20W-COMSCICM124-1 Prelim Exam Version 1

TOTAL POINTS

9.99 / 11

QUESTION 1

110/1

- Opts Correct

√ - 1 pts Wrong

QUESTION 2

221/1

√ - 0 pts Correct

- 1 pts Wrong

QUESTION 3

331/1

√ - 0 pts Correct

- 1 pts Wrong

QUESTION 4

441/1

√ - 0 pts Correct

- 1 pts Wrong

QUESTION 5

551/1

√ - 0 pts Correct

- 1 pts Wrong

QUESTION 6

661/1

√ - 0 pts Correct

- 1 pts Wrong

QUESTION 7

771/1

√ - 0 pts Correct

- 1 pts Wrong

QUESTION 8

881/1

√ - 0 pts Correct

- 1 pts Wrong

QUESTION 9

991/1

√ - 0 pts Correct

- 1 pts Wrong

QUESTION 10

10 10 1 / 1

√ - 0 pts Correct

- 1 pts Wrong

QUESTION 11

11 11 0.99 / 1

√ - 0.01 pts Correct

- 0.03 pts Wrong

- 0.02 pts No answer

Preliminary Exam, Version 1

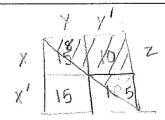
- Each question must have an explanation in the box below the answer selection. This explanation must convince the reader that you had a valid reason for your answer, and that your choice was not random. If you do not justify your answer, the problem will be marked as incorrect.
- The exam is open notes, but electronics of any kind are not allowed except for simple calculators that cannot access the internet.
- There is one extra credit question. If you correctly answer this question, you get 10 extra points. If you incorrectly answer this question, you will get deducted 5 points from your total points.
- If you correctly answer 10 out of 10 questions your grade will be 100.
- If you correctly answer 9 out of 10 questions your grade will be 95.
- If you correctly answer 8 out of 10 questions your grade will be 90.
- If you correctly answer 7 out of 10 questions your grade will be 85.
- If you correctly answer 6 out of 10 questions your grade will be 80.
- If you correctly answer 5 out of 10 questions your grade will be 70.
- If you correctly answer 4 out of 10 questions your grade will be 60.
- If you correctly answer 3 out of 10 questions your grade will be 45.
- If you correctly answer 2 out of 10 questions your grade will be 30.
- If you correctly answer 1 out of 10 questions your grade will be 15.
- If you answer 0 out of 10 questions your grade will be 0.

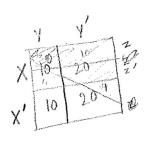
Name and ID:	
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- 1. Which of the following statements is true for any random variables X and Y?
 - \square (a) X and Y have zero correlation, then X and Y are independent. x
 - \square (b) Let Z be any random variable. Suppose X and Y are independent, then P(X,Y|Z)=P(X|Z)P(Y|Z).
 - X and Y are independent, then P(X+Y)=P(X)+P(Y)
 - ☐ (d) Choice (b) and (c)
 - \square (e) None of the above.

(c) This is always true
The others are incorrect (b) will only hold is
the variables are conditionally independent.

So answer ÌS





$$P(x,Y|Z) = \frac{3}{23}$$

$$P(x|Z) = \frac{18}{23}$$

$$P(y|z) = \frac{8}{23}$$

2. You flip a fair coin 10 times. X is the number of heads you get. What can you say about X? Select all that apply.

 \Box (a) X = 5 \Box (b) E(X) = 5 \Box (c) Var(X) = 1 \Box (d) Var(X) = 2.5 \Box (e) None of the above

 $E(X) = np = 10 \times 0.5 = 5$

 $Var(x) = npq = 10 \times 0.5 \times 0.5 = 2.5$

So answers are

b and a

- 3. You roll two fair, six-sided dice separately. Let X be the sum of the two results. What is E(X)?
 - \square (a) 3.5 \square (b) 6 \square (c) 7 \square (d) 12 \square (e) None of the above

$$E(x) = E(A+B) = E(A) + E(B) = 3.5 + 3.5$$

(where $E(A) = \frac{1}{6}(1+2+3+4+5+6) = \frac{24}{6} = 3.5$)

- 4. Let X and Y be independent. Suppose $X \sim \mathcal{N}(1,1)$ and $Y \sim \mathcal{N}(2,3)$. Find $E(2XY^2)$.
 - □ (a) 6 **□** (b) 14 □ (c) 16 □ (d) 18 □ (e) None of the above

$$E(2XY^2) = E(2X)E(Y^2)$$

$$E(2x) = 2E(x) = 2$$

- 5. Suppose that $X \sim N(0,2)$, and we are interested in computing $E(e^x)$. Directly computing $\int e^x f_X(x) dx$ is hard; in this problem, we will approximate it. Taylor series approximation for f(x) at a is $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n$. Use Taylor series to expand e^x at a=0 into polynomial degree 2. What is the expectation of your Taylor approximation? Answer is
 - \square (a) 0 \square (b) 1 \square (c) 2 \square (d) 3 \square (e) None of the above

$$f(x) = 1 + x + \frac{x^2}{2}$$

$$E(f(x)) = E(1+x+x^{2}) = E(x)+E(x^{2})+1$$

$$= 0+\frac{1}{2}(Var(x)+[E(x)]^{2})+1$$

$$= 0+\frac{1}{3}(2)+1 = \boxed{2}$$

- 6. How can you efficiently find x that minimizes $f(x) = xe^{-2x}$ where x is from 0 to 1.
 - \Box (a) Solve the equation $(1-2x)e^{-2x}=0$
 - \Box (b) Solve the equation $(1+x)e^{-2x}=0$
 - \square (c) Estimate f(x) at each value between [0,1].
 - Show that f''(x) < 0 in [0,1], and then pick the minimum of f(0) and f(1).
 - (e) None of the above

$$\frac{d}{dx}(xe^{-2x}) = e^{-2x} + -2xe^{-2x}$$

$$= (1-2x)(e^{-2x}) = 0 \qquad f'(y) = 0$$

$$f''(x) = -2(e^{-2x}) + (1-2x)(-2e^{-2x})$$

$$= -4e^{-2x} + 4xe^{-2x} = 4e^{-2x}(x-1)$$
Which is negative in (0,1)

80 we pick the minimum of the boundary
$$f(0) = 0 \qquad f(1) = \frac{1}{e^{-2}} > 0$$

- 7. Find the rank of the matrix
 - 2 5 0 2 4
 - $\begin{bmatrix} 2 & 3 & 8 \\ 5 & 1 & 7 \end{bmatrix}$
 - 1 7
 - (b) 2 \square (c) 3 \square (d) 4 \square (e) None of the above □ (a) 1

We know that $V_1 & V_2$ are linearly independent However $V_3 = 2V_2 + V_1$

So rank is 2

Ans

8. Which of the following is eigenvector of

 $\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$

 \square (a) [1,1] \square (b) [1,2] \square (c) [2,2] \square (d) [1,1] and [2,2] \square (e) None of the above

 $\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix} = 4 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ Yes

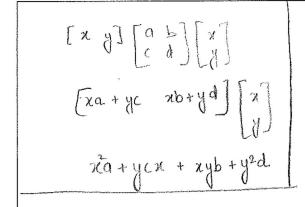
 $\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ No

 $\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ 8 \end{bmatrix} = 4 \begin{bmatrix} 2 \\ 2 \end{bmatrix} \text{ Yes}$

However [1], [2] correspond to the same eigenvalue (4)

Ans is a

- 9. Let A be a positive definite matrix; that is, for any vector x, we have $x^T A x > 0$. Which of the following is **not always** true?
 - All elements of A are positive. \Box (b) A has full rank.
 - \square (c) All eigenvalues of A are positive \square (d) A has a positive determinant. \curlyvee
 - ☐ (e) None of the above



A matrix can be positive definite without all elements being positive.

Ans is a

[1-1] positive definite with negative
$$-1$$
 definite with -1 dement

$$(1-\lambda)(6-\lambda)+1=0$$

$$6+\lambda^2-1+\lambda+1=0$$

$$\lambda^2-1+\lambda+1=0$$

7 1 149-28 = both are

10. Which of the options below is true for the matrix

$$\begin{bmatrix} -2 & 6 \\ 1 & -3 \end{bmatrix}$$

- \square (a) A is invertible. \square (b) A is negative definite. \square (c) A is negative semi-definite.
- ☐ (d) Choice (a) and (b) ☐ (e) None of the above

$$\det \begin{pmatrix} -2-\lambda & 6 \\ 1 & -3-\lambda \end{pmatrix} = \begin{pmatrix} (2+\lambda)(3+\lambda) - 6 & = 6+5\lambda+\lambda^2-6 \\ & = 5\lambda+\lambda^2=0 \end{pmatrix}$$

$$\lambda = 0$$
, -5

11. Extra credit question. If you correctly answer this question, you get 10 extra points. If you incorrectly answer this question, you will get deducted 5 points from your total points. Let a be any real number, and let:

$$A = \begin{bmatrix} 1 & 0 & a \\ 0 & 1+a & 0 \\ a & 0 & 1 \end{bmatrix}$$

What are the distinct eigenvalues of A?

(a)
$$1-a, 1+a$$
 \Box (b) $0, 1, 1+a$ \Box (c) $1-a, 0, 1+a$ \Box (d) $1-a, 1+a, 2+a$ \Box (e) None of the above.

$$\det \begin{pmatrix} 1-\lambda & 0 & a \\ 0 & 1+a-\lambda & 0 \\ a & 0 & 1-\lambda \end{pmatrix} = (1-\lambda)^2 (1+a-\lambda) - a(a)(1+a-\lambda)$$

$$= (1-\lambda+a) (1+\lambda^2-2\lambda-a^2) = 0$$

$$\delta = (1-\lambda+a) = 0 \quad \text{or} \quad 1+\lambda^2-2\lambda-a^2 = 0$$

$$\lambda = 1+a \quad \lambda^2-2\lambda+(1-a^2) = 0$$

$$+2 + \sqrt{4-4(1-a^2)}$$

$$\frac{2+2a}{2}, \frac{2-2a}{2}$$

$$\lambda = (1+a), (1-a)$$

$$\delta = (1+a), (1-a)$$