# CS 174A – Introduction to Computer Graphics, Fall 2017

## Midterm ExamSolutions

Nov 14, 2017

# A

***Name:*** ___________________________________

***Student ID:*** ______________________________

- Only students registered in the course may take this exam.
- The exam is closed book.
- You are not permitted to use any electronic equipment during the exam.
- No partial credit will be given for the multiple choice questions.
- ***Points will be deducted*** (50% the value of the question) for wrong answers on multiple-choice questions, so ***don't guess blindly***.
- Read all the questions first and start with the ones that seem easier to you.
- Make sure to circle your answers clearly.
- You must turn in the entire exam, including any extra sheets attached at the end.

| Question | Your Score | Maximum Possible Score |
|---|---|---|
| 1-4 | | 18 points |
| 5 | | 14 points |
| 6 | | 12 points |
| 7 | | 9 points |
| 8 | | 14 points |
| 9 | | 6 points |
| 10 | | 5 points |
| 11 | | 4 points |
| 12 | | 6 points |
| 13 | | 12 points |
| **Total** | | **100 points** |
| **Extra credit** | | **15 points** |

Name:

1) **(12 pts)** True/False:

- **T** / **F** – Generally, a graphics artist uses tools like Maya to create animations, and a computer graphics software engineer develops tools like Maya.
- **T** / **F** - The homogeneous points (2,2,2,4) and (4,4,4,4) map to the same Cartesian point after homogenization.
- **T** / **F** - A non-uniform scaling transformation leaves the w coordinate of a homogeneous point unchanged.
- **T** / **F** - Non-uniform scaling is in the class of affine transformations, but it is not a linear transformation.
- **T** / **F** - Affine transformations can change the origin of the local coordinate frame.
- **T** / **F** - Moving the camera 4 units forward in z is indistinguishable from moving the world 4 units backward in z.
- **T** / **F** - Perspective division happens after the viewing transformation and before the projection transformation.
- **T** / **F -** After perspective division, all points have been projected onto the image plane.
- **T** / **F** - Specifying the eye point, lookat point, and up vector completely determines a projection transformation
- **T** / **F -** Perspective transformations (including the perspective division) is a linear transformation.
- **T** / **F -** Both oblique and orthographic projections have projections perpendicular to the projection plane.
- **T** / **F** - The transformation from an orthographic view volume to the normalized device coordinate system depends on the size of the viewport.

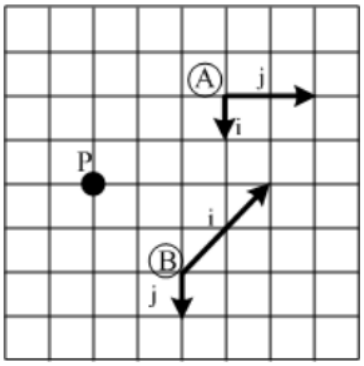
2) **(2pts)** Mass-spring-damper systems are most useful for modeling and animating
   **a)** Deformable solids
   **b)** Fluids
   **c)** Animals and humans

3) **(2 pts)** Lennard-Jones potentials are most useful for modeling and animating
   **a)** Deformable solids
   **b)** Fluids
   **c)** Animals and humans

4) **(2 pts)** Articulated body dynamics is most useful for modeling and animating
   **a)** Deformable solids
   **b)** Fluids
   **c)** Animals and humans

Name:

**5) (14 pts) (a) (4 pts)** Express point P in each of the two coordinate systems and vector $\mathbf{i}_A$ in terms of frame B




$P_a = [\ 2, -1.5, 1]^T$ , $P_b = [\ -1, -4, 1]^T$

$$\mathbf{i}_a = \left( \begin{bmatrix} .5 \\ -2 \\ 1 \end{bmatrix} - \begin{bmatrix} .5 \\ -3 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

**b) (2 pts)** What is a coordinate frame made up of?

basis vectors and an origin; [**i j k** O]

**c) (8 pts)** Derive the 3×3 homogeneous transformation matrix which takes a point from frame A coordinates and expresses it in terms of frame B coordinates using your answers from part (a) . Do not use simple transforms to represent the transformation matrix.

$$\begin{bmatrix} -1 \\ -4 \\ 1 \end{bmatrix} = -2\left( \begin{bmatrix} .5 \\ -2 \\ 1 \end{bmatrix} - \begin{bmatrix} .5 \\ -3 \\ 1 \end{bmatrix} \right) - 1.5\left( \begin{bmatrix} 1.5 \\ -1 \\ 1 \end{bmatrix} - \begin{bmatrix} .5 \\ -3 \\ 1 \end{bmatrix} \right) + \begin{bmatrix} .5 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} & 1 & .5 \\ 1 & 2 & -3 \\ & & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -1.5 \\ 1 \end{bmatrix}$$

Right-multiplying this matrix by (2, -1.5, 1) gives the solution (-1,-4,1).

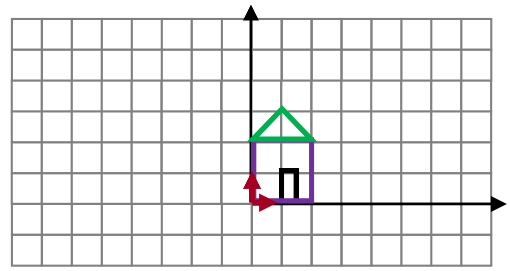
Note that the goal is not to reconstruct a series of basic transformations.

2 pt deduction if there is no derivation

Name:

**6) (12 pts)** a) **(6 pts)** Write the mathematical formula which is equivale nt to applying the following OpenGL transformations to the object, $\mathbf{P}_{house}$ and results in $\mathbf{P}'_{house}$

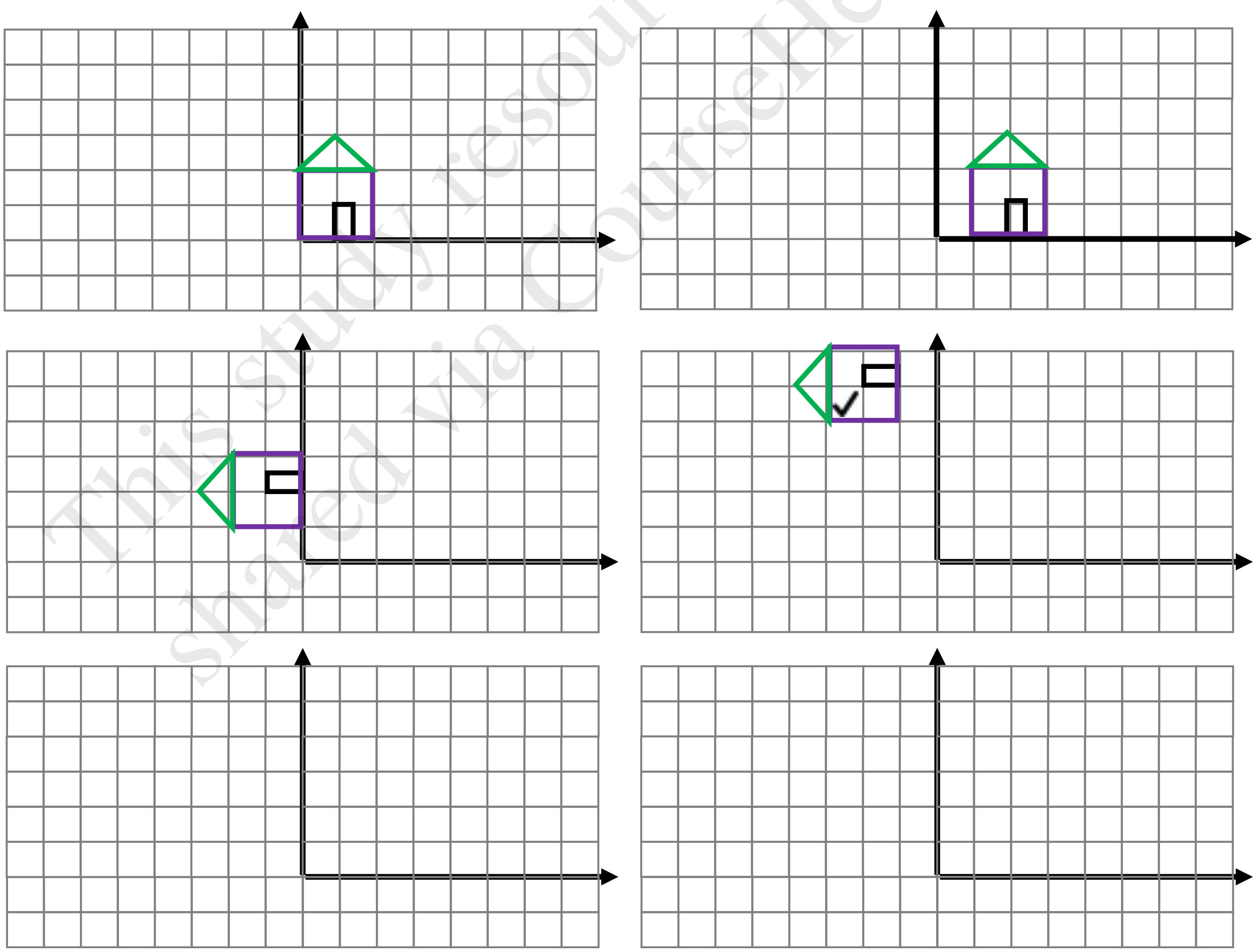
```
modelMatrix.setAsIdentity();
modelMatrix = modelMatrix * Translate(-1,3,0);
matrixStack.push(modelMatrix);
modelMatrix = modelMatrix * Rotate(90);
modelMatrix = modelMatrix * Translate(1,0,0);
modelMatrix = matrixStack.pop();
modelMatrix = modelMatrix * Rotate(90);
modelMatrix = modelMatrix * Translate(1,0,0);
drawHouse();
```



**P_house'=T(-1,3,0)*R_z(90)*T(1,0,0)*P_house**

**b) (6 pts)** Sketch the result of applying the prior OpenGL transformations in terms of point transformations. Show intermediate results.

**P_house'=T(-1,3,0)*R_z(90)*T(1,0,0)*P_house**



Name:

**7) (9 pts)** Given the following transformation M=C*B*A

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

**a) (6 pts)** Give the sequence of OpenGL transformations that would produce the same transformation matrix M

**A**=**R**x(-90), **B**=**S**(2,1,1), **C**=**T**(0,1,0)

```
modelMatrix.setAsIdentity();
modelMatrix = modelMatrix * Translate(0,1,0);
modelMatrix = modelMatrix * Scale(2,1,1);
modelMatrix = modelMatrix * RotateX(-90);
```

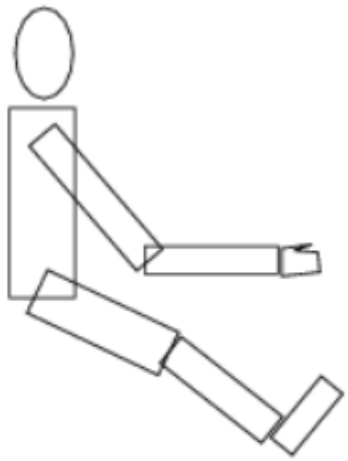
**b) (3 pts)** What is the inverse of this transformation matrix M? Express it in terms of elementary transformations.

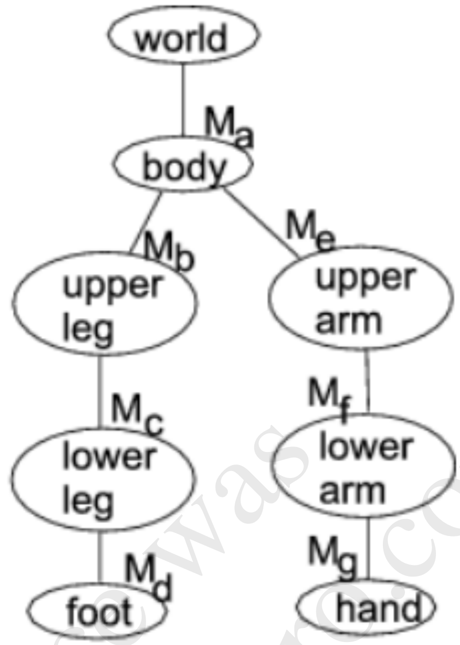
$$R^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} S^{-1} = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} T^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{M}^{-1} = \mathbf{R}^{-1}\mathbf{S}^{-1}\mathbf{T}^{-1}$$

Name:

**8)** **(14 pts)** The transformation matrices in the following scene graph define the relative transformations of each body part with respect to its parent.






**(7 pts)** Determine an expression for the composite transformation matrix that would be used to draw the hand, i.e., that takes a point from the hand coordinate frame to the world coordinate frame.

$P_{\text{in\_world\_coord}}$ **= A E F G** $P_{\text{in\_hand\_coord}}$ *(7 pts).*

*3 pts awarded if the multiplications were correct but the graph walk was backwards.*

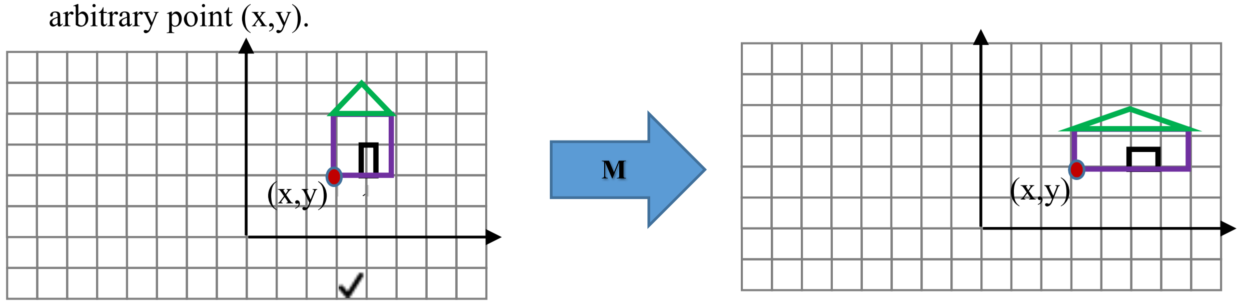
**(7 pts)** Determine an expression for the transformation matrix that takes a point from the hand coordinate frame to the foot coordinate frame.

$P_{\text{in\_foot\_coord}} = \mathbf{D^{-1}\, C^{-1}\, B^{-1}\;\, E\, F\, G}\; P_{\text{in\_hand\_coord}}$ (7 pts).

*3 pts awarded if the multiplications were correct but the graph walk was backwards.*

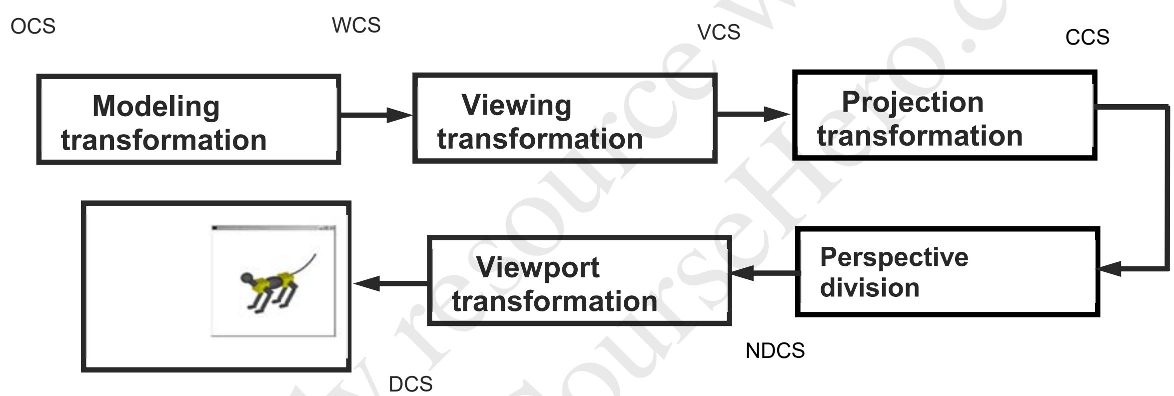
Name:

**9)** **(6 pts)** Give the 2D transformation matrix, **M**, for non-uniform scaling (sx, sy) around an arbitrary point (x,y).

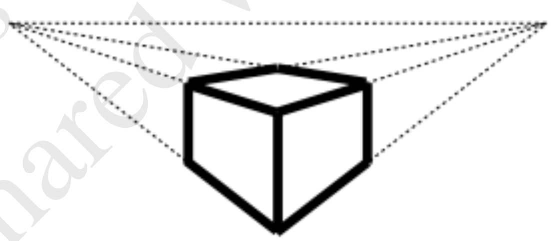



**M=T**(x,y)**S(**sx,sy)**T**(-x,-y)

**10)** **(5 pts)** Sketch a block diagram of the transformations involved in transforming a point from object coordinates to device coordinates. Label the intermediate coordinate frames.




**11)** **(4 pts)** Sketch a two-point perspective view of a cube. How would one produce such a photograph or image?



Drawing should depict a cube with two vanishing points or one obvious axis of which edges are parallel.

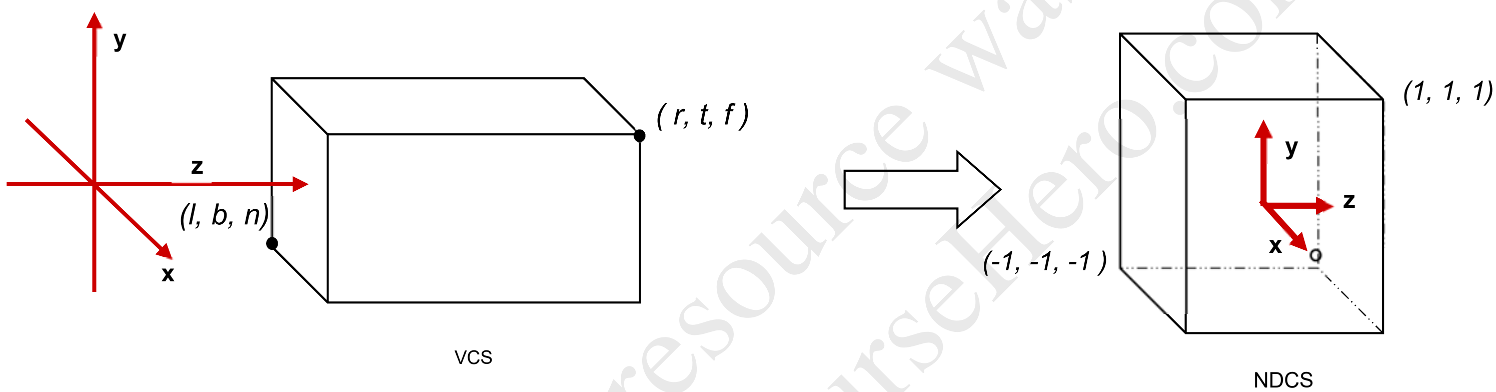
Explanation: align image plane with aforementioned parallel axis (2 pts).

Name:

**12) (6 pts)** A point P is at location (50,50) in DCS (display coordinates), with a viewport of width 100 and height 100. Give its x and y location in NDCS (normalized device coordinates).

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{100} & & & 0 \\ & \frac{2}{100} & & 0 \\ & & 1 & 0 \\ & & & 1 \end{bmatrix} \begin{bmatrix} 1 & & & -\frac{100-1}{2} \\ & 1 & & -\frac{100-1}{2} \\ & & 1 & 0 \\ & & & 1 \end{bmatrix} \begin{bmatrix} 50 \\ 50 \\ -1 \\ 1 \end{bmatrix}$$

**13) (12 pts)** Derive the orthographic matrix in terms of elementary transformations that transform objects from viewing coordinate system to normalized coordinate system. (Hint: Unlike the lecture, this VCS is a left handed coordinate system.)




$$\mathbf{M}_O = \begin{bmatrix} \frac{2}{r-l} & & & \\ & \frac{2}{t-b} & & \\ & & \frac{2}{f-n} & \\ & & & 1 \end{bmatrix} \begin{bmatrix} 1 & & & -\frac{l+r}{2} \\ & 1 & & -\frac{t+b}{2} \\ & & 1 & -\frac{f+n}{2} \\ & & & 1 \end{bmatrix}$$