19F-COMSCIM146-1 Midterm

WILSON JUSUF

TOTAL POINTS

98 / 100

QUESTION 1

1 Name 2 / 2

✓ **- 0 pts** **Correct**

QUESTION 2

Short Questions 27 pts

2.1 True/False 10 / 10

✓ **- 0 pts** **Correct**

- 2 pts a) incorect

- 2 pts b) incorrect

- 2 pts c) incorrect

- 2 pts d)incorrect

- 2 pts e) incorrect

2.2 Multiple Choice 9 / 9

✓ **- 0 pts** **Correct**

- 3 pts a) incorrect

- 3 pts b) NE[X}

- 3 pts c) 0.6

2.3 Regression 7 / 8

Write down the J(w)

✓ **+ 2 pts** **correct write down the square error**

+ 1 pts partly wrong of the error formulation

correct gradient w.r.t. w1

✓ **+ 2 pts** **all three terms correct**

+ 1 pts partly correct (pos/neg sign)

+ 0 pts wrong gradient

correct gradient w.r.t. w2

✓ **+ 2 pts** **all three terms are correct**

+ 1 pts partly correct (+/-)

+ 0 pts wrong gradient

correct simplified result for w1

+ 1 pts correct result for w1

✓ **+ 0.5 pts** **correct result based on the wrong gradient calculation or partly correct simplification; or no final substituation at all (get equation w1=sth. but has w2 there)**

+ 1 pts correct closed form result

+ 0.5 pts trying to use closed form but wrong calculation

+ 0 pts no solution for w1 or completely wrong answer

correct simplified result for w2

+ 1 pts correct final result

✓ **+ 0.5 pts** **partly wrong because the wrong gradient calculation or wrong simplification procedure; or no final simplification at all (get w2=some equation but still have w1/w2 there)**

+ 1 pts correct matrix formulation

+ 0.5 pts using closed form but wrong formulation

+ 0 pts no solution for w2 or completley wrong answer

Directly use the closed form

+ 8 pts correct w result

+ 2 pts correct closed form equation

+ 3 pts correctly write down the X^TX

+ 3 pts correctly write down the (X^Ty)

+ 1 pts wrong shape of the matrix

+ 1 pts partly mistake in the last final step

+ 1 pts partly correct closed format

+ 1 pts partly correct substituation of the matrix

+ 0 pts nothing

QUESTION 3

3 Decision Tree 15 / 15

Q1

- **2 pts** 100% wrong formula
- **0 pts** Wrong answer, we didn't ask to compute exact number, so as long as entropy formulation is correct, we don't remove points.
- **1 pts** Half wrong formula
- **3 pts** Anything else not above

Q2

- **2 pts** 100% wrong formula
- **0 pts** Wrong answer, we didn't ask to compute exact number, so as long as entropy formulation is correct, we don't remove points.
- **1 pts** Half wrong formula
- **3 pts** Anything else not above

Q3

- **2 pts** 100% wrong formula
- **0 pts** Wrong answer, we didn't ask to compute exact number, so as long as entropy formulation is correct, we don't remove points.
- **1 pts** Half wrong formula
- **3 pts** Anything else not above

Q4

- **1 pts** Wrong tree but correct use of entropy and reasoning in Info Gain
- **2 pts** Wrong tree without any reasoning. Remove points when entropy formula and numbers are correct. Conditioned on Q1-3, students should know how to use Info Gain. This is important concept.
- **1 pts** Draw 2 trees but did not say which one is chosen
- **1.5 pts** Correct reasons, but incorrectly use entropy formula
- **1 pts** Wrong application of entropy formula, and wrong tree without explanation.
- **1 pts** Not consistent with entropy computation.
- **1 pts** write algorithm but not say what is the tree, or say using info gain

Q5

- **2 pts** Anything absolutely wrong. We do not take points off Q5 if its answer depends on Q4.
- **3 pts** Does not attempt at all.

✓ **- 0 pts 100% correct everything**

QUESTION 4

4 Perceptron 20 / 20

✓ **+ 2 pts Q1 [2 pts]: answer is "d".**

Q2 [3 pts]

✓ **+ 1 pts Answer is No.**

✓ **+ 2 pts The weight will not change and stay the same as the initial value.**

+ 0 pts Note: weight can be initialized as any value and not necessarily to be zero.

- 0.5 pts Partially Correct for the second checkpoint: not only the weight not converged, the weights actually will not be updated

+ 0 pts Note: I think you understand this question, although we shouldn't allow the parameter not updated.

Q3 [5 pts]

✓ **+ 2 pts Before Update: y_i w^T x_i <= 0; After Update: we want y_i (w + \alpha y_i x_i)^T x_i > 0**

✓ **+ 2 pts \alpha > -y_i w^T x_i / ||x_i||^2 y_i^2**

✓ **+ 0.5 pts Since y_i = +1 or -1, y_i^2 = 1**

✓ **+ 0.5 pts Therefore: \alpha > -y_i w^T x_i / ||x_i||^2**

- 1 pts Sign is reversed for checkpoint 1.

- 0.5 pts Lack of explanation for the second, third and fourth checkpoints.

+ 0 pts We can actually choose any learning rate larger than -yi w xi / ||xi||^2, instead of only the marginal value.

+ 0 pts Note: I assume you know the third checkpoint: y_i^2 = 1, but you should actually clarify it.

- 0.5 pts Note: you cannot delete x or y from nominator and denominator, as it's matrix multiplication

- 0.5 pts Note: x not necessarily only have two dimension.

- 0.5 pts Note: It's ||x||^2, not ||x^2||. Please see the definition of norm.

- 0.5 pts Note: miss a x for norminator

- 0.5 pts Sign is reversed for checkpoint 2

Q4(a) [3 pts]

✓ **+ 1 pts** **u^T w^t = u^T (w^{t-1} + \alpha y_{I_t} x_{I_t}).**

✓ **+ 1 pts** **\forall i, y_i(u^Tx_i) >= \gamma.**

✓ **+ 1 pts** **induction and initial state (w^0 = 0).**

- 1 pts Missing alpha for the first point, and more alpha in the second checkpoint.

- 0.5 pts Lack of explanation for the third checkpoint on initial state (w^0=0)

- 1 pts Didn't combine the first two checkpoints and get correct bound.

+ 0 pts Note: in 4(b) you mention w^0 = 0, here I assume you know it, but you should actually clarify it.

- 0.5 pts Didn't finish the proof

Q4(b) [3 pts]

✓ **+ 0.5 pts** **||w^t||^2 = ||w^{t-1} + \alpha y_{I_t} x_{I_t}||^2 = ||w^{t-1}||^2 + 2\alpha y_{I_t} (w^{t-1})^T x_{I_t} + (\alpha y_{I_t})^2 ||x_{I_t}||^2.**

✓ **+ 0.5 pts** **Since the perceptron with parameter (w^t) makes mistake on data point (x_{I_t}, y_{I_t}): y_{I_t} (w^{t-1})^T x_{I_t} < 0.**

✓ **+ 0.5 pts** **Since y_i = +1 or -1, y_i^2 = 1.**

✓ **+ 0.5 pts** **By definition, ||x_t||^2 <= R^2**

✓ **+ 1 pts** **induction and initial state (w^0 = 0).**

- 0.5 pts Lack of explanation for the second, third and fourth checkpoints (how to derive ||w^t||^2 - ||w^{t-1}||^2 <= alpha^2 R^2).

- 0.5 pts Lack of explanation for the fifth checkpoint on initial state (w^0=0)

+ 2.5 pts Another proof. I think it's correct, except that you should clarify w^0=0. Also, better to clarify why ||sum(yi*xi)||^2 <= max(||yixi||^2)

+ 2.5 pts Another proof. I think it's correct, except that you should clarify ||yi||=1.

Q4(c) [3 pts]

✓ **+ 1 pts** **Cauchy Schwarz Inequality: w^t \cdot u <= ||u|||w^t||.**

✓ **+ 1 pts** **||u|| = 1.**

✓ **+ 1 pts** **mistake bound is R^2/gamma^2.**

- 0.5 pts Lack of explanation for the first and second checkpoints.

+ 0 pts Note: ⊥ means vertical, and // means parallel. The in-equation w \cdot u = ||w|| only holds when u // w, instead of u ⊥ w. And also this inequality is cauchy schwarz inequality, better to clarify it.

✓ **+ 1 pts** **Q4(d) [1 pts]: No, as \alpha doesn't appear in the mistake bound.**

QUESTION 5

5 Logistic Regression 23 / 24

- 3 pts Logistic Regression Q1 : correct ans is 1. Your ans is incorrect.

- 3 pts Logistic Regression Q2. correct ans is 0. Your ans is incorrect.

- 3 pts Logistic Regression Q3: correct ans is 1/2. Your ans is incorrect.

Logistic Regression Q4. Given gamma = 0

- 1 pts sigma = 1 when w^Tx>0 (not mentioned)

✓ **- 1 pts** **sigma = 1/2 when w^Tx=0 (not mentioned)**

- 1 pts sigma = 0 when w^Tx<0 (not mentioned)

- 0 pts You have slight mistake

Logistic Regression Q5.

- 1 pts Did not mention Probability condition unchanged or P(y=1|x) >=0.5

- 2 pts Did not derive From P(y=1|x) >=0.5, to w^Tx >= 0

- 1 pts w^Tx >= 0

+ 2 pts Simply mention that From P(y=1|x) >=0.5, it can be derived that w^Tx >= 0 but does not derive it.

- 0 pts You have slight mistake in calculation.

- 4 pts Logistic Regression Q6. Correct ans is 1-P(y=1|x) = 1/(1+exp(w^Tx/gamma)). Your answer is incorrect.

Logistic Regression Q7.

- 1 pts Need to consider both cases when y=1 and y=-1.

- 2 pts incorrect/no mention of product of P(y|x) or sum of log(p(y|x))

- 1 pts incorrect/no mention of (i) max of -log function or (ii) min of +log or max of P or (iii) product/sum but mention P or log P

✓ **- 0 pts You have slight mistake. or Your answer is unclear.**

- 1 pts either in the equation (1) You used sigma in place of y or (2) did not use any y or (3) you forgot to use log

- 0 pts Logistic Regression: All correct

QUESTION 6

6 Maximum Likelihood 12 / 12

1)

+ **1.5 pts** Product decomposition
+ **1.5 pts** Correct indices/expression

2)

+ **1.5 pts** Log of joint distribution
+ **1.5 pts** Correct log transformation

3)

+ **2 pts** Partial with respect to lambda
+ **1 pts** mistake on partial with respect to lambda
+ **1 pts** set to zero + wrong math
+ **2 pts** Setting to zero
+ **2 pts** Correct final expression
+ **1 pts** Final expression minor mistake (sign, N missing)
+ **0 pts** No attempt/wrong

✓ **+ 12 pts All correct**

CM146: Introduction to Machine Learning — Fall 2019

Midterm Solution

Nov. 4^{th}, 2019

- Please do not open the exam unless you are instructed to do so.
- This is a closed book and closed notes exam.
- Everything you need in order to solve the problems is supplied in the body of this exam.
- Mark your answers ON THE EXAM ITSELF. If you make a mess, clearly indicate your final answer (box it).
- If you think something about a question is open to interpretation, please make a note on the exam.
- If you run out of room for your answer in the space provided, you can write it down in the last page and indicate clearly that you've done so.
- You may ask TA for scratch paper or scratch in the last page of the exam.
- You have 1 hour 30 minutes (90 minutes).
- **Besides having the correct answer, being concise and clear is very important. For full credit, you must show your work and explain your answers.**

Good Luck!

Legibly write your name and UID in the space provided below to earn **2 points**.

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Short Questions (27pts)

1. (10 pts) True OR False (check the box).

   (a) Decision tree with a larger depth is more likely to generalize better to new data points.
   ☐ True ☑ False

   (b) 5-NN (KNN with K=5) is more robust to outliers than 1-NN.
   ☑ True ☐ False

   (c) Comparing to stochastic gradient descent, gradient descent can always find the global minimum.
   ☐ True ☑ False

   (d) A classifier that attains 100% accuracy on the training set is always better than a classifier that attains 70% accuracy on the same training set.
   ☐ True ☑ False

   (e) If data is not linearly separable, K-NN algorithm cannot reach training error zero.
   ☐ True ☑ False (put $k = 1$)

2. (9 pts) Multiple Choice (check the box).

   $N_1 \geq 10$.

   (a) Suppose we want to compute 10-Fold Cross-Validation error on 1000 training examples. We need to compute error N_1 times, and the Cross-Validation error is the average of the errors. To compute each error, we need to build a model with data of size N_2, and test the model on the data of size N_3. What are the appropriate numbers for N_1, N_2, N_3?

   ☑ A. $N_1 = 10, N_2 = 900, N_3 = 100$
   ☐ B. $N_1 = 1, N_2 = 800, N_3 = 200$
   ☐ C. $N_1 = 10, N_2 = 1000, N_3 = 100$
   ☐ D. $N_1 = 1, N_2 = 1000, N_3 = 1000$

   (b) Let $X_1, \ldots, X_N$ are i.i.d. random variables with the same distribution of a random variable X. Let $E[X]$ to be the expectation of X. What is the expectation of $X_1 + X_2 + \ldots + X_N$? (by linearity of expectation)
   ☐ A. $E[X]$ ☑ B. $NE[X]$ ☐ C. $N^2E[X]$ ☐ D. 0

   (c) A coin is tossed 100 times and lands heads 60 times. What is the maximum likelihood estimate for the probability of heads.
   ☐ A. 1 ☐ B. 0.2 ☑ C. 0.6 ☐ D. 0

   $\binom{100}{60} \theta^{60} (1-\theta)^{40}$

3. (8 pts) We are given two-dimensional inputs x_i and their corresponding output y_i. We denote $x_{i,1}$ and $x_{i,2}$ to be the first and second dimension of x_i. We use the following linear regression model to predict y:

$$y_i = w_1 x_{i,1} + w_2 x_{i,2}.$$

Given a data set $\{(x_i, y_i)\}, i = 1, \ldots, N$, derive the best w_1 and w_2 that minimize the square error. To simplify the answer, you can use the following notations:

$$\alpha_1 = \sum_{i=1}^{N} x_{i,1}^2, \quad \alpha_2 = \sum_{i=1}^{N} x_{i,2}^2, \quad \alpha_{12} = \sum_{i=1}^{N} x_{i,1} x_{i,2}, \quad \beta_1 = \sum_{i=1}^{N} x_{i,1} y_i, \quad \beta_2 = \sum_{i=1}^{N} x_{i,2} y_i.$$

we want to minimize $J(w) = \frac{1}{2}\sum_i (y_i - w^T x_i)^2$ where $w \in \mathbb{R}^2$ & $w = (w_1, w_2)$.

so note that

$$\frac{\partial J(w)}{\partial w_1} = \sum_i (y_i - w^T x_i) \frac{\partial (y_i - w^T x_i)}{\partial w_1} = -\sum_i (y_i - w^T x_i) x_{i,1}$$

we want $\frac{\partial J(w)}{\partial w_1} = 0$ for minimal $J(w)$ since J convex.

$$-\sum_i (y_i - w^T x_i) x_{i,1} = 0 \Rightarrow -\sum_i y_i x_{i,1} + \sum_i (w^T x_i) x_{i,1} = 0$$

$$\Rightarrow -\sum_i (y_i x_{i,1} + w_1 x_{i,1}^2 + w_2 x_{i,2} x_{i,1}) = \beta_1$$

$$\Rightarrow w_1 \sum_i x_{i,1}^2 = \beta_1 - \sum_i w_2 x_{i,2} x_{i,1} \Rightarrow w_1 = \frac{\beta_1 - w_2 \alpha_{12}}{\alpha_1}$$

similarly,

$$\frac{\partial J(w)}{\partial w_2} = -\sum_i (y_i - w^T x_i) x_{i,2} = 0$$

$$\Rightarrow -\sum_i y_i x_{i,2} + \sum_i (w_1 x_{i,1} + w_2 x_{i,2}) x_{i,2} = 0$$

$$\Rightarrow -\beta_2 + w_1 \sum_i x_{i,1} x_{i,2} + w_2 \sum_i x_{i,2}^2 = 0$$

$$\Rightarrow w_1 \alpha_{12} + w_2 \alpha_2 = \beta_2 \Rightarrow w_2 = \frac{\beta_2 - w_1 \alpha_{12}}{\alpha_2}$$

we solve simultaneously.

$$w_2 = \frac{\beta_2 - \left(\frac{\beta_1 - w_2 \alpha_{12}}{\alpha_1}\right)}{\alpha_2} = \frac{\beta_2}{\alpha_2} - \frac{\beta_1}{\alpha_1 \alpha_2} + \frac{w_2 \alpha_{12}}{\alpha_1 \alpha_2} \Rightarrow w_2 - \frac{w_2 \alpha_{12}}{\alpha_1 \alpha_2} = \frac{\beta_2}{\alpha_2} - \frac{\beta_1}{\alpha_1 \alpha_2}$$

$$\Rightarrow w_2 \left(1 - \frac{\alpha_{12}}{\alpha_1 \alpha_2}\right) = \frac{\beta_2}{\alpha_2} - \frac{\beta_1}{\alpha_1 \alpha_2}$$

$$\Rightarrow w_2 = \frac{\frac{\beta_2}{\alpha_2} - \frac{\beta_1}{\alpha_1 \alpha_2}}{1 - \frac{\alpha_{12}}{\alpha_1 \alpha_2}}$$

similarly for w_1

$$w_1 = \frac{\beta_1 - \left(\frac{\frac{\beta_2}{\alpha_2} - \frac{\beta_1}{\alpha_1 \alpha_2}}{1 - \frac{\alpha_{12}}{\alpha_1 \alpha_2}}\right) \alpha_{12}}{\alpha_1}$$

Decision Tree (15 pts)

Consider the following training dataset with 2 features (Age and Weight), and the outcome is Diabetes. You don't have to simplify your answer and note $\log_2 3 \approx 1.6, \log_2 5 \approx 2.3$.

Patient	*Age*	*Weight*	*Diabetes*
1	Young	Heavy	No
2	Young	Heavy	No
3	Young	Heavy	No
4	Young	Heavy	No
5	Young	Light	No
6	Young	Light	No

Patient	*Age*	*Weight*	*Diabetes*
7	Young	Light	No
8	Young	Light	No
9	Old	Heavy	Yes
10	Old	Heavy	Yes
11	Old	Light	No
12	Old	Light	No

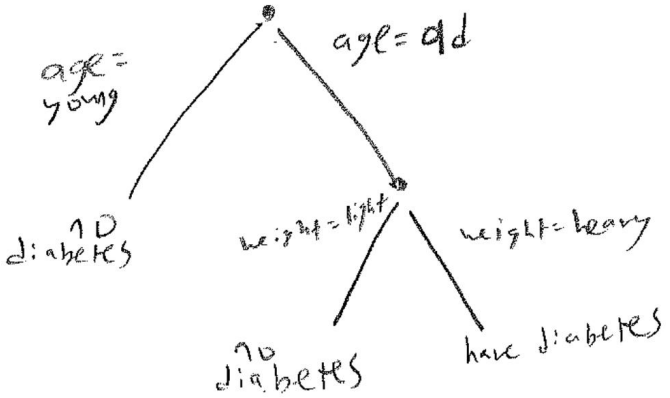
1. (3 pts) What is the entropy H(Diabetes)?
   Hint: $H(S) = -\sum_{v=1}^{K} P(S = a_v) \log_2 P(S = a_v)$

2. (3 pts) What is the information gain if we partition the data on the attribute *Age*?
   Hint: $\text{Gain}(S, A) = H(S) - \sum_{v \in \text{Value}(A)} \frac{|S_v|}{|S|} H(S_v)$.

3. (3 pts) What is the information gain if we partition the data on the attribute *Weight*?

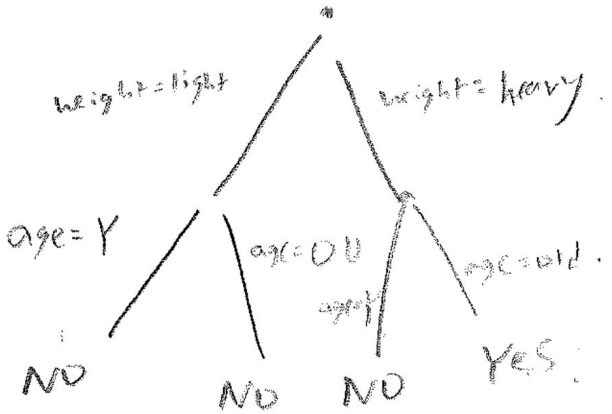
4. (3 pts) ~~Apply the ID3 algorithm to build the tree using both features Age and Weight~~

we would therefore split on Age first since it has highest gain.



5. (3 pts) Find another tree that yields the same training error as the tree built by ID3.

The tree above has training error 0%, so:



this also has 0% training error.

Perceptron (20 pts)

In this problem we consider various variants of Perceptron and explore their properties.

1. (2 pts) First, complete the Line 3 of the Perceptron algorithm by choosing from the following options. (Hint: Given a data point (x_i, y_i), if the current model parametrized by w makes a wrong decision, the model will update.)
   (a) $w^T x_i \geq 0$ (b) $y_i = 1$ (c) $y_i w^T x_i \geq 0$ (d) $y_i w^T x_i \leq 0$
   (e) $w^T x_i < 0$ (f) $y_i = -1$

Algorithm 1 Perceptron with learning rate α

1: Initialize $w = \vec{0}$
2: **for** each $(x_i, y_i) \in \mathbb{D}_{train}$, where $y_i = 1$ or -1 **do**
3: **if** $y_i w^T x_i \leq 0$ **then** (d)
4: $w \leftarrow w + \alpha y_i x_i$
5: **end if**
6: **end for**
7: **return** w

2. (3 pts) Can we set $\alpha = 0$? First answer yes/no then explain your answer.
   no. If we do so, then $w_{t+1} = w_t \ \forall t$, and our weights will stay the same.

3. (5 pts) Choose a learning rate such that when the algorithm sees two consecutive occurrences of the same example, it will never make a mistake on the second occurrence. Prove your answer is correct. (Hint: before update, statement in Line 3 is True. After the update, it has to be False)

for some x_i: we want $y_i w_t^T x_i \leq 0 \Rightarrow y_i w_{t+1}^T x_i > 0$.

note that $w_{t+1} = w_t + \alpha y_i x_i$

by multiplying y_i, we obtain

$y_i w_{t+1} = y_i w_t + \alpha x_i y_i^2$

note that $y_i^2 = 1$. So,

$\alpha > \frac{-y_i w_t^T x_i}{(x_i)^T x_i} = \frac{-y_i w_t^T x_i}{\|x_i\|^2}$

So we can choose any α greater than this, just before the update for $w_{t+1} = w_t + \alpha y_i x_i$,

Then we multiply with x_i:

$(y_i w_{t+1}^T) x_i = (y_i w_t + \alpha x_i y_i^2)^T x_i = y_i w_t^T x_i + \alpha (x_i y_i^2)^T x_i > 0$

(since we are using w_{t+1}^T)

4. We prove the convergence of Perceptron algorithm in class. In the following, we will derive the mistake bound of the Perceptron algorithm with learning rate α. We assume (1) there exist a vector u and $\gamma > 0$ such that $||u|| = 1$ and for all data $(x_i, y_i) \in \mathbb{D}_{train}$, $y_i(u^T x_i) \geq \gamma$; (2) there exist $R > 0$ such that $||x_i|| \leq R$. Complete the following proof.

   (a) (3 pts) Let w^t represent the weight vector w after t updates. We further assume $w^0 = \vec{0}$ (i.e., w is initialized with a vector with all zeros). Prove $w^t \cdot u \geq t\alpha\gamma$.

   (b) (3 pts) Show that after t updates, $||w^t||^2 \leq t\alpha^2 R^2$.

   (c) (3 pts) What is the mistake bound of the Perceptron algorithm with learning rate α? Prove your answer.

   (d) (1 pts) Does the choice of α affect the mistake bound? (Yes/No)

Logistic Regression (24 pts)

Remember we mentioned in the lecture, for a binary classification problem $y = 1, -1$ logistic regression model $P(y = 1|x)$ by $P(y = 1|x) = \sigma(w^T x) = 1/(1 + \exp(-w^T x))$. In the following, we consider a variant of logistic regression and model $P(y = 1|x)$ with

$$\sigma_\gamma(w^T x) = \frac{1}{1 + \exp(-w^T x/\gamma)},$$

where $\gamma > 0$ is a hyper-parameter that can be tuned. Answer the following questions.

1. (3 pts) When $w^T x \to \infty$, what is the value of $\sigma_\gamma(w^T x)$?

$\sigma_\gamma \to 1$

2. (3 pts) When $w^T x \to -\infty$, what is the value of $\sigma_\gamma(w^T x)$?

$\sigma_\gamma(w^T x) \to 0$.

3. (3 pts) What happen when $\gamma \to \infty$?

then $\exp(-w^T x/\gamma) \to \exp(0) = 1$

so $\sigma_\gamma(w^T x) \to \frac{1}{2}$

4. (3 pts) What happen when $\gamma \to 0$?

then $\exp(-w^T x/\gamma) = \exp(\pm\infty) =$ either ∞, or 0.

if $w^T x < 0$, $\gamma \to 0$ means $\sigma_\gamma(w^T x) \to 0$

if $w^T x > 0$, $\gamma \to 0$ means $\sigma_\gamma(w^T x) \to 1$

5. (4 pts) Show that for any γ the decision boundary is a linear function.

note that

$$\sigma_\gamma(w^T x) = \frac{1}{1 + \exp(-w^T x/\gamma)} \geq \frac{1}{2} \iff -\frac{w^T x}{\gamma} \geq 0$$

for any $\gamma > 0$, $-\frac{w^T x}{\gamma} \geq 0 \iff -w^T x \geq 0$

which resembles a linear decision boundary.

6. (4 pts) Write down $P(y = -1|x)$.

$$P(y=-1|x) = 1 - \frac{1}{1+\exp(-w^T x/\gamma)} = \frac{\exp(-w^T x/\gamma)}{1+\exp(-w^T x/\gamma)} = \frac{1}{1+\exp(w^T x/\gamma)}$$

7. (4 pts) Given a dataset $(x_i, y_i), i = 1, \ldots, N$. Write down the optimization problem maximizing the log-likelihood of the above model.

we can instead minimize log likelihood.

$$\underset{w}{\operatorname{argmax}} \prod_i P(y=y_i|x_i,w) = \underset{w}{\operatorname{argmin}} \left(-\sum_i \log P(y=y_i|x_i,w) \right)$$

$$= \underset{w}{\operatorname{argmin}} \left(-\sum_i \log \left([y_i = 1] \frac{1}{1+\exp(-w^T x/\gamma)} + [y_i = -1] \frac{1}{1+\exp(w^T x/\gamma)} \right) \right)$$

where $[y_i = j] = 1$ if $y_i = j$, and 0 otherwise.
(indicator function)

we find w by using the above objective function and ~~taking~~ doing GD w.r.t to w vector entries.

Maximum Likelihood (12 pts)

Let $x_1, \ldots, x_N$ are i.i.d. random samples from the exponential distribution with the probability density function (pdf):

$$P(x) = \lambda \exp(-\lambda x).$$

Answer the following questions.

1. (3 pts) Write down the joint probability of $P(x_1, x_2, \ldots, x_N)$.

$$P(x_1, x_2, \ldots, x_N) = P(x_1) P(x_2) \cdots P(x_N) = \prod P(x_i)$$
$$= \lambda^N \exp\left(-\lambda \left(\sum_{i=1}^{N} x_i\right)\right)$$

2. (3 pts) What is the log likelihood of λ given the dataset $\{x_1, x_2, \ldots, x_N\}$?

$$\ln P(x_1, \ldots, x_N) = N \ln \lambda + -\lambda \left(\sum_{i=1}^{N} x_i\right) = L(\lambda)$$

(name this $L(\lambda)$)

3. (6 pts) Derive the maximum likelihood estimator of λ (i.e., find the λ that maximizes the likelihood).

we want $\frac{\partial L(\lambda)}{\partial \lambda} = 0$ for above to be maximized

$$\frac{\partial L(\lambda)}{\partial \lambda} = \frac{N}{\lambda} - \sum_{i=1}^{N} x_i = 0 \quad \rightarrow \frac{N}{\lambda} = \sum_{i=1}^{N} x_i$$

so $$\lambda = \frac{N}{\left(\sum_{i=1}^{N} x_i\right)}$$

If you run out of room in answering questions, you can continued your answer here. Please indicate clearly that the answer is in the last page.