PROBLEM 1 19 /20 PTS

*A. **True or False questions (1 PTS each)**. Indicate if each statement is true (T) or false (F) by checking the T and F columns. If you change your mind, make sure to make it clear and legible.*

No.	T	F	Statement
1		✓	Friction factor is always dependent on *Re* and surface roughness.
2	✓		The Fanning friction factor can be interpreted as the ratio of viscous energy loss in duct flow relative to fluid kinetic energy.
3		✓	The skin friction coefficient (c_f) is the ratio of wall shear stress relative to kinetic energy per unit volume ($0.5\ \rho U^{1/2}$), where U is the average velocity in the boundary layer
4		✓	In dimensional analysis, scaling parameters must be selected so that they themselves can form a dimensionless number without other variables.
5	✓		If flow conditions between model and prototype systems have kinematic similarity, they must also have geometric similarity.
6		✓	The Reynolds number does not appear in the dimensionless form of the Navier-Stokes equation.
7	✓		If fully developed flow in a circular pipe has a Reynolds number of 10^3, pipe surface roughness has no impact on the friction factor. Laminar
8	✓		One would expect that the loss coefficient K in a pipe section will be higher in sudden versus gradual contraction.
9		✓	A manufacturer recommends that its flow sensor works best when the flow is fully developed. Putting this flow sensor immediately after a 90° bend should be OK as long as there is no sudden expansion or extraction.
10		✓	To enhance radial mixing, it is recommended to design a piping system (i.e., with circular pipe) to have a Reynolds number of around 2300.
11	✓		In the transition flow regime, there is instability of laminar motion with intermittent bursts of turbulence.
12	✓		Flow separation occurs when there is excessive momentum loss near wall as fluid flows downstream against increasing pressure.
13	✓		If the boundary layer is laminar in a fully developed flow in a long straight pipe, the boundary layer will be always be laminar downstream.
14		✓	If the boundary layer of flow past a flat plate is laminar near the leading edge (x=0), the boundary layer will always be laminar downstream (x>0).
15	✓		In external flow, streamlining of body shape is important for delaying flow separation and thus reducing the pressure drag.

Problem 1 continues to next page

B. _Multiple choice questions (1 PTS each)._ *Choose the best answer. You must use the table below to indicate your answer. If you change your mind, make sure to make it clear and legible.*

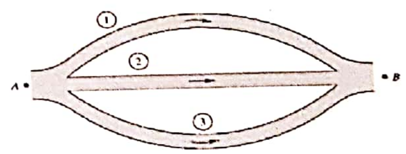
No.	A	B	C	D	E
16.	✓				
17			✓	✓	
? 18				✓	
? 19		✓			
? 20					✓

16. In __________ pipe flow, friction factor __________ with increasing surface roughness.
    (A) *turbulent; increases*
    (B) *turbulent; decreases*
    (C) *both turbulent and laminar; increases*
    (D) *laminar; increases*
    (E) *laminar;decreases*

17. Friction factor correlations for circular pipe can sometimes be used to estimate head loss in non-circular pipe, especially under the following condition.
    (A) *Incompressible fluid*
    (B) *Constant viscosity*
    (C) *Laminar low*
    (D) *Turbulent flow*
    (E) *Constant density*

Current = Q ✓ Press =

18. Consider a pipe section with water flowing from point A to point B through three separate pipe branches (1-3). The following **must** be true about the pipe branches:



    (A) *different pressure drop*
    (B) *different flow rates*
    (C) *same flow rates*
    (D) *same pressure drops*
    (E) *different head losses*

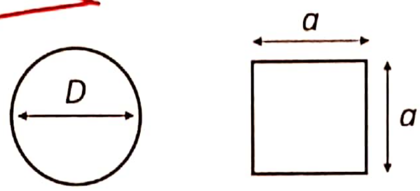
19. For flow past a thin flat plate, the boundary layer theory works best under the following conditions **EXCEPT**: (Note: plate parallel to the x-axis and perpendicular to the y-axis)
    (A) *No flow separation*
    (B) *The flow is laminar with Re<1*
    (C) *y-component velocity is small*
    (D) *x-component velocity gradients are small*
    (E) *Pressure is a function of x only*

20. Why do the surface of golf balls have dimples?
    (A) *To induce turbulence in the boundary layer*
    (B) *To reduce pressure drag*
    (C) *To streamline the body .*
    (D) *B and C*
    (E) *A and B*

PROBLEM 2 17 /25 PTS

A straw manufacturer makes two types of drinking straw: (1) the typical round shape and (2) a square cross-sectional shape (2). The amount of material in each straw is to be the same (i.e., the same length l and the same cross-sectional perimeter l_p). For the same pressure drop Δp between the inlet and outlet of each straw type, what is the ratio of the flow rates through the straw (Q_1/Q_2)?

- Assume the drink is viscous to ensure laminar flow. In this case, the Darcy friction factor
  $$f = C/\mathrm{Re}_{Dh} \qquad \text{(Eq. 1)}$$
  where $C = 64$ for round straw and $C = 56.9$ for square straw.
- Assume that potential head change is negligible relative to frictional head loss.



(1) (2)

PROBLEM 2 ANSWER

$$\frac{P_{in}}{\gamma} + \cancel{z_{in}} + \frac{V_{in}^2}{2g} = \frac{P_{out}}{\gamma} + z_{out} + \frac{V_{out}^2}{2g} + h_f - h_{pump} + h_{turbine}$$

for both straws:

$z_{in} = 0 \qquad V_{in} = 0$

$h_{pump} = h_{turbine} = 0$

$z_{out} = l$

$$\frac{P_{in}}{\gamma} - \frac{P_{out}}{\gamma} - z_{out} - h_f = \frac{V_{out}^2}{2g}$$ 3/5

$$\frac{\Delta P}{\gamma} - l - h_f = \frac{V_{out}^2}{2g}$$

$$f = \frac{C}{Re_{DH}} \qquad h_f = f\,\frac{L}{D_H}\,\frac{V_{out}^2}{2g}$$

-15

PROBLEM 2 ANSWER (CONT'D)

$$D_h = \frac{4 \cdot A_c}{L_p}$$ for circular straw:

$$A_c = \frac{\pi D^2}{4} \quad L_p = l_p \quad D_h = \frac{\pi D^2}{l_p}$$

for square straw:

$$A_c = a^2 \quad L_p = l_p \quad D_h = \frac{4a^2}{l_p}$$

- circular straw:

$$h_f = f \frac{l \cdot l_p}{\pi D^2} \frac{V_{out}^2}{2g}$$

- square straw:

$$h_f = f \left(\frac{l \cdot l_p}{4a^2} \right) \frac{V_{out}^2}{2g}$$

$$f = \frac{C}{Re_{Dh}} \quad , \quad Re_{Dh} = \frac{\rho V D_h}{\mu}$$

- circular straw:

+3

$$f = \frac{64}{Re_{Dh}} = \frac{64\mu}{\rho V D_h} = \frac{64 \mu l_p}{\rho V \pi D^2}$$

$$h_f = 64 l \left(\frac{l_p}{\pi D^2} \right)^2 \frac{V_{out}^2}{2g} = 32 l \left(\frac{l_p}{\pi D^2} \right)^2 \frac{V_{out}^2}{g}$$

- square straw:

$$f = \frac{56.9 l_p}{4a^2}$$

$$h_f = 28.45 l \left(\frac{l_p}{4a^2} \right)^2 \frac{V_{out}^2}{g}$$

$$\frac{\Delta p}{\gamma} - \cancel{z} - h_f = \frac{V_{out}^2}{2g}$$

potential head loss change negligible o.k. +1

PROBLEM 2 ANSWER (CONT'D)

$$\frac{\Delta p}{\gamma} - h_f = \frac{V_{out}^2}{2g}$$

To get Q from V:

$$Q = AV \rightarrow V = \frac{Q}{A}$$

circular straw:

$$V = \frac{Q}{\frac{\pi}{4}D^2} = \frac{4Q}{\pi D^2}$$

square straw:

$$V = \frac{Q}{a^2}$$

$$\sqrt{2g\left(\frac{\Delta p}{\gamma} - h_f\right)} = V_{out}$$ +5

circular:

$$\sqrt{2g\left(\frac{\Delta p}{\gamma} - h_f\right)} = \frac{4Q}{\pi D^2} \rightarrow Q_1 = \frac{\pi D^2}{4}\sqrt{2g\left(\frac{\Delta p}{\gamma} - h_f\right)}$$

square:

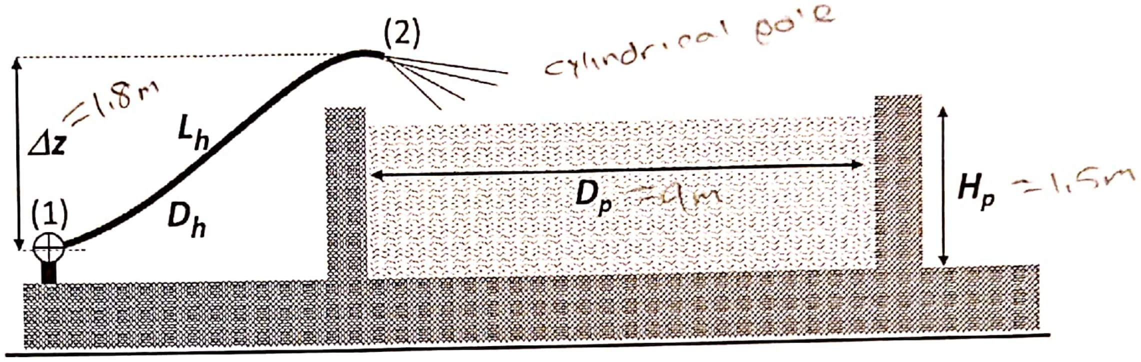
$$\sqrt{2g\left(\frac{\Delta p}{\gamma} - h_f\right)} = \frac{Q}{a_2} \rightarrow Q_2 = a^2\sqrt{2g\left(\frac{\Delta p}{\gamma} - h_f\right)}$$

$$\boxed{\frac{Q_1}{Q_2} = \frac{\pi D^2}{4a^2}\sqrt{\frac{2g\frac{\Delta p}{\gamma} - 2gh_{f,c}}{2g\frac{\Delta p}{\gamma} - 2gh_{f,s}}} \qquad h_{f,c} = 32\ell\left(\frac{\ell_p}{\pi D^2}\right)^2\frac{V_{out}^2}{g} \qquad h_{f,s} = 28.45\ell\left(\frac{\ell_p}{4a^2}\right)^3\frac{V_{out}^2}{g}}$$

2/5

~~24~~ 23 /25 PTS

PROBLEM 3



An above ground swimming pool of D_p= 9 m diameter and H_p=1.5 m depth is to be filled using a garden hose. The hose has smooth interior of length L_h=30 m and diameter D_h=0.015 m. If the pressure at the hose inlet remains at 55 psig (379, 212 Pa; 1 Pa = 1 N/m^2), estimate the **t_{fill} hours** (1 hr = 3,600 s) it would take to fill the pool. The water exits the hose as a free jet at Δz=1.8 m above the faucet. Note: g= 9.8 m/s^2, $\upsilon = 10^{-6}$ m^2/s , γ = 9,800 N/m^3 . **State your assumptions regarding minor head losses.** (*HINT: start with guess f_D of 0.0197*)

PROBLEM 3 ANSWER

$$\frac{P_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + h_f + h_{ml}$$

minor losses

Assumption: only minor head loss occurs at the end of the hose where a sharp exit occurs: $K_{exit} = 1$

$P_2 = 0$ (relative to P_1) $\quad P_1 = 379,212\ Pa$

$\gamma = 9800\ N/m^3$

$z_1 = 0$

$z_2 = 1.8\ m$

$V_1 = 0$

V_2 = unknown

$$h_f = f_D \frac{L}{D_h}\frac{V^2}{2g} = f_D \frac{L}{D}\frac{V^2}{2g}$$

$$D_h = \frac{4\cdot\frac{\pi}{4}D^2}{2\pi\frac{D}{2}} = D$$

+4/5

minor head loss

$$0 = \frac{-P_1}{\gamma} + z_2 + \frac{V_2^2}{2g} + \left(f_D\frac{L}{D} + 1\right)\frac{V_2^2}{2g}$$

PROBLEM 3 ANSWER (CONT'D)

$$0 = \frac{-P_1}{\gamma} + z_2 + \frac{V_2^2}{2g}\left(1 + 1 + f_D \frac{L}{D}\right)$$

plug in to make V_2 easier to find when iterating f_D

$$0 = \frac{-379{,}212\,Pa}{9800\,N/m^3} + 1.8\,m + \frac{V_2^2}{2 \cdot 9.81\,m/s^2}\left(2 + f_D\left(\frac{30\,m}{0.015\,m}\right)\right)$$

$$0 = -36.895 + \frac{V_2^2}{19.62\,m/s^2}\left(2 + 2000 f_D\right)$$

$$\frac{723.882}{2 + 2000 f_D} = V_2^2$$

$$V_2 = \sqrt{\frac{723.882}{2 + 2000 f_D}}$$

initial guess: $f_D = 0.0197$

$V_2 = 4.182\,m/s$; next page

unsure how to iterate w/o colebrook eqn or moody chart:

w/ this velocity, time to fill pool:

$$Q_2 = A_2 V_2 = \frac{\pi}{4} D_{hose}^2 V_2 = \frac{\pi}{4}(0.015\,m)^2 V_2$$
$$= 7.389 \times 10^{-4}\,m^3/s$$

volume of pool:

$$\pi\left(\frac{D^2}{4}\right) \cdot h = \pi\left(\frac{9\,m^2}{4}\right) \cdot 1.5\,m = 95.426\,m^3$$

$$\frac{95.426\,m^3}{7.389 \times 10^{-4}\,m^3/s} \times \frac{1\,hr}{3600\,s} = \boxed{35.87\,hrs.}$$

PROBLEM 3 ANSWER (CONT'D)

Iterate using Reynold's #

$$Re_D = \frac{\rho V D}{\mu} = \frac{VD}{\nu} = \frac{4.182\ m/s \cdot 0.015\ m}{10^{-6}\ m^2/s}$$

$$Re_D = 62730$$

✓

↓

use Blasius Eqn to iterate since

$$4000 < Re_D < 100{,}000$$

$$f_D = \frac{0.316}{(62730)^{1/4}} = 0.019967$$

+10

$$Q_2 = A_2 V_2 = \frac{\pi}{4} D_{hose}^2 V_2$$

$$\text{new } V_2 = \sqrt{\frac{723.882}{2+2000(0.019967)}}$$

$$V_2 = 4.1548\ m/s$$

+5

$$Q_2 = \frac{\pi}{4}(0.015m)^2(4.1548\ m/s)$$

$$Q_2 = 7.342 \times 10^{-4}\ m^3/s$$

volume of pool:

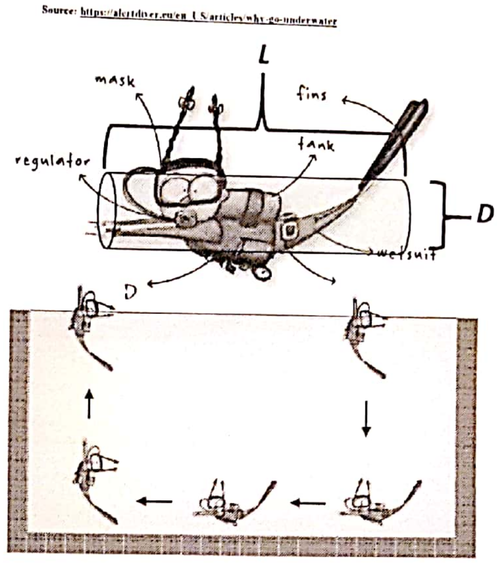
$$\pi\left(\frac{D^2}{4}\right) \cdot h = \pi\left(\frac{9m^2}{4}\right) \cdot 1.5m = 95.426 m^3$$

+4/5

$$\frac{95.426 m^3}{7.342\times10^{-4} m^3/s} \cdot \frac{1hr}{3600s} = \boxed{36.1\ hrs}$$

PROBLEM 4

21.5 ______ /30 PTS



Sally Bruin, a student scuba diver, is learning how to use a buoyancy control device (BCD) in a deep pool. The BCD is essentially an air-filled jacket that can be partially inflated or deflated in a controlled manner in order to establish and maintain neutral buoyancy.

As a first exercise, Sally must first slowly dive into the bottom of the pool by partially deflating the BCD to create negative buoyancy. Once at the bottom, she must then orient her body horizontally and then slowly inflate the BCD until neutral buoyancy is established (i.e., Sally's overall weight (F_W) exactly matches the upward buoyant force F_B). Sally can then move around the pool at a constant depth by propelling her body horizontally using fins attached to her feet. Whenever necessary, she can maintain neutral buoyancy by making small inflation/deflation adjustments of the BCD. To finish the exercise, Sally must ascend to the surface in a slow and controlled manner by first orienting her body vertically and then propelling her body upward (at a force F_P) using her fins while still maintaining neutral buoyancy. Indeed, a cardinal rule of scuba diving is to never ascend to the surface by inflating the BCD; it can be extremely difficult to control the ascent rate with positive buoyancy. An ascent rate of more than 0.3 m/s risks decompression sickness, which can be fatal.

Assume that you can approximately model Sally as a cylinder with an effective body diameter of $D = 0.4\ m$, $L/D = 4$, and an effective bulk density of ρ_b. Assume that the water density ρ is $1{,}000\ kg/m^3$ and kinematic viscosity υ is $10^{-6}\ m^2/s$.

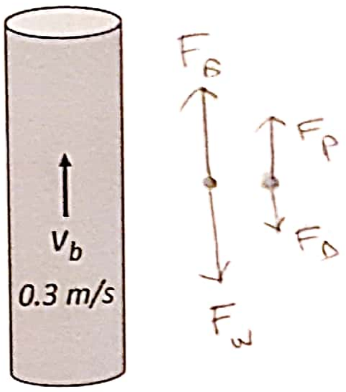
a) **(8 POINTS)** Consider the case when Sally, oriented vertically, is neutrally buoyant and is moving upward at a constant speed v_b. Sketch below the forces acting on Sally and their directions. Also indicate the direction of water velocity U relative to Sally. What is the net force acting on Sally, the effective bulk density ρ_b, *and* the value of U?

b) **(9 POINTS)** When Sally, is ascending at a constant velocity v_b, the drag force (F_D) acting on Sally (by the fluid) should also be dependent on the kinematic viscosity υ and density ρ of the fluid as well as Sally's effective diameter (D). Use the Buckingham Pi method to show that flow of fluid past Sally can be described by two dimensionless variables: one that is proportional to the drag coefficient ($\Pi_1 \propto C_D$) and another one that is proportional to the Reynolds number ($\Pi_2 \propto Re$)

c) **(9 POINTS)** How much work (in **calories**) must Sally expend in order to propel herself upward to the surface from a depth of H=10 m at constant velocity of v_b= 0.3 m/s. Is the drag significant? As a comparison, normal walking (in air) is about 100 calories/mile = 0.62 cal/10 m. Note: 1 $kg\text{-}m/s^2$ = 0.24 calories.
(HINT: With BCD, Sally remains neutrally buoyant as she ascends so she does not need to do work against her own weight).

F_w, F_D, F_B, F_p

Problem 4 Answer Part (a) +8



$\sum F =$ 0 N

$\rho_b =$ $\rho = 1000$ kg/m³

$U = -V_t$ m/s

$F_B = F_w$

$F_p = F_D$

since speed is constant

~~+3~~ +17.5

Problem 4 Answer Parts (b) and (c)

c) 5 variables: F_D, V_t, ν, ρ, D

scaling parameters

F_D	V_b	ν	ρ	D
$[MLT^{-2}]$	$[LT^{-1}]$	$[L^2T^{-1}]$	$[ML^{-3}]$	$[L]$

3 symbols:

$5 - 3 = 2$ π-terms

+2

$$\pi_1 = F_D \overset{V_b}{\nu^a} \rho^b D^c = M^0 L^0 T^0$$

$$[MLT^{-2}][L^{2a}T^{-a}][M^b L^{-3b}][L^c] = M^0L^0T^0$$

$M: 1 + b = 0 \rightarrow b = -1$

$L: 1 + 2a - 3b + c = 0 \rightarrow 1 - 4 + 3 + c = 0 \rightarrow c = 0$

$T: -2 - a = 0 \rightarrow a = -2$

$$\frac{MLT^{-2}}{L^4 T^{-2} M L^{-3}}$$

Problem 4 Answer Parts (b) and (c)

$$\boxed{\Pi_1 = \frac{F_D}{\nu^2 \rho}} \rightarrow C_D = \frac{2 F_D}{\rho u^2 A}$$

$$\Pi_2 = V_t \, \nu^a \rho^b D^c = M^0 L^0 T^0$$

$$[LT^{-1}][L^{2a} T^{-a}][M^b L^{-3b}][L^c] = M^0 L^0 T^0$$

$L: \; 1 + 2a - 3b + c = 0$

$T: \; -1 - a = 0 \quad a = -1$

$M: \; b = 0$

$\cancel{1 + 2a + c = 0 \rightarrow c = -1 - 2a}$

$\cancel{-1 - a + c = 0 \rightarrow -1 - a - 1 - 2a = 0}$

$\cancel{-2 - 3a = 0}$

$1 + 2(-1) + c = 0$

$c = 1$

$$\boxed{\Pi_2 = \frac{V_t D}{\nu} = Re}$$

+5.5

Problem 4 Answer Parts (b) and (c) +6

c) She only needs to work against the drag force

$\Sigma F = F_D = 0$ +2

$F_D = \frac{1}{2} \rho u^2 \cdot A \cdot C_D$ +2

$= \frac{1}{2} \cdot 1000\, kg/m^3 \cdot (0.3\, m/s)^2 \cdot \frac{\pi}{4}(0.4m)^2 \cdot C_D$

assume $C_D = \cancel{1}$ 0.85 from table

$F_D = 5.655\, N$

$Power = F_D \cdot \cancel{0.3\, m/s} \cdot 10m$ +2

$\cancel{= 1.5964\, \frac{kg-m}{s^2}}$

$= 56.55\, W$

$= 13.572\, cals$

significant drag +1