

# CS180 Sample Midterm

1. For each of the following problems answer True or False and briefly justify your answer.

- (a)  $2^n = \Omega(3^n)$
- (b)  $f(n) = O(g(n))$  if and only if  $f(n)^2 = O(g(n)^2)$
- (c) In every instance of the Stable Matching Problem, there is a solution containing a pair  $(m, w)$  such that  $m$  is ranked first on the preference list of  $w$  and  $w$  is ranked first on the preference list of  $m$ .
- (d) A directed graph is strongly connected if and only if a DFS started from any vertex will visit every vertex in the graph.

2. Design a data structure to support finding the  $k$ -th largest number for an input stream. More specifically, your data structure needs to support the following two operations:

- Insert: insert a new number
- Find- $k$ -th-largest: output the current  $k$ -th largest number.

Note that both operations have to run in  $O(\log k)$  time.

3. Given a connected undirected graph  $G = (V, E)$ , design an  $O(|V| + |E|)$  time algorithm to find a vertex in  $G$  whose removal does not disconnect  $G$ . Prove the correctness of your algorithm. (Hint: think about the two end nodes of the diameter of the graph).

4. Consider the interval scheduling problem we talked in the class: there are  $n$  jobs and each job has its starting time  $s_j$  and finishing time  $f_j$ . The goal is to find the maximum number of jobs such that no pair of jobs overlap. Instead of using the algorithm we taught in the class, let's consider another greedy algorithm:

Let  $S$  denotes the set of these  $n$  intervals. Repeat until the job set  $S$  is empty:

- (a) Select the interval  $I$  that overlaps the least number of other intervals in  $S$ .
- (b) Add  $I$  to the final set  $S'$ .
- (c) Remove all intervals from  $S$  that overlap with  $I$ .

Prove or disapprove that this algorithm solves the problem.

5. Assume we are given an array that contains all but one of the integers from 1 to  $n + 1$ . Assume the array is sorted in increasing order, give an  $O(\log n)$  time algorithm to find the missing integer.