Student ID:

# CS 112 1st Mid-term Spring 2017

4:00 am–5:50 pm    April 24th

## Notes:

1: The mid-term is closed book, closed notes.
2: Calculator allowed. No other electronic devices allowed.
3: One 8.5x11 paper both sides can be used cheat sheet is allowed.

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|---|---|
| Problem 1 (5 Points) | 5 |
| Problem 2 (5 Points) | 5 |
| Problem 3 (5 Points) | 5 |
| Problem 4 (5 Points) | 5 |
| Problem 5 (5 Points) | 5 |
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| Total (25 Points) | 25 |

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**Problem 1. (5 points)**
Multiple Choice Questions. Clearly select the appropriate answer for each of the following questions.

(1) In an exam with 4 multiple choice questions, Each question have 3 different choices. Assume you select the answer of each question by selecting one of the answers at random with equal probability and the choice you make for every question is independent from choices you make for others. What is the probability of answering all questions right and what is the probability of answering all of them wrong ?

- $\frac{1}{4}^3, \frac{2}{4}^3$
- $\frac{1}{3}^4, \frac{2}{3}^4$
- $\binom{4}{2}\frac{1}{3}^3\frac{2}{3}^1, (\frac{2}{3})^4$

(2) A well-shuffled deck of 52 cards is dealt evenly to two players (26 cards each). What is the probability that player 1 gets all the aces.

- $\frac{\binom{48}{22}}{\binom{52}{26}}$
- $\frac{4!\binom{48}{22}}{\binom{52}{26}}$
- $\frac{48!}{22!}\frac{52!}{26!}$

(3) Which one is the solution for the following differential equation:

$$\frac{dy}{dt} - 3y = e^{2t}, \quad \text{subject to } y(0) = 1$$

- $3e^{2t} - 2e^{-2t}$ $\quad (6e^{2t}+4e^{-2t}) \quad -9e^{2t}+6e^{-2t} = -3e^{2t}+10e^{-2t}$ ✗
- $2e^{3t} - e^{2t}$ $\quad (\cancel{6e^{3t}}-2e^{2t}) \quad -\cancel{6e^{3t}}+3e^{2t} = e^{2t}$ ✓
- $3e^{2t} - 2e^{3t}$ $\quad (6e^{2t}-\cancel{6e^{3t}}) \quad -9e^{2t}+\cancel{6e^{3t}} = -3e^{2t}$ ✗

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**Problem 2. (5 points)**
Assume that packets arriving at your computer from the network belong to one of four different types : A, B, C, or D. The type of each packet is independent from the types of other packets. The probability of packet type to be type A is 0.3, type B is 0.4, type C is 0.2, and type D probability is 0.1.

- What is the probability that among the first 8 packets you receive, there will be equal number of packets from each type ?
- What is the probability that the first packet of type $C$ is is recieved on or after $9^{\text{th}}$ one ?

(1) $P[A=B=C=D=2]$

$= \binom{8}{2,2,2,2} 0.3^2\, 0.4^2\, 0.2^2\, 0.1^2 = \frac{8!}{2!2!2!2!} 0.3^2\, 0.4^2\, 0.2^2\, 0.1^2 \boxed{\approx 0.0145}$

(2) P[first packet of type C received on or after 9th one]

= P[no packets of type C among first 8 packets]

$= (1-0.2)^8 \boxed{\approx 0.1678}$

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**Problem 3. (5 points)**
You are developing a machine learning classifier to detect pedestrians from a camera fixed on a car. You came up with a powerful pedestrian recognition algorithm that when there is a pedestrian in front of the car it will be able to recognize her with 98.5% accuracy.

(1) When tested on the road, What is the probability that the first person it fails to recognize is the 10th person ?

(2) If the total number of pedestrians the algorithms will be tested with = 1000, what is the probability that it will be able to recognize more than 95% of the test subjects.

(1) Let $X \sim Geo(r = 1 - 0.985 = 0.015)$ be the first pedestrian the classifier fails to recognize.

$P[X = 10] = (1 - 0.015)^{10-1}\, 0.015 = 0.985^9\, 0.015 \boxed{\approx 0.0131}$

(2) Let $X \sim Bin(n = 1000, p = 0.985)$ be the number of pedestrians correctly recognized, out of 1000.

$P[X > 0.95 \times 1000] = P[X > 950] = \sum_{x=951}^{1000} \binom{1000}{x} 0.985^x (1-0.985)^{1000-x} \boxed{\approx 1.000}$

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**Problem 4. (5 points)**
Among visitors to an e-commerce web site that sells Books, CDs and Drawings, 40% of the visitors buy books, 35% request CDS, and 25% request Drawings. Of those customers requesting books, only 30% request fast shipping. Of those customers requesting CDs, 60% fill request fast shipping, while of those requesting Drawings, 50% request fast shipping. If the next customer asks for fast shipping, what is the probability that he is buying books ? Assume a customer will buy only one type of goods a time.

Let B = books, C = CDs, D = drawings, F = fast shipping.

By Bayes' Rule:

$$P[B|F] = \frac{P[B \wedge F]}{P[F]}$$

$$= \frac{P[F|B]P[B]}{P[F|B]P[B] + P[F|C]P[C] + P[F|D]P[D]}$$

$$= \frac{0.3 \times 0.4}{0.3 \times 0.4 + 0.6 \times 0.35 + 0.5 \times 0.25}$$

$$\boxed{= \frac{24}{91} \approx 0.2637}$$

**Problem 5. (5 points)** 4

Using z-transform, prove that the merging of $n$ mutually independent Poisson streams with parameters $\lambda_1, \lambda_2, ..., \lambda_n$ produces a Poisson stream with parameter $\sum_{i=1}^{n} \lambda_i$.

Let $X_i \sim Poi(\lambda_i)$, $1 \le i \le n \Rightarrow P[X_i = n] = e^{-\lambda_i} \frac{\lambda_i^n}{n!}$.

$$Z[X_i] = \sum_n P[X_i = n] z^n \qquad \text{(definition of z-transform)}$$

$$= \sum_{n=0}^{\infty} e^{-\lambda_i} \frac{\lambda_i^n}{n!} z^n$$

$$= e^{-\lambda_i} \sum_{n=0}^{\infty} \frac{(\lambda_i z)^n}{n!}$$

$$= e^{-\lambda_i} e^{\lambda_i z}$$

$$= e^{\lambda_i (z-1)}$$

$$Z\left[\sum_i X_i\right] = \prod_i Z[X_i] \qquad \text{(convolution property)}$$

$$= \prod_{i=1}^{n} e^{\lambda_i (z-1)}$$

$$= e^{\sum_{i=1}^{n} \lambda_i (z-1)}$$

↑

This form is the z-transform $Z[Y]$ for $Y \sim Poi\left(\sum_{i=1}^{n} \lambda_i\right)$.

$\therefore \sum_i X_i \sim Poi\left(\sum_{i=1}^{n} \lambda_i\right)$.