UNIVERSITY OF CALIFORNIA, LOS ANGELES
Civil and Environmental Engineering Department

CEE 110 Introduction to Probability and Statistics for Engineers

Midterm Exam
(4:00-5:50 pm Wednesday May 2, 2018)

- Students are allowed to bring one page (one side, approximately 8.5×11 inches) of summary notes with them into the examination room. No other material such as textbooks, class notes, or homework is permitted.
- The exam has ***five*** problems with multiple parts. The credit for each part is indicated.
- Before you start, make sure all pages of your examination are legible.
- If you have to leave the room for any reason during the exam, please do so quietly.
- If you have a question during the exam, raise your hand and, if you are in the center of a row, come to the side isle.
- Full credit will be given for answers that are worked out correctly. You may leave your answer in terms of a correct symbolic equation if you like (partial credits). To obtain credit, be sure to show as much work as possible! You should draw a box around your final answer to each problem.
- Useful tables are included at the end of the exam.
- Students may use any calculator during the exam, but the entire answer process must be clear on the written answer sheet. All problems must be worked as if only a basic calculator was available.

Make Sure Your Name Is On All Exam Sheets

You Must Turn In Your Cheat Sheet With The Exam And Make Sure Your Name Is On It

Good Luck!

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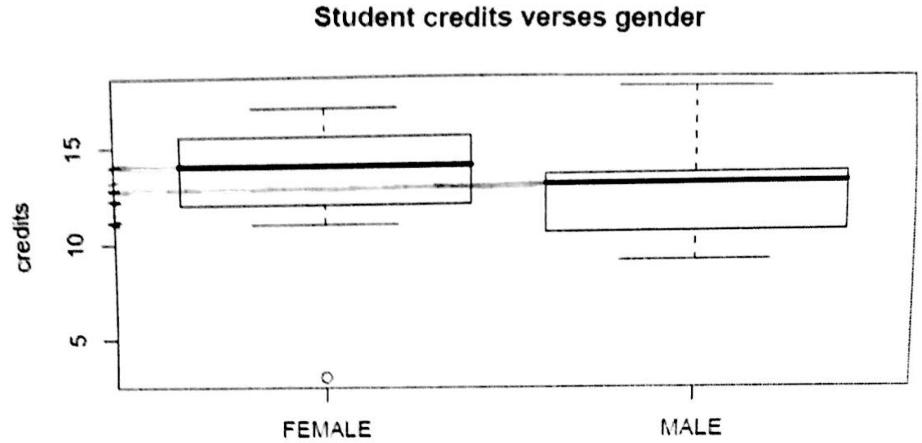
1. The following box and whisker plot shows the number of credits that the College of Engineering students earned based on their gender.

14



a. What is the median number of credits for each gender? Which gender has a higher median number of credits? (5 pt)

Female median : 14 credits
Male median : 12.5 credits
The female gender has a higher median number of credits

b. What is the approximate IQR (=fourth spread) of each gender? What percent of the data falls within the IQR? (5 pt)

Female : 4 credits (16-12)
Male : 2.5 credits (13 - 10.5)
50% of the data falls within the IQR

c. Which gender has a larger range of credits? (5 pt)

Male

d. If someone reported the mean as the typical number of credits, would it be valid in this case? Justify your answer. (5 pt)

For females it would be close to valid as the data does not look skewed, so the mean would better represent the typical number.
For males it would not be valid as the data is clearly skewed, and the median is not close to the middle of the box.

Outlier!

-1

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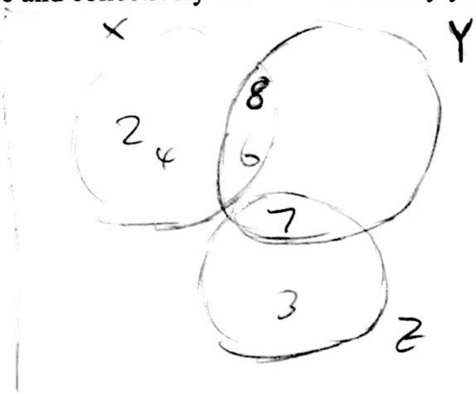
2. The sample space of an experiment consists of a total of eight outcomes: S={1,2,3,4,5,6,7,8}. The events of X, Y, Z are defined as: X={even numbers}, Y={>5}, Z={3, 7}

   a. Draw the Venn diagram of the sample space and the events. Are X, Y, Z are mutually exclusive and collectively exhaustive? Justify your answers. (5 pt)

10



X,Y,Z are not mutually exclusive as they have common elements.

X,Y,Z are not collectively exhaustive as they do not contain all elements in S, like 1, 5.

Suppose we know that P(X∩Y)=1/3; X and Y are independent events; and probabilities of all even numbered outcomes are equal, in other words, p(2)=p(4)=p(6)=p(8).

   b. Find P(X). (5 pt).

p(6) and p(8) are in X∩Y, and if they add to 1/3, then

$p(8) = p(6) = \frac{P(X \cap Y)}{2} = \frac{1}{6}$

$p(2) + p(4) + p(6) + p(8) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$

$P(X) = \frac{2}{3}$

   c. Find P(Y). (5 pt)

$P(7) + P(3) = 1 - P(X) = 1/3 = P(Z)$

since X and Y are independent

$P(X \cap Y) = P(X)P(Y)$

$P(Y) = \frac{P(X \cap Y)}{P(X)} = \frac{1/3}{2/3} = \frac{1}{2}$

$P(Y) = \frac{1}{2}$

   d. Find P(X∪Y). (5 pt)

$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

$= 2/3 + 1/2 - 1/3$

$= 1/3 + 1/2$

$P(X \cup Y) = 5/6$

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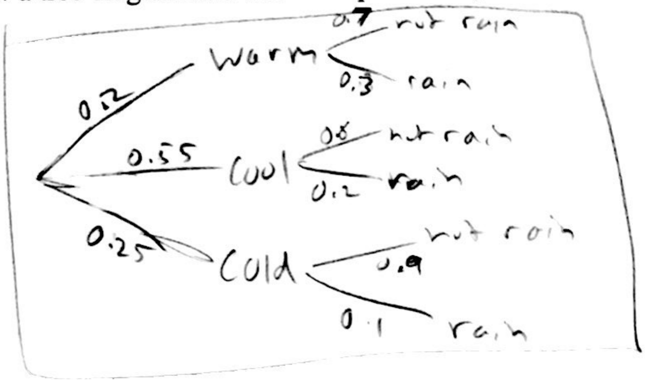
3. In Los Angeles, the weather on a spring day is classified as either cold, cool, or warm. The probability that it is cold is 0.25, the probability that it is cool is 0.55 and the probability that it is warm is 0.2. In addition, on cold days the probability that it will rain is 0.1 and on cool days the probability that it will rain is 0.2 and on warm days the probability that it will rain is 0.3.

   a. Draw a tree diagram and show the probability with the associated outcomes. (5 pt)

+4



   b. Find the probability that it will rain. (5 pt)

+5

$P(W \cap R)$ $\quad$ $P(C \cap R)$ $\quad$ $P(D \cap R)$

$(0.2 \times 0.3) + (0.55 \times 0.2) + (0.25 \times 0.1) = 0.195$

$P(R) = 0.195$

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c. If it rains, what is the probability that it is either cold or cool? Round the values to two decimal places if needed. (5 pt)

$P(CD \mid R) = \frac{P(CD \cap R)}{P(R)} = \frac{(0.1)(0.25)}{0.195} = 0.128$ +5

$P(CO \mid R) = \frac{P(CO \cap R)}{P(R)} = \frac{(0.55)(0.12)}{0.195} = 0.564$

0.69

d. If it is not raining on a particular day, what is the probability that it is cold? Round the values to two decimal places if needed. (5 pt)

+5

$P(NR) = 1 - P(R) = 1 - 0.195 = 0.805$

$P(CD \mid NR) = \frac{P(CD \cap NR)}{P(NR)} = \frac{(0.25)(0.9)}{0.805} = 0.279$

$P(CD \mid NR) = 0.28$

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+20

4. Suppose the sample space of discrete random variable is {0, 1, 2, 3, 4}. The pmf of the random variable is p(x)=cx, in other words p(0)=c×0, p(1)=c×1, p(2)=c×2, p(3)=c×3, p(4)=c×4.

a. What is the value of c? (5 pt)

$1 = c(0) + c + 2c + 3c + 4c$

$1 = 10c$

$c = 0.1$

b. What is the cdf? (5 pt)

x	0	1	2	3	4
p(x)	0	0.1	0.2	0.3	0.4

$$cdf(x) \begin{cases} 0; & x < 1 \\ 0.1; & x < 2 \\ 0.3; & x < 3 \\ 0.6; & x < 4 \\ 1; & x < 5 \end{cases}$$

c. Find E(X) and V(X). (5 pt)

$E(x) = 0(0) + 0.1(1) + 2(0.2) + 3(0.3) + 4(0.4) = 3$

$E(x) = 3$

$V(x) = E(x^2) - (E(x))^2 = 10 - (3)^2 = 1$

$V(x) = 1$

$E(x^2) = 0(0) + 0.1(1) + 4(0.2) + 9(0.3) + 16(0.4) = 10$

d. Suppose Y = 2X-5, find E(Y) and V(Y). (5 pt)

$E(Y) = E(2x-5) = a(E(x)) + b = 2(E(x)) - 5 = 2(3) - 5$

$E(Y) = 1$

$V(Y) = V(2x-5) = a^2 V(x) = 2^2(1) = 4$

$V(Y) = 4$

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5. A certain large shipment comes with a guarantee that it contains no more than 15% defective items. If the proportion of defective items in the shipment is greater than 15%, the shipment may be returned. You draw a random sample of 10 items. Let x be the number of defective items in the sample.

   a. If in fact 15% of the items in the shipment are defective (so that the shipment is good, but just barely), what is the probability of at least seven defective items in the sample? (5 pt)

$P(x \geq 7) = P(7) + P(8) + P(9) + P(10)$ $\quad p = 0.15$

$P(7) = \binom{10}{7} p^7 (1-p)^3 = \binom{10}{7} 0.15^7 0.85^3 = 0.000126$

$P(8) = \binom{10}{8} p^8 (1-p)^2 = \binom{10}{8} 0.15^8 0.85^2 = 0.000008$

$P(9) = \binom{10}{9} p^9 (1-p)^1 = \binom{10}{9} 0.15^9 0.85^1 = 0.0000003$

$P(10) = \binom{10}{10} p^{10} (1-p)^0 = \binom{10}{10} 0.15^{10} 0.85^0 = 0.000000006$

$P(x \geq 7) = 0.000135$

   b. Based on the answer to part a, if 15% of the items in the shipment are defective, would 7 defectives in a sample of size 10 be an unusually large number? If you found that 7 of the 10 sample items were defective, would this be convincing evidence that the shipment should be returned? Explain. (5 pt)

Yes, 7 defectives in a sample size of 10 is an unusually large number, as the chances of it happening is nearly 0. This would be convincing evidence to return the shipment, as the chance of it satisfying the 15% requirement and having 7 defectives in 10 items is extremely low.

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c. If in fact 15% of the items in the shipment are defective, what is the probability of at least two defectives in the sample? (5 pt)

$P(x \geq 2) = 1 - P(x < 2) = 1 - (P(1) + P(0))$

$P(1) = \binom{10}{1} 0.15^1 0.85^9 = 0.347$

$P(0) = \binom{10}{0} 0.15^0 (0.85)^{10} = 0.197$

$P(x<2) = 0.5443$

$P(x \geq 2) = 1 - P(x<2) = 1 - 0.5443 = 0.4557$

$\boxed{P(x \geq 2) = 0.4557}$

d. Based on the answer to part c, if 15% of the items in the shipment are defective, would 2 defectives in a sample of size 10 be an unusually large number? If you found that 2 of the 10 sample items were defective, would this be convincing evidence that the shipment should be returned? Explain. (5 pt)

No, 2 defectives in a sample size of 10 would not be an unusually large number, as it is little less than half as likely to occur. It would not be convincing evidence to return the shipment, as the chance of it satisfying the 15% requirement and finding two defectives is quite high.

TABLE: Cumulative Binomial Distribution - 1

n	x	p = .01	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50
2	0	0.9801	0.9025	0.8100	0.7225	0.6400	0.5625	0.4900	0.4225	0.3600	0.3025	0.2500
	1	0.9999	0.9975	0.9900	0.9775	0.9600	0.9375	0.9100	0.8775	0.8400	0.7975	0.7500
	2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	0	0.97030	0.85738	0.729	0.61413	0.512	0.42187	0.343	0.27463	0.216	0.16638	0.125
	1	0.99970	0.99275	0.972	0.93925	0.896	0.84375	0.784	0.71825	0.648	0.57475	0.500
	2	1.00000	0.99988	0.999	0.99663	0.992	0.98437	0.973	0.95713	0.936	0.90887	0.875
	3	1.00000	1.00000	1.000	1.00000	1.000	1.00000	1.000	1.00000	1.000	1.00000	1.000
4	0	0.96060	0.81451	0.6561	0.52201	0.4096	0.31641	0.2401	0.17851	0.1296	0.09151	0.0625
	1	0.99941	0.98598	0.9477	0.89048	0.8192	0.73828	0.6517	0.56298	0.4752	0.39098	0.3125
	2	1.00000	0.99952	0.9963	0.98802	0.9728	0.94922	0.9163	0.87352	0.8208	0.75852	0.6875
	3	1.00000	0.99999	0.9999	0.99949	0.9984	0.99609	0.9919	0.98499	0.9744	0.95899	0.9375
	4	1.00000	1.00000	1.0000	1.00000	1.0000	1.00000	1.0000	1.00000	1.0000	1.00000	1.0000
5	0	0.95099	0.77378	0.59049	0.44371	0.32768	0.23730	0.16807	0.11603	0.07776	0.05033	0.03125
	1	0.99902	0.97741	0.91854	0.83521	0.73728	0.63281	0.52822	0.42842	0.33696	0.25622	0.18750
	2	0.99999	0.99884	0.99144	0.97339	0.94208	0.89648	0.83692	0.76483	0.68256	0.59313	0.50000
	3	1.00000	0.99997	0.99954	0.99777	0.99328	0.98437	0.96922	0.94598	0.91296	0.86878	0.81250
	4	1.00000	1.00000	0.99999	0.99992	0.99968	0.99902	0.99757	0.99475	0.98976	0.98155	0.96875
	5	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
6	0	0.94148	0.73509	0.53144	0.37715	0.26214	0.17798	0.11765	0.07542	0.04666	0.02768	0.01563
	1	0.99854	0.96723	0.88573	0.77648	0.65536	0.53394	0.42017	0.31908	0.23328	0.16357	0.10938
	2	0.99998	0.99777	0.98415	0.95266	0.90112	0.83057	0.74431	0.64709	0.54432	0.44152	0.34375
	3	1.00000	0.99991	0.99873	0.99411	0.98304	0.96240	0.92953	0.88258	0.82080	0.74474	0.65625
	4	1.00000	1.00000	0.99994	0.99960	0.99840	0.99536	0.98906	0.97768	0.95904	0.93080	0.89062
	5	1.00000	1.00000	1.00000	0.99999	0.99994	0.99976	0.99927	0.99816	0.99590	0.99170	0.98437
	6	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
7	0	0.93207	0.69834	0.47830	0.32058	0.20972	0.13348	0.08235	0.04902	0.02799	0.01522	0.00781
	1	0.99797	0.95562	0.85031	0.71658	0.57672	0.44495	0.32942	0.23380	0.15863	0.10242	0.06250
	2	0.99997	0.99624	0.97431	0.92623	0.85197	0.75641	0.64707	0.53228	0.41990	0.31644	0.22656
	3	1.00000	0.99981	0.99727	0.98790	0.96666	0.92944	0.87396	0.80015	0.71021	0.60829	0.50000
	4	1.00000	0.99999	0.99982	0.99878	0.99533	0.98712	0.97120	0.94439	0.90374	0.84707	0.77344
	5	1.00000	1.00000	0.99999	0.99993	0.99963	0.99866	0.99621	0.99099	0.98116	0.96429	0.93750
	6	1.00000	1.00000	1.00000	1.00000	0.99999	0.99994	0.99978	0.99936	0.99836	0.99626	0.99219
	7	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
8	0	0.92274	0.66342	0.43047	0.27249	0.16777	0.10011	0.05765	0.03186	0.01680	0.00837	0.00391
	1	0.99731	0.94276	0.81310	0.65718	0.50332	0.36708	0.25530	0.16913	0.10638	0.06318	0.03516
	2	0.99995	0.99421	0.96191	0.89479	0.79692	0.67854	0.55177	0.42781	0.31539	0.22013	0.14453
	3	1.00000	0.99963	0.99498	0.97865	0.94372	0.88618	0.80590	0.70640	0.59409	0.47696	0.36328
	4	1.00000	0.99998	0.99957	0.99715	0.98959	0.97270	0.94203	0.89391	0.82633	0.73962	0.63672
	5	1.00000	1.00000	0.99998	0.99976	0.99877	0.99577	0.98871	0.97468	0.95019	0.91154	0.85547
	6	1.00000	1.00000	1.00000	0.99999	0.99992	0.99962	0.99871	0.99643	0.99148	0.98188	0.96484
	7	1.00000	1.00000	1.00000	1.00000	1.00000	0.99998	0.99993	0.99977	0.99934	0.99832	0.99609
	8	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

TABLE: Cumulative Binomial Distribution - 2

n	x	.01	.05	.10	.15	.20	p .25	.30	.35	.40	.45	.50
9	0	0.91352	0.63025	0.38742	0.23162	0.13422	0.07508	0.04035	0.02071	0.01008	0.00461	0.00195
	1	0.99656	0.92879	0.77484	0.59948	0.43621	0.30034	0.19600	0.12109	0.07054	0.03852	0.01953
	2	0.99992	0.99164	0.94703	0.85915	0.73820	0.60068	0.46283	0.33727	0.23179	0.14950	0.08984
	3	1.00000	0.99936	0.99167	0.96607	0.91436	0.83427	0.72966	0.60889	0.48261	0.36138	0.25391
	4	1.00000	0.99997	0.99911	0.99437	0.98042	0.95107	0.90119	0.82828	0.73343	0.62142	0.50000
	5	1.00000	1.00000	0.99994	0.99937	0.99693	0.99001	0.97471	0.94641	0.90065	0.83418	0.74609
	6	1.00000	1.00000	1.00000	0.99995	0.99969	0.99866	0.99571	0.98882	0.97497	0.95023	0.91016
	7	1.00000	1.00000	1.00000	1.00000	0.99998	0.99989	0.99957	0.99860	0.99620	0.99092	0.98047
	8	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99998	0.99992	0.99974	0.99924	0.99805
	9	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
10	0	0.90438	0.59874	0.34868	0.19687	0.10737	0.05631	0.02825	0.01346	0.00605	0.00253	0.00098
	1	0.99573	0.91386	0.73610	0.54430	0.37581	0.24403	0.14931	0.08595	0.04636	0.02326	0.01074
	2	0.99989	0.98850	0.92981	0.82020	0.67780	0.52559	0.38278	0.26161	0.16729	0.09956	0.05469
	3	1.00000	0.99897	0.98720	0.95003	0.87913	0.77588	0.64961	0.51383	0.38228	0.26604	0.17188
	4	1.00000	0.99994	0.99837	0.99013	0.96721	0.92187	0.84973	0.75150	0.63310	0.50440	0.37695
	5	1.00000	1.00000	0.99985	0.99862	0.99363	0.98027	0.95265	0.90507	0.83376	0.73844	0.62305
	6	1.00000	1.00000	0.99999	0.99987	0.99914	0.99649	0.98941	0.97398	0.94524	0.89801	0.82812
	7	1.00000	1.00000	1.00000	0.99999	0.99992	0.99958	0.99841	0.99518	0.98771	0.97261	0.94531
	8	1.00000	1.00000	1.00000	1.00000	1.00000	0.99997	0.99986	0.99946	0.99832	0.99550	0.98926
	9	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99997	0.99990	0.99966	0.99902
	10	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
11	0	0.89534	0.56880	0.31381	0.16734	0.08590	0.04224	0.01977	0.00875	0.00363	0.00139	0.00049
	1	0.99482	0.89811	0.69736	0.49219	0.32212	0.19710	0.11299	0.06058	0.03023	0.01393	0.00586
	2	0.99984	0.98476	0.91044	0.77881	0.61740	0.45520	0.31274	0.20013	0.11892	0.06522	0.03271
	3	1.00000	0.99845	0.98147	0.93056	0.83886	0.71330	0.56956	0.42555	0.29628	0.19112	0.11328
	4	1.00000	0.99989	0.99725	0.98411	0.94959	0.88537	0.78970	0.66831	0.53277	0.39714	0.27441
	5	1.00000	0.99999	0.99970	0.99734	0.98835	0.96567	0.92178	0.85132	0.75350	0.63312	0.50000
	6	1.00000	1.00000	0.99998	0.99968	0.99803	0.99244	0.97838	0.94986	0.90065	0.82620	0.72559
	7	1.00000	1.00000	1.00000	0.99997	0.99976	0.99881	0.99571	0.98776	0.97072	0.93904	0.88672
	8	1.00000	1.00000	1.00000	1.00000	0.99998	0.99987	0.99942	0.99796	0.99408	0.98520	0.96729
	9	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99995	0.99979	0.99927	0.99779	0.99414
	10	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99996	0.99985	0.99951
	11	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
12	0	0.88638	0.54036	0.28243	0.14224	0.06872	0.03168	0.01384	0.00569	0.00218	0.00077	0.00024
	1	0.99383	0.88164	0.65900	0.44346	0.27488	0.15838	0.08503	0.04244	0.01959	0.00829	0.00317
	2	0.99979	0.98043	0.88913	0.73582	0.55835	0.39068	0.25282	0.15129	0.08344	0.04214	0.01929
	3	1.00000	0.99776	0.97436	0.90779	0.79457	0.64878	0.49252	0.34665	0.22534	0.13447	0.07300
	4	1.00000	0.99982	0.99567	0.97608	0.92744	0.84236	0.72366	0.58335	0.43818	0.30443	0.19385
	5	1.00000	0.99999	0.99946	0.99536	0.98059	0.94560	0.88215	0.78726	0.66521	0.52693	0.38721
	6	1.00000	1.00000	0.99995	0.99933	0.99610	0.98575	0.96140	0.91537	0.84179	0.73931	0.61279
	7	1.00000	1.00000	1.00000	0.99993	0.99942	0.99722	0.99051	0.97449	0.94269	0.88826	0.80615
	8	1.00000	1.00000	1.00000	0.99999	0.99994	0.99961	0.99831	0.99439	0.98473	0.96443	0.92700