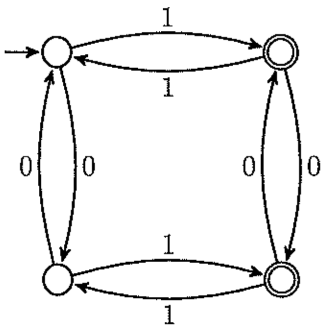
s) **1** Give a simple verbal description of the language recognized by the following DFA.



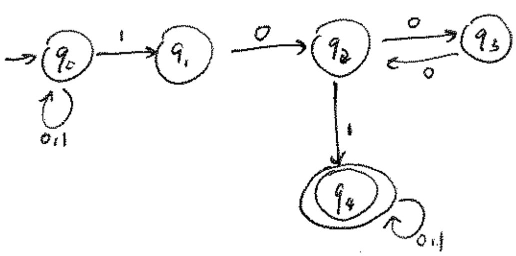
Odd number of 1's in the string.

2 Draw NFAs for the following languages, taking full advantage of nondeterminism:

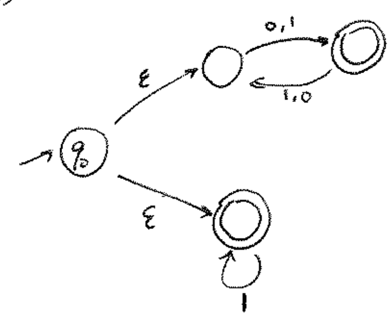
s) **a.** binary strings in which there is a pair of 1s separated by an odd number of 0s;

s) **b.** binary strings that have odd length or do not contain a 0.

a).



b)



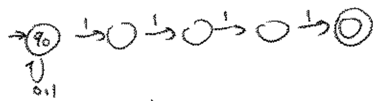
3 Prove that the following languages over the binary alphabet are regular:

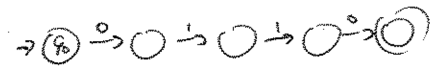
(2 pts) **a.** strings that contain as a substring 1111 or 0110 but not both;

(2 pts) **b.** nonempty strings in which the first and last symbols are different;

(2 pts) **c.** strings in which every 1 is immediately followed by 00.

a). String that contain 1111, 0110,





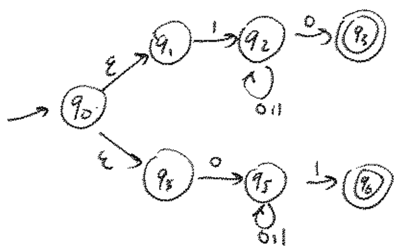
can be written in ~~D~~ NFA above. => They are regula[r]

Let them be L_1, L_2 respectively. The language is

$L = L_1 \cup L_2 \setminus (L_1 \cap L_2)$ is ~~re~~ therefore, regular.

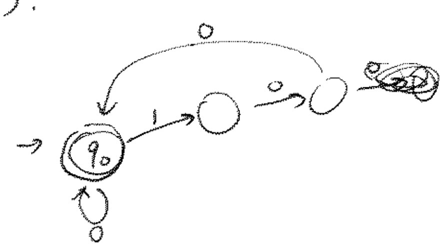
∴ It's regular

b).



It can be written as NFA like on th[e] right. It's therefore, regular.

c).



It can be written as NFA on the right. It's therefo[re] regular.

(3 pts) **4** Let L be a regular language over the binary alphabet. Let L' be the set of all strings in L that contain a 0 in an odd-numbered position. Prove that L' is regular.

Since L is a regular language, let

$\{Q, \Sigma, \delta, q_0, F\}$ be DFA for L.

Let $\{Q \times \{0,1\} \times \{0,1\}, \Sigma, \Delta, (q_0, 0, 0), F \times \{0,1\} \times \{1\}\}$

The first 0, 1 represent whether the current string length is odd. (0 even, 1 odd).

The second 0,1 represent have we found 0 in odd position (1: found, 0: not found).

~~$\Delta((q_i, a, 0), \sigma)$~~

$$\Delta((q_i, x, y), \sigma) = \begin{cases} (\delta(q_i, \sigma), \bar{x}, 1) & \text{if } (y=1) \text{ or } (x=0 \text{ and } \sigma=0) \\ (\delta(q_i, \sigma), \bar{x}, 0) & \text{otherwise} \end{cases}$$

Therefore, we have constructed an ~~NF~~ DFA D' for L'

(3 pts) 5 Fix a DFA D. A string w is called a *hot potato* for D if, starting from the initial state, D experiences a total of $|w|$ state changes as it processes w. Thus, every symbol of w causes a change of state. Prove that D's hot potatoes form a regular language.

For each state q_i in D,

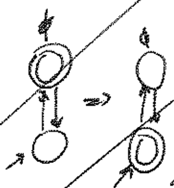
remove all the self-pointing arrow.

that forms a new DFA D'.

D' recognizes all hot potato in D.

(3 pts) **6** Your friend gives you a DFA D with the binary alphabet. Describe a sequence of steps that will allow you to determine whether there exists a string of even length that D accepts. You must announce your answer in a finite number of steps.

~~First, change every~~ ~~state from~~ ~~accept state to non-accept state. change every non-accept state to accept state. We have a DFA D' that recongize D~~ ~~com~~ ~~complement.~~



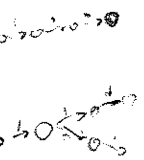
~~Use BFS to find the accepted state with distance of odd number. in D'.~~

~~If such~~

① First. check if there is any loop in D ~~with BFS~~
Ignore ~~Remove~~ all loop with even length.
If there is still a loop with odd length and some state on the loop ~~in D~~ can reach F, there must be a even length string accepted. Let $w = xy^*z$. $\begin{cases} \text{if } |xz| = \text{even} & \text{Let } y^* = y^2 \\ \quad\;\; = \text{odd} & \text{Let } y^* = y. \end{cases}$

② If there is no odd length loop that can lead to F ~~Then~~, we are safe to ignore the effect of loops.
We may use BFS and calculate if th

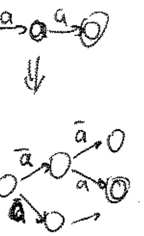
(3 pts) **7** Let L be a regular language over the binary alphabet. Consider the language L' of all strings obtained by taking a string in L and flipping an odd number of symbols in it. Prove that L' is regular.



Proof. ~~Let~~ Since L is a regular language.

Let $\{Q, \Sigma, \delta, q_0, F\}$ be DFA for L.

Then. Let



$\{Q \times \{0,1\}, \Sigma, \Delta, (q_0, 0), \cancel{F \times \{0,1\}}\ F \times \{1\}\}$ be ~~DFA~~ NFA for L'

where for each σ, there will be two arrows.

$\Delta((q_i, 0), \sigma) = \{(\delta(q_i, \sigma), 0) \text{ or } (\delta(q_i, \bar{\sigma}), 1)\}$.

$\Delta((q_i, 1), \sigma) = (\delta(q_i, \sigma), 1)$ or $= (\delta(q_i, \bar{\sigma}), 0)$

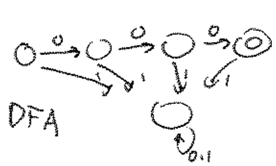
$\{0,1\}$ here record how many symbols have been flipped

0: even #

1: odd #

We have constructed a NFA for L', and thus L' is reg.

Ex. $L = \{00\}$



L'