20W-COMSCICM124-1 Prelim Exam Version 1

TOTAL POINTS

9.99 / 11

QUESTION 1

1 1 **0 / 1**

- **- 0 pts** Correct
- ✓ **- 1 pts** Wrong

QUESTION 2

2 2 **1 / 1**

- ✓ **- 0 pts** Correct
- **- 1 pts** Wrong

QUESTION 3

3 3 **1 / 1**

- ✓ **- 0 pts** Correct
- **- 1 pts** Wrong

QUESTION 4

4 4 **1 / 1**

- ✓ **- 0 pts** Correct
- **- 1 pts** Wrong

QUESTION 5

5 5 **1 / 1**

- ✓ **- 0 pts** Correct
- **- 1 pts** Wrong

QUESTION 6

6 6 **1 / 1**

- ✓ **- 0 pts** Correct
- **- 1 pts** Wrong

QUESTION 7

7 7 **1 / 1**

- ✓ **- 0 pts** Correct
- **- 1 pts** Wrong

QUESTION 8

8 8 **1 / 1**

- ✓ **- 0 pts** Correct
- **- 1 pts** Wrong

QUESTION 9

9 9 **1 / 1**

- ✓ **- 0 pts** Correct
- **- 1 pts** Wrong

QUESTION 10

10 10 **1 / 1**

- ✓ **- 0 pts** Correct
- **- 1 pts** Wrong

QUESTION 11

11 11 **0.99 / 1**

- ✓ **- 0.01 pts** Correct
- **- 0.03 pts** Wrong
- **- 0.02 pts** No answer

CM124 CM224: Machine Learning Applications in Genetics	Winter 2020
Preliminary Exam, Version 1	

- **Each question must have an explanation in the box below the answer selection.** This explanation must convince the reader that you had a valid reason for your answer, and that your choice was not random. If you do not justify your answer, the problem will be marked as incorrect.
- The exam is open notes, but electronics of any kind are not allowed except for simple calculators that cannot access the internet.
- There is one extra credit question. If you correctly answer this question, you get 10 extra points. If you incorrectly answer this question, you will get deducted 5 points from your total points.
- If you correctly answer 10 out of 10 questions your grade will be 100.
- If you correctly answer 9 out of 10 questions your grade will be 95.
- If you correctly answer 8 out of 10 questions your grade will be 90.
- If you correctly answer 7 out of 10 questions your grade will be 85.
- If you correctly answer 6 out of 10 questions your grade will be 80.
- If you correctly answer 5 out of 10 questions your grade will be 70.
- If you correctly answer 4 out of 10 questions your grade will be 60.
- If you correctly answer 3 out of 10 questions your grade will be 45.
- If you correctly answer 2 out of 10 questions your grade will be 30.
- If you correctly answer 1 out of 10 questions your grade will be 15.
- If you answer 0 out of 10 questions your grade will be 0.

Name and ID:

1. Which of the following statements is true for any random variables X and Y?

☐ (a) X and Y have zero correlation, then X and Y are independent. ✗
☐ (b) Let Z be **any** random variable. Suppose X and Y are independent, then $P(X, Y|Z) = P(X|Z)P(Y|Z)$.
■ (c) X and Y are independent, then $P(X+Y) = P(X) + P(Y)$ ✓
☐ (d) Choice (b) and (c)
☐ (e) None of the above. ✗

(c) This is always true

The others are incorrect. (b) will only hold is the variables are conditionally independent.

So answer is $\boxed{c}$

	Y	Y'
X	~~15~~ 18	10
X'	15	~~10~~ 5

Z

$P(x, y | z) = \frac{8}{23}$

$P(x|z) = \frac{18}{23}$

$P(y|z) = \frac{8}{23}$

	Y	Y'
X	10	20
X'	10	20

Z Z'

$\frac{6}{30}$

$\frac{1}{3}$

$\frac{6}{30}$ $\frac{26}{30} \times \frac{6}{30}$

2. You flip a fair coin 10 times. X is the number of heads you get. What can you say about X? Select all that apply.

☐ (a) $X = 5$ ■ ~~(b)~~ $E(X) = 5$ ☐ (c) $Var(X) = 1$
■ ~~(d)~~ $Var(X) = 2.5$ ☐ (e) None of the above

$E(X) = np = 10 \times 0.5 = 5$

$Var(X) = npq = 10 \times 0.5 \times 0.5 = 2.5$

So answers are

$\boxed{b}$ and $\boxed{d}$

3. You roll two fair, six-sided dice separately. Let X be the sum of the two results. What is $E(X)$?

☐ (a) 3.5 ☐ (b) 6 ■ (c) 7 ☐ (d) 12 ☐ (e) None of the above

Let A be the first die roll
B be the second die roll

$$E(X) = E(A+B) = E(A) + E(B) = 3.5 + 3.5$$

$$\left(\text{where } E(A) = \frac{1}{6}(1+2+3+4+5+6) = \frac{21}{6} = 3.5\right)$$

$$E(X) = 7$$

Answer is $\boxed{C}$

4. Let X and Y be independent. Suppose $X \sim \mathcal{N}(1,1)$ and $Y \sim \mathcal{N}(2,3)$. Find $E(2XY^2)$.

☐ (a) 6 ■ (b) 14 ☐ (c) 16 ☐ (d) 18 ☐ (e) None of the above

$E(2XY^2) = E(2X)\,E(Y^2)$

$E(2X) = 2E(X) = 2$

$E(Y^2) = Var(Y) + [E(Y)]^2 = 3 + 2^2 = 7$

$E(2XY^2) = 2 \times 7 = 14$

Answer is $\boxed{b}$

5. Suppose that $X \sim N(0,2)$, and we are interested in computing $E(e^x)$. Directly computing $\int e^x f_X(x)dx$ is hard; in this problem, we will approximate it. Taylor series approximation for $f(x)$ at a is $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$. Use Taylor series to expand e^x at $a = 0$ into polynomial degree 2. What is the expectation of your Taylor approximation? Answer is

☐ (a) 0 ☐ (b) 1 ■ (c) 2 ☐ (d) 3 ☐ (e) None of the above

$f(0) = 1$
$f(1) = 1$
$f(2) = 1$

$f(x) = 1 + x + \frac{x^2}{2}$

$E(f(x)) = E\left(1 + x + \frac{x^2}{2}\right) = E(x) + E\left(\frac{x^2}{2}\right) + 1$

$= 0 + \frac{1}{2}\left(Var(x) + [E(x)]^2\right) + 1$

$= 0 + \frac{1}{2}(2) + 1 = \boxed{2}$

Ans is $\boxed{c}$

6. How can you **efficiently** find x that minimizes $f(x) = xe^{-2x}$ where x is from 0 to 1.

☐ (a) Solve the equation $(1-2x)e^{-2x} = 0$
☐ (b) Solve the equation $(1+x)e^{-2x} = 0$
☐ (c) Estimate $f(x)$ at each value between [0,1].
■ (d) Show that $f''(x) < 0$ in [0,1], and then pick the minimum of f(0) and f(1).
☐ (e) None of the above

$$\frac{d}{dx}\left(xe^{-2x}\right) = e^{-2x} + -2xe^{-2x}$$

$$= (1-2x)(e^{-2x}) = 0 \qquad f'(1/2) = 0$$

$$f''(x) = -2(e^{-2x}) + (1-2x)(-2e^{-2x})$$

$$= -4e^{-2x} + 4xe^{-2x} = 4e^{-2x}(x-1)$$

which is negative in (0,1)

so we pick the minimum of the boundary

$$\boxed{f(0) = 0} \qquad f(1) = \frac{1}{e^{-2}} > 0$$

7. Find the rank of the matrix

$$\begin{bmatrix} 1 & 2 & 5 \\ 0 & 2 & 4 \\ 2 & 3 & 8 \\ 5 & 1 & 7 \end{bmatrix}$$

☐ (a) 1 ■ ~~(b)~~ 2 ✓ ☐ (c) 3 ☐ (d) 4 ☐ (e) None of the above

Let $V_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 5 \end{bmatrix}$ $V_2 = \begin{bmatrix} 2 \\ 2 \\ 3 \\ 1 \end{bmatrix}$ $\ldots$ $V_3 = \begin{bmatrix} 5 \\ 4 \\ 8 \\ 7 \end{bmatrix}$

We know that V_1 & V_2 are linearly independent However $V_3 = 2V_2 + V_1$

So rank is 2

Ans is $\boxed{b}$

8. Which of the following is eigenvector of

$$\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

☐ (a) [1,1] ☐ (b) [1,2] ☐ (c) [2,2] ■ (d) [1,1] and [2,2] ☐ (e) None of the above

$$\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix} = 4\begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \text{Yes}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix} \neq \lambda\begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \text{No}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ 8 \end{bmatrix} = 4\begin{bmatrix} 2 \\ 2 \end{bmatrix} \quad \text{Yes}$$

However $\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ correspond to the same eigenvalue (4)

Ans is $\boxed{d}$

9. Let A be a positive definite matrix; that is, for any vector x, we have $x^T Ax > 0$. Which of the following is **not always** true?

■ (a) All elements of A are positive. ☐ (b) A has full rank.
☐ (c) All eigenvalues of A are positive. Y ☐ (d) A has a positive determinant. Y
☐ (e) None of the above

$$[x \;\; y]\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} xa + yc & xb + yd \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$x^2 a + ycx + xyb + y^2 d$$

A matrix can be positive definite without all elements being positive.

Ans is $\boxed{a}$

$$\begin{bmatrix} 1 & -1 \\ 1 & 6 \end{bmatrix}$$

positive definite with negative element

$$\begin{matrix} 1-\lambda & -1 \\ 1 & 6-\lambda \end{matrix}$$

$$(1-\lambda)(6-\lambda) + 1 = 0$$

$$6 + \lambda^2 - 7\lambda + 1 = 0$$

$$\lambda^2 - 7\lambda + 7 = 0$$

$$\frac{7 \pm \sqrt{49-28}}{2} = \text{both are } +\text{ve}$$

10. Which of the options below is true for the matrix
$$\begin{bmatrix} -2 & 6 \\ 1 & -3 \end{bmatrix}$$

☐ (a) A is invertible. ☐ (b) A is negative definite. ■ ✓(c) A is negative semi-definite.
☐ (d) Choice (a) and (b) ☐ (e) None of the above

$$\det\begin{pmatrix} -2-\lambda & 6 \\ 1 & -3-\lambda \end{pmatrix} = (2+\lambda)(3+\lambda) - 6 = 6+5\lambda+\lambda^2-6$$
$$= 5\lambda+\lambda^2 = 0$$
$$\lambda = 0, -5$$

So A is negative semi-definite

11. **Extra credit question.** If you correctly answer this question, you get 10 extra points. If you incorrectly answer this question, you will get deducted 5 points from your total points. Let a be any real number, and let:

$$A = \begin{bmatrix} 1 & 0 & a \\ 0 & 1+a & 0 \\ a & 0 & 1 \end{bmatrix}$$

What are the distinct eigenvalues of A?

■ (a) $1-a, 1+a$ ☐ (b) $0, 1, 1+a$ ☐ (c) $1-a, 0, 1+a$
☐ (d) $1-a$, $1+a$, $2+a$ ☐ (e) None of the above.

$$\det\begin{pmatrix} 1-\lambda & 0 & a \\ 0 & 1+a-\lambda & 0 \\ a & 0 & 1-\lambda \end{pmatrix} = (1-\lambda)^2(1+a-\lambda) - a(a)(1+a-\lambda)$$

$$= (1-\lambda+a)(1+\lambda^2-2\lambda-a^2) = 0$$

So $1-\lambda+a = 0$ or $1+\lambda^2-2\lambda-a^2 = 0$

$\lambda = 1+a$

$\lambda^2 - 2\lambda + (1-a^2) = 0$

$$\frac{+2 \pm \sqrt{4-4(1-a^2)}}{2}$$

$$\frac{2+2a}{2}, \frac{2-2a}{2}$$

$\lambda = (1+a), (1-a)$

So $\lambda = (1+a), (1-a)$