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CS170A — Mathematical Models and Methods for Computer Science Spring 2013 D.S. Parker, Scott G-H. Tu

Midterm Examination
OPEN BOOK, OPEN NOTES, CLOSED NETWORK
Wednesday, May 8, 4:00pm-5:50pm

Do not cheat.

Problem	Points
1	4/25
2	5 /25
3	\ \ /25
4	23 /25
Total	170/100

Throughout this exam we will make use of the following dataset D of height and weight values:

i	h_i	w_i		_	1							
1	58	115 117	7			-	78	11		14	12	
2	59	117		0	2)	10		`		1 +	30
3	60	120	3			7						
4	61	123	3				15					
5	62	126	3					2	4			
6	63	129	3		7-		116	10. 32.35		40		
7	64	132	3		17		8	1		sasic	57	
8	65	135	3		5		123	x				77

Let h be the column vector of h_i values, and w be the column vector of w_i values. For the column vectors h and w, the least squares fit to w = ah + b is a = 2.917, b = -54.75.

For each of the following matrices, determine (yes/no) whether the matrix is: unitary, hermitian, invertible, positive definite.

-1 1-1	1	matrix	unitary?		herm	itian?	invertible?		positive	definite
1	$\frac{1}{\sqrt{2}}$	$\begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}$	True _	False	True _	False ⊠	True 🔁	False \square	True 🗀	False 🔀
- 12 - 12	$\frac{1}{\sqrt{2}}$	$\begin{pmatrix} -1_{i_1} & 1_{j_2} \\ 1_{i_2} & 1_{j_3} \end{pmatrix}$	True _	False	True 📐	False _	True 🖂	False _	True 🗀	False 🚖
1 1-1	$\frac{1}{\sqrt{2}}$	$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	True _	False 🖂	True _	False 📉	True 🔀	False _	True 🗀	False 🔀

For this examination, assume:

Definition: A is positive definite if it is Hermitian and all its eigenvalues are positive real values. For each of the following equations, mark whether the equation is True (valid for all specified matrices A) or False. Assume that A' denotes the hermitian transpose of A, and $i = \sqrt{-1}$.

For full credit, if you mark it True, you must explain how you derived this. If you mark it False, you must give a counterexample.

(a) True False

If X and Y are real matrices such that X + iY is unitary, then $\begin{pmatrix} X & -Y \\ Y & X \end{pmatrix}$ is orthogonal.

Explanation:

X +iy => X unitary - Y unitary > (x -4) orthogral

(b) True False

If both the determinant and trace of a 2×2 Hermitian matrix A are positive real values, then A is positive definite.

(Hint: the trace of A — the sum of its diagonal elements — is also the sum of its eigenvalues.)

Explanation: A: (2) I eigenvalue 3, -1 ... not position detinhe

(c) True False If H is Hermitian, then $(I+iH)(I-iH)^{-1}$ is unitary, where I is the identity.

(Hint: $(A^{-1})' = (A')^{-1}$).

 $\frac{([I+iH)(I-iH)^{-1})^{-1}}{([I-iH)^{-1})^{-1}} = \frac{2}{([I+iH)([I-iH)^{-1})^{-1}}$ $\frac{([I-iH)^{-1})^{-1}}{([I-iH)^{-1})^{-1}} = \frac{2}{([I-iH)^{-1})^{-1}}$ $\frac{([I-iH)^{-1})^{-1}}{([I-iH)^{-1})^{-1}} = \frac{2}{([I-iH)^{-1})^{-1}}$ Explanation: I +14 is hernetten I-14 = [1+14]

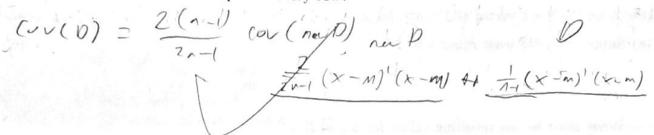
(d) True False If C is a correlation matrix with SVD C = USV', then U = V and S is a diagonal matrix with positive values on the diagonal

Explanation: The SUB of C is idential to 14 the decomposition / six a C is real and symmetric. Therefore the 5 vb 15 in the form of an eyendecomposition: C=USU'=QLQ' (Shas pishin adves on day offe, S is show whis + 11.1

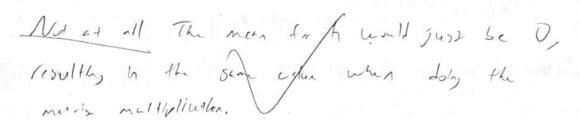
Covariance

Assume D is the 8×2 matrix with the two columns h and w mentioned earlier.

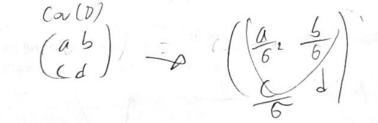
(a) What differences are there between the covariance matrices for D and for the dataset that is like D, but has two identical copies of every row?



(b) Suppose that we changed the vector h by subtracting its mean $\bar{h} = mean(h)$ from each of its entries. How would cov(D) change, if at all?

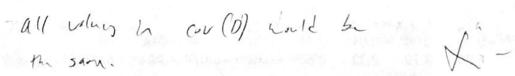


(c) Suppose that we changed the vector h by multiplying each of its entries by $1/\sigma$, where σ is the standard deviation of h. How would cov(D) change, if at all?



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(d) Suppose that we changed w to be h. How would cov(D) change, if at all?



(e) If we plot a least squares line fit through the (h, w) points, will the point made up of the mean values (\bar{h}, \bar{w}) also be on the line?

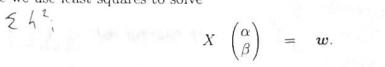
 $\frac{\sqrt{h}}{\sqrt{m}} = \frac{61.5}{10.5} = \frac{120.5}{8}$ $\frac{\sqrt{0}}{\sqrt{m}} = \frac{2.917 \cdot 61.5}{12.5} = \frac{54.25}{12.5}$ $= \frac{12.5.543}{12.5} = \frac{54.25}{12.5}$ 2 105 68

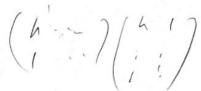
3. Least Squares

As stated earlier, for the column vectors h and w, the least squares fit to w = ah + b is a = 2.917, b = -54.75.

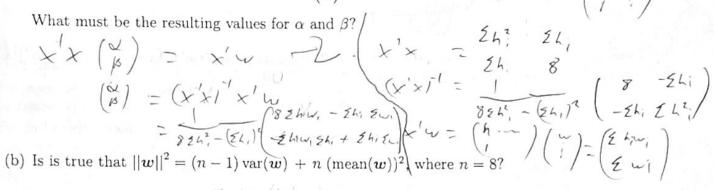
Let '1' be the 8×1 vector of 1's, and let X be the matrix with the two columns h and 1.

(a) Suppose we use least squares to solve





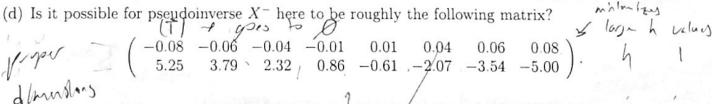
$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} x' x' | x' | w \\ 82 hw, -2h; 2w; \end{pmatrix}$$





(c) Estimate the R² value for this least squares fit.





- (e) Give a matrix A for which $c = A^- w$ is the set of coefficients $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ of a quadratic polynomial fitting the dataset D using least squares.

The covariance matrices for each of the four matrices D is:

$$cov(D) = \begin{pmatrix} 6.0 & 17.50 \\ 17.5 & 51.13 \end{pmatrix}$$

 $cov(D) = \begin{pmatrix} 6.0 & 17.50 \\ 17.5 & 51.13 \end{pmatrix} \qquad A - \geq \underline{1} \begin{pmatrix} \frac{77}{21} & \frac{7}{15} & \frac{7}{15} \\ \frac{77}{15} & \frac{7}{15} & \frac{7}{15} & \frac{7}{15} \\ \frac{7}{15} & \frac{7}{15} & \frac{7}{15} & \frac{7}{15} \\ \frac{7}{15} & \frac{7}{15} & \frac{7}{15} & \frac{7}{15} & \frac{7}{15} \\ \frac{7}{15} & \frac{7}{15} & \frac{7}{15} & \frac{7}{15} & \frac{7}{15} & \frac{7}{15} \\ \frac{7}{15} & \frac{7}{15} & \frac{7}{15} & \frac{7}{15} & \frac{7}{15} & \frac{7}{15} \\ \frac{7}{15} & \frac{7}{15$

and each has the SVD

$$cov(D) = \begin{pmatrix} -0.32 & -0.95 \\ -0.95 & 0.32 \end{pmatrix} \begin{pmatrix} 57.12 & 0 \\ 0 & 0.0088 \end{pmatrix} \begin{pmatrix} -0.32 & -0.95 \\ -0.95 & 0.32 \end{pmatrix}$$

(a) What is the largest eigenvalue of cov(D)? What is the first principal component of D?

What is the first principal component of
$$D$$
?

$$|a|ger 2 = 57.12$$

elsewhere
$$|a|ger 2 = 57.12$$

$$|a|ger 3 = 57.12$$

$$|a|ger 4 = 57.12$$

Suppose that we changed the vector h by subtracting is supposed that we changed the vector h by subtracting is supposed in a supposed that we changed the vector h by subtracting is supposed in a supposed in the vector h by subtracting is supposed in a supposed in the vector h by subtracting is supposed in a supposed in the vector h by subtracting is supposed in a supposed in the vector h by subtracting is supposed in the vector h by subtracting in h by subtracting in the vector h by subtracting in h by sub (b) Suppose that we changed the vector h by subtracting its mean from each of its entries. How

(c) Suppose that we changed the vectors h and w by multiplying each of their entries by 2. How would the first principal component of D change, if at all?





(d) Suppose that we switched w and h, so that the columns of D are reordered. How would the first principal component of D change, if at all?



(e) Suppose we added a new point $(h_9, w_9) = (60, 100000)$ to D. How would the principal components for resulting $\mathfrak{A} \times 2$ dataset D change?

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