CS M146 Midterm

Jonathan Quach

TOTAL POINTS

78 / 100

QUESTION 1

Short Questions 40 pts

1.1 True/False 18 / 21

- **- 0 pts** all correct
- **- 3 pts** a incorrect
- ✓ **- 3 pts b incorrect**
- **- 3 pts** c incorrect
- **- 3 pts** d incorrect
- **- 3 pts** e incorrect
- **- 3 pts** g incorrect
- **- 3 pts** h incorrect

1.2 Model Evaluation 9 / 9

- ✓ **- 0 pts Correct**
- **- 3 pts** One incorrect answer
- **- 6 pts** Two incorrect answers
- **- 1 pts** One incorrect explanation
- **- 2 pts** Two incorrect explanations
- **- 0 pts** Click here to replace this description.

1.3 Decision Boundaries 5 / 10

- **- 0 pts** Correct
- ✓ **- 5 pts one incorrect answer**
- **- 10 pts** Two incorrect answers
- **- 1 pts** No mark which region is positive/negative
- **- 2 pts** 1 minor mistake

QUESTION 2

Perceptron 20 pts

2.1 algorithm 12 / 12

- **- 12 pts** 4 wrong answers
- **- 9 pts** 3 wrong answers
- **- 6 pts** 2 wrong answers
- **- 3 pts** 1 wrong answer
- ✓ **- 0 pts Correct**

2.2 seperability 4 / 4

- ✓ **- 0 pts Correct (answer no)**
- **- 2 pts** Answer true and show the data is linearly seperable
- **- 4 pts** incorrect answer

2.3 data augmentation 4 / 4

- ✓ **- 0 pts Correct: either describe how to extend w and x; or provide a correct 3-d weight vector.**
- **- 2 pts** minor error: either forget to describe how to extend either w or x; or provide an incorrect 3-d weight vector with some explanation
- **- 4 pts** Incorrect answer: provide 2-d weight vector; or provide an incorrect 3-d weight vector with no explanation

In your gragh, you have to exchange the position of "b" and "1".

QUESTION 3

Decision Tree 18 pts

3.1 H(Passed) 4 / 4

- ✓ **- 0 pts Correct**
- **- 2 pts** minor mistake
- **- 4 pts** incorrect
- **- 1 pts** forget negative sign in the entropy
- **- 0 pts** Correct formulations, but Incorrect calculation

3.2 G(passed, GPA) 0 / 4

- **- 0 pts** Correct
- **- 2 pts** minor mistake
- ✓ **- 4 pts incorrect answer**
- **- 1 pts** tiny mistake
- **- 0 pts** Correct formulations, but Incorrect calculation

3.3 G(passed, study) 0 / 4

- **- 0 pts** Correct

- **2 pts** minor mistake
- ✓ **- 4 pts incorrect**
- **1 pts** tiny mistake
- **0 pts** Correct formulation, but Incorrect calculation

3.4 Tree **4 / 6**

- **0 pts** Correct
- **6 pts** incorrect
- **2 pts** split when labels are pure
- ✓ **- 2 pts Split on attribute with lower information gain**
- **4 pts** Only one split

QUESTION 4

Linear Regression 20 pts

4.1 application **6 / 6**

- ✓ **- 0 pts Correct**
- **1 pts** minor mistake
- **2 pts** description is unclear
- **6 pts** incorrect

4.2 optimization algorithm **6 / 6**

- ✓ **- 0 pts Correct (GD,SGD, or analytic solution)**
- **2 pts** missing or incorrect gradient
- **6 pts** incorrect
- **4 pts** no attempt at gradient

4.3 global optimality **4 / 8**

- **0 pts** Correct
- ✓ **- 4 pts Incomplete answer, with arguements and derivation attempt**
- **1 pts** Almost correct (with a missing step)
- **6 pts** Argues PSD or squared function
- **8 pts** Incorrect

Hessian is incorrect

QUESTION 5

5 Name & ID **2 / 2**

- ✓ **- 0 pts Correct**
- **2 pts** No name and ID

gradescope

CM146: Introduction to Machine Learning	Winter 2018

Midterm

Feb. 13^{th}, 2018

- This is a closed book exam. Everything you need in order to solve the problems is supplied in the body of this exam.
- This exam booklet contains **four** problems.
- You have 90 minutes to earn a total of 100 points.
- **Besides having the correct answer, being concise and clear is very important. For full credit, you must show your work and explain your answers.**

Good Luck!

Name and ID: (2 Point) Jonathan Quach 604 595 720

Name		/2
Short Questions		/40
Perceptron		/20
Decision Tree		/18
Regression		/20
Total		/100

Short Questions [40 points]

1. [21 points] True/False Questions (Add 1 sentence to justify your answer if the answer is "False".)

   TA said ↗ bigger

   (a) When the hypothesis space is richer, over-fitting is more likely.

   True

   (b) Nearest neighbors is more efficient at training time than logistic regression.

   False both are equally efficient during training time. Since they all are batch learning

   (c) Perception algorithms can always stop after seeing γ^2/R^2 number of examples if the data is linearly separable, where γ is the size of the margin and R is the size of the largest instance.

   False, perceptron algorithm can stop after!! seeing $\frac{R^2}{\gamma^2}$ number of examples if data is linearly separable

   (d) Instead of maximizing a likelihood function, we can minimizing the corresponding negative log-likelihood function.

   True

   (e) If data is not linearly separable, decision tree can not reach training error zero.

   False decision tree is capable of modeling nonlinear data as it is a nonlinear classifier.

   (g) If data is not linearly separable, logistic regression can not reach training error zero.

   True

   (h) To predict the probability of an event, one would prefer a linear regression model trained with squared error to a classifier trained with logistic regression.

   False logistic regression is better for predicting an event since linear regression outputs real values and

logistic regression outputs some value in $[0,1]$.

(Kai-Wei and Sai taught me better)

2. [9 points] You are a reviewer for the International Conference on Machine Learning, and you read papers with the following claims. Would you accept or reject each paper? Provide a one sentence justification if your answer is "reject".

- **accept/reject** "My model is better than yours. Look at the training error rates!"

  $\boxed{\text{Reject}}$, a model can overfit training data and generalize poorly.

- **accept/reject** "My model is better than yours. After tuning the parameters on the test set, my model achieves lower test error rates!"

  Reject, a model should have tuned with a validation data set in order to avoid overfitting test data set.

- **accept/reject** "My model is better than yours. After tuning the parameters using 5-fold cross validation, my model achieves lower test error rates!"

  Accept

3. [10 points] On the 2D dataset of Fig. 1, draw the decision boundaries learned by logistic regression and 1-NN (using two features x and y). Be sure to mark which regions are labeled positive or negative, and assume that ties are broken arbitrarily. (Assuming a data point could be its own neighbor)

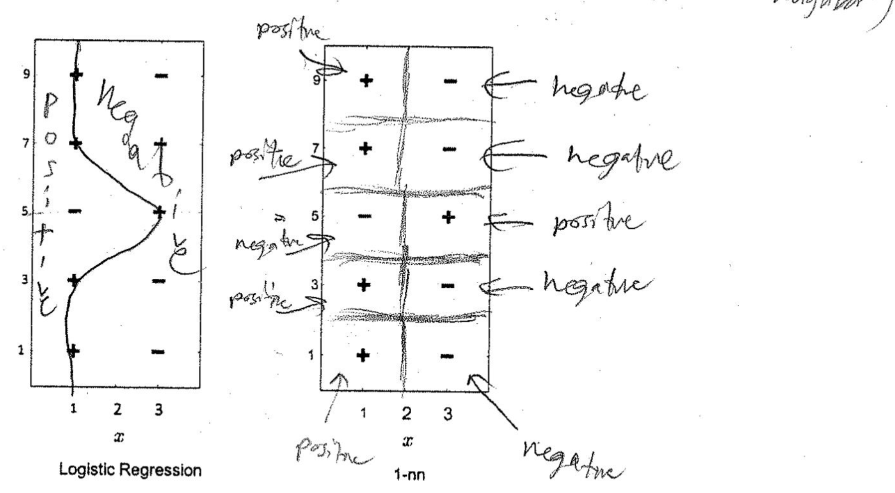


Figure 1: Example 2D dataset for question

Perceptron [20 points]

Recall that the Perceptron algorithm makes an updates when the model makes a mistake. Assume now our model makes prediction using the following formulation:

$$y = \begin{cases} 1 & \text{if} \quad w^T x \geq 1, \\ -1 & \text{if} \quad w^T x < 1. \end{cases} \tag{1}$$

1. [12 points] Finish the following Perceptron algorithm by choosing from the following options.

(a) $w^T x_i \geq 0$	(b) $y_i = 1$	(c) $w^T x \geq \cancel{0}\,1$ and $y_i = 1$	(d) $w^T x \geq \cancel{0}\,1$ and $y_i = -1$
(e) $w^T x_i < 0$	(f) $y_i = -1$	~~(g)~~ $w^T x < \cancel{0}\,1$ and $y_i = 1$	(h) $w^T x < \cancel{0}\,1$ and $y_i = -1$
(i) x_i	(j) - x_i	~~(k)~~ $w + x_i$	~~(l)~~ $w - x_i$
(m) $y_i(w + x_i)$	(n) $-y_i(w + x_i)$	(o) $w^T x_i$	(p) $-w^T x_i$

Given a training set $D = \{x_i, y_i\}_{i=1}^m$

Initialize $w \leftarrow 0$.

For $(x_i, y_i) \in D$:

if (g) negative should be positive

$w \leftarrow$ (k)

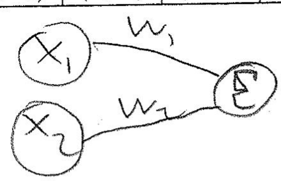
if (d) positive should be negative

$w \leftarrow$ (l)

Return w

2. [4 points] Let w to be a two dimensional vector. Given the following dataset, can the function described in (1) separate the dataset?

Instance	1	2	3	4	5	6	7	8
Label y	+1	-1	+1	+1	+1	-1	-1	+1
Data (x_1, x_2)	(2, 0)	(2, 4)	(-1, 1)	(1, -1)	(-1, -1)	(4, 0)	(2, 2)	(0, 2)



(1) $2w_1 \geq 1$

(2) $2w_1 + 4w_2 < -1$

(3) $-w_1 + w_2 \geq 1$

(4) $w_1 - w_2 \geq 1$

No

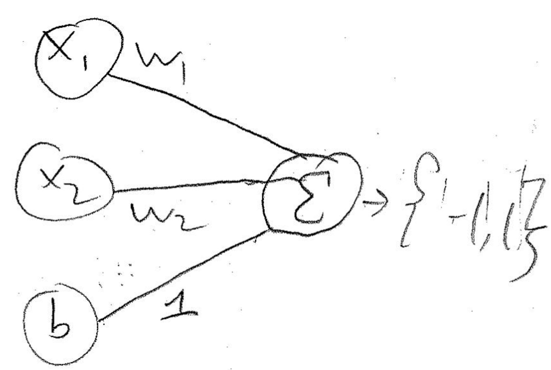
as adding (3) and (4) results in $0 \geq 2$, which is false

Instance	1	2	3	4	5	6	7	8
Label y	+1	-1	+1	+1	+1	-1	-1	+1
Data (x_1, x_2)	(2, 0)	(2, 4)	(-1, 1)	(1, -1)	(-1, -1)	(4, 0)	(2, 2)	(0, 2)

3. [4 points] If your answer to the previous question is "no", please describe how to extend w and data points x into 3-dimensional vectors, such that the data can be separable. If your answer to the previous question is "yes", write down the w that can separate the data.

We need a bias term b such that

$$w = \begin{bmatrix} w_1 \\ w_2 \\ b \end{bmatrix}.$$



Decision Tree [18 points]

We will use the dataset below to learn a decision tree which predicts if people pass machine learning (Yes or No), based on their previous GPA (High, Medium, or Low) and whether or not they studied.

GPA	Studied	Passed
L	F	F
L	T	T
M	F	F
M	T	T
H	F	T
H	T	T

For this problem, you can write your answers using $\log_2$, but it may be helpful to note that $\log_2 3 \approx 1.6$ and entropy $H(S) = -\sum_{v=1}^{K} P(S=v)\log_2 P(S=v)$. The information gain of an attribute A is $G(S,A) = H(S) - \sum_{v \in Value(A)} \frac{|S_v|}{|S|} H(S_v)$, where S_v is the subset of S for which A has value v.

1. [~~4 points] What is the~~ entropy H(Passed)?

$$H(Passed) = -\frac{2}{3}\log_2\frac{2}{3} - \frac{1}{3}\log_2\frac{1}{3}$$

$$\log_2 2^{-1} = -\log_2 2 = -1$$

2. [4 points] What is the entropy G(Passed, GPA)? info gain

$$H(passed) - \left[\left(\frac{1}{3}\cdot\frac{1}{2}\log_2\frac{1}{2}\right) + \left(\frac{1}{3}\cdot\frac{1}{2}\log_2\frac{1}{2}\right) + \left(\frac{1}{3}\cdot\frac{1}{1}\log_2\frac{1}{1}\right)\right]$$

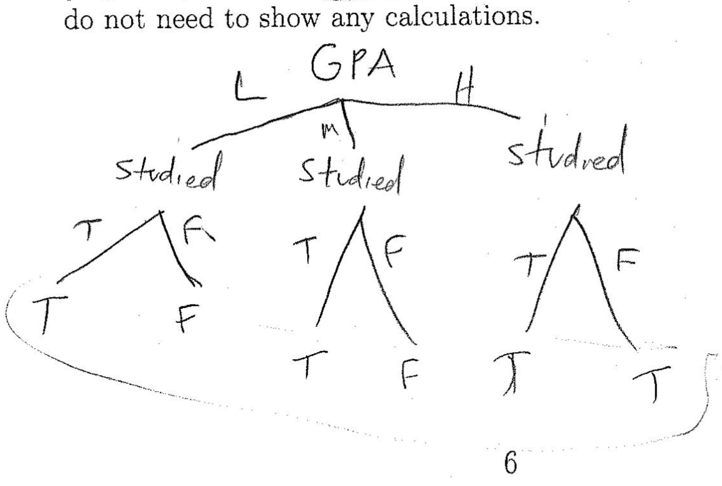
$$= H(passed) - \frac{1}{6} + \frac{1}{6} = \boxed{-\frac{2}{3}\log_2\frac{2}{3} - \frac{1}{3}\log_2\frac{1}{3} - \left(\frac{1}{3}\right)}$$

3. [4 points] What is the entropy G(Passed, Studied)? info gain

$$H(passed) - \left[\frac{1}{2}\cdot\frac{1}{1}\log_2 1 + \frac{1}{2}\cdot\frac{1}{3}\log_2\frac{1}{3}\right]$$

$$= -\frac{2}{3}\log_2\frac{2}{3} - \frac{1}{3}\log_2\frac{1}{3} - \frac{1}{6}\log_2\frac{1}{6}$$

4. [6 points] Draw ~~the full~~ decision tree that would be learned for this dataset. You do not need to show any calculations.



where Leaves are whether or not people passed

Linear Regression [20 points]

1. [6 points] Describe one application of linear regression. Please define clearly what are your input, output, and features.

Linear regression could be used to calculate the cost of a house with some features (I'll use 2 features for example)

Input: X, where X holds m training examples $x_1, x_2, \dots x_m$ and each $x_i \in X$ has say two features. x_{i1} and x_{i2} (say the first feature is # of bedrooms and second feature is crime rate of the location the house is in). Of course there could be n features, but this is an application/example!

Output: Price of the house in some range (e.g. [\$0.00, \$10,000,000]).

2. [6 points] Given a dataset $\{(x^{(i)}, y^{(i)})\}_{i=1}^{M}$ in a two dimensional space. The objective function of linear regression with square loss is

$$J(w_1, w_2) = \frac{1}{2}\sum_{i=1}^{M}(y_i - (w_1 x_1^{(i)} + w_2 x_2^{(i)}))^2, \tag{2}$$

where w_1 and w_2 are feature weight to be learned. Write down one optimization procedure that can learn w_1 and w_2 from data. Please be as explicit as possible.

We could take the gradient of $J(w_1, w_2)$ and have w_1 and w_2 be added with the negative of said gradient. Learning rate should also be multiplied by the gradient in order to have higher chance of convergence. This will result in a lower cost value evaluated in the next iteration, which is good

$W_1 = W_1 - \alpha \nabla J$ (where α is learning rate)

$W_2 = W_2 - \alpha \nabla J$

because the model will have better accuracy at predicting values for new/unseen data instances.

$(y - w^T x)^2$ $\frac{dw}{dx}$ $y = 3x$ $\frac{dy}{dx} = 3$ $\frac{\partial}{\partial \theta}$

3. [8 points] Prove that Eq. (2) has a global optimal solution. (Full points if the proof is mathematically correct. 4 points if you can describe the procedure for proving the claim.)

$$J(w_1, w_2) = \frac{1}{2} \sum_{i=1}^{M} \left(y_i - \left(w_1 x_1^{(i)} + w_2 x_2^{(i)} \right) \right)^2$$

$$\nabla J = \left\langle \sum_{i=1}^{M} \left(y_i - \left(w_1 x_1^{(i)} + w_2 x_2^{(i)} \right) \right) \cdot X_1, \sum_{i=1}^{M} \left(y_i - \left(w_1 x_1^{(i)} + w_2 x_2^{(i)} \right) \right) \cdot X_2 \right\rangle$$

get Hessian of $J(w_1, w_2)$ and show that it is positive semi definite, which shows that $J(w_1, w_2)$ is convex, thus having global optimal solution.

$$H = \begin{bmatrix} W_1 X_1 + \sum_{i=1}^{M} \left(y_i - \left(w_1 x_1^{(i)} + w_2 x_2^{(i)} \right) \right) & X_2 X_1 \\ X_1 X_2 & W_2 X_2 + \sum_{i=1} \left(y_i - \left(w_1 x_1^{(i)} + w_2 x_2^{(i)} \right. \right. \end{bmatrix}$$

(which mean no value is nonnegative)

Since all elements are squared values, we get that

For any $z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$, $z^T H z \geq 0$, so the function is convex and is guaranteed to have a global minimum as H is positive semi definite.