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CS 112 1st Mid-term Spring 2017

4:00 am–5:50 pm April 24th

Notes:

1: The mid-term is closed book, closed notes.
2: Calculator allowed. No other electronic devices allowed.
3: One 8.5x11 paper both sides can be used cheat sheet is allowed.

Problem 1 (5 Points)	5
Problem 2 (5 Points)	5
Problem 3 (5 Points)	4
Problem 4 (5 Points)	5
Problem 5 (5 Points)	4
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Total (25 Points)	23

$$\binom{4}{4}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^0 = \left(\frac{1}{3}\right)^4$$

$$\binom{4}{4}\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^0 = \left(\frac{2}{3}\right)^4$$

Problem 1. (5 points) 5

Multiple Choice Questions. Clearly select the appropriate answer for each of the following questions.

(1) In an exam with 4 multiple choice questions, Each question have 3 different choices. Assume you select the answer of each question by selecting one of the answers at random with equal probability and the choice you make for every question is independent from choices you make for others. What is the probability of answering all questions right and what is the probability of answering all of them wrong ?

- $\frac{1}{4}^3$, $\frac{2}{4}^3$
- $\frac{1}{3}^4$, $\frac{2}{3}^4$
- $\binom{4}{2}\frac{1}{3}^3\frac{2}{3}^1$, $(\frac{2}{3})^4$

All right $= \frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3} = \left(\frac{1}{3}\right)^4$

All wrong $= \frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3} = \left(\frac{2}{3}\right)^4$

(2) A well-shuffled deck of 52 cards is dealt evenly to two players (26 cards each). What is the probability that player 1 gets all the aces.

- $\frac{\binom{48}{22}}{\binom{52}{26}}$
- $\frac{4!\binom{48}{22}}{\binom{52}{26}}$
- $\cancel{\frac{48!\,52!}{22!\,26!}}$

- Out of $\binom{52}{26}$ ways to give cards to one player
- $\binom{48}{22}$ ways to pick the other 22 cards
- order does not matter, so no need for 4!

(3) Which one is the solution for the following differential equation:

$$\frac{dy}{dt} - 3y = e^{2t}, \quad \text{subject to } y(0) = 1$$

- $3e^{2t} - 2e^{-2t}$
- $2e^{3t} - e^{2t}$
- $3e^{2t} - 2e^{3t}$

$$y' - 3y = e^{2t}$$

* Can solve by plugging in.

$$\frac{dy}{dt}\left(3e^{2t} - 2e^{-2t}\right) = 6e^{2t} + 4e^{-2t} - 9e^{2t} + 6e^{-2t}$$

$$\frac{dy}{dt}\left(2e^{3t} - e^{2t}\right) = 6e^{3t} - 2e^{2t} - 6e^{3t} + 3e^{2t}$$

$$\frac{dy}{dt}\left(3e^{2t} - 2e^{3t}\right) = 6e^{2t} - 6e^{3t} - 9e^{2t} + 6e^{3t}$$

$$\cancel{6e^{3t}} - 2e^{2t} - \cancel{6e^{3t}} + 3e^{2t} = e^{2t}$$

* Note: I did not have a calculator, so all answers were left in unsimplified form.

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Problem 2. (5 points)
Assume that packets arriving at your computer from the network belong to one of four different types : A, B, C, or D. The type of each packet is independent from the types of other packets. The probability of packet type to be type A is 0.3, type B is 0.4, type C is 0.2, and type D probability is 0.1.

- What is the probability that among the first 8 packets you receive, there will be equal number of packets from each type ?
- What is the probability that the first packet of type C is is recieved on or after 9th one ?

a) This can be modeled using a general Bernoulli distribution with the following parameters.

$P_A = .3 \quad n_A = 2 \qquad N = 8$

$P_B = .4 \quad n_B = 2$

$P_C = .2 \quad n_C = 2$

$P_D = .1 \quad n_D = 2$

Thus, the probability that the 8 packets will consist of equal amounts of the 4 packet types is $\left(\frac{8!}{2!2!2!2!}\right)(.3^2)(.4^2)(.2^2)(.1^2)$

b) This can be determined using a sum of geometric random variables. The probability that the first C-packet is recived on or after the 9th received equals $(1 - P(\text{first C packet was the 1st through 8th to arrive}) = 1 - \sum_{n=1}^{8} (1-.2)^{n-1} .2$

$$\begin{array}{r} \overset{9\,9\,10}{\cancel{1.000}} \\ -\ .985 \\ \hline .015 \end{array}$$

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Problem 3. (5 points)

You are developing a machine learning classifier to detect pedestrians from a camera fixed on a car. You came up with a powerful pedestrian recognition algorithm that when there is a pedestrian in front of the car it will be able to recognize her with 98.5% accuracy.

(1) When tested on the road, What is the probability that the first person it fails to recognize is the 10th person ?

(2) If the total number of pedestrians the algorithms will be tested with = 1000, what is the probability that it will be able to recognize more than 95% of the test subjects.

1) This can be modeled with a geometric random variable with parameters $n=10$ and $p=.015$. Thus, the probability that the first person the algorithm fails to recognize is the 10th person is $(1-.015)^9(.015) = \cancel{(.985)^9}(.015)$

2) Since the algorithm can recognize pedestrians with 98.5% accuracy, we would expect the algorithm will recognize 98.5% of the 1000 people, or 985 people. This expected value allows us to model this situation using a Poisson random variable, with $\lambda = 985$. Thus, the probability that the algorithm will recognize more than 950 people is equal to the sum

$$\sum_{n=951}^{1000} \frac{e^{-985}\,985^n}{n!}$$

← this is approximate

⟹ use binomial for exact solut

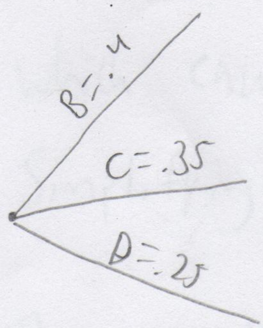
$$\begin{array}{r} .35 \\ .06 \\ \hline .0216 \end{array} \qquad 4 \qquad \cancel{\begin{array}{r} 125 \\ 021 \\ 120 \\ \hline 266 \end{array}} \qquad \begin{array}{r} 125 \\ 210 \\ 120 \\ \hline .455 \end{array}$$

Problem 4. (5 points)
Among visitors to an e-commerce web site that sells Books, CDs and Drawings, 40% of the visitors buy books, 35% request CDS, and 25% request Drawings. Of those customers requesting books, only 30% request fast shipping. Of those customers requesting CDs, 60% fill request fast shipping, while of those requesting Drawings, 50% request fast shipping. If the next customer asks for fast shipping, what is the probability that he is buying books ? Assume a customer will buy only one type of goods a time.

$P(B) = .4 \qquad P(FS \mid B) = .3$

$P(C) = .35 \qquad P(FS \mid C) = .6$

$P(D) = .25 \qquad P(FS \mid D) = .5$



$P(FS \cap B) = .4 \cdot .3 = .120$

$P(FS \cap C) = .35 \cdot .6 = .210$

$P(FS \cap D) = .25 \cdot .5 = .125$

$P(FS) = .12 + .210 + .125 = .455$

$$\boxed{\frac{P(FS \cap B)}{P(FS)} = \frac{.12}{.455}}$$

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Problem 5. (5 points)
Using z-transform, prove that the merging of n mutually independent Poisson streams with parameters $\lambda_1, \lambda_2, ..., \lambda_n$ produces a Poisson stream with parameter $\sum_{i=1}^{n} \lambda_i$.

In the notes, it was proved that the z-transform of the Poisson random variable is $e^{-\lambda(1-z)}$.

By the sum to convolution property, we also know that ~~$f \otimes g = F(z)\,G(z)$. In this case, we can use~~ this to multiply the transformed Poisson streams to obtain $f_1 \otimes f_2 \cdots f_n = F_1(z)\,F_2(z) \cdots F_n(z)$ which equals $e^{-\lambda_1(1-z)}\, e^{-\lambda_2(1-z)} \cdots e^{-\lambda_n(1-z)}$.

Simplifying this produces the result $e^{-\sum_{i=1}^{n} \lambda_i (1-z)}$.

This result can then be inverted back from $H(z) \leftrightarrow h(t)$, producing the distribution for a Poisson random variable with a parameter $\lambda = \sum_{i=1}^{n} \lambda_i$