CEE 110—Probability and Statistics Midterm
April 30,2020

Name:____ Student ID: ___________________

1. Suppose the following table shows January rainfall in Los Angeles/

Year	JAN
2011	3.00
2012	1.62
2013	1.12
2014	2.05
2015	1.25
2016	2.84
2017	5.92
2018	4.82
2019	3.06
2020	4.20

a) Calculate the mean and the standard deviation of the rainfall. Round the values to the nearest hundredth if needed. (5 pts)

Mean = $\frac{3+1.62\ldots+4.20}{10} = 2.99$

Variance = $S = \frac{\sum (x_i - \bar{x})^2}{n-1}$

$= \frac{(3-2.99)^2+(1.62-2.99)^2\ldots+(4.2-2.99)^2}{10-1} = 2.52$

STD= $\sqrt{2.52} = 1.59$

b) What is the median of rainfall?
What are lower and upper quartiles?
Round the values to the nearest hundredth if needed. (5 pts)

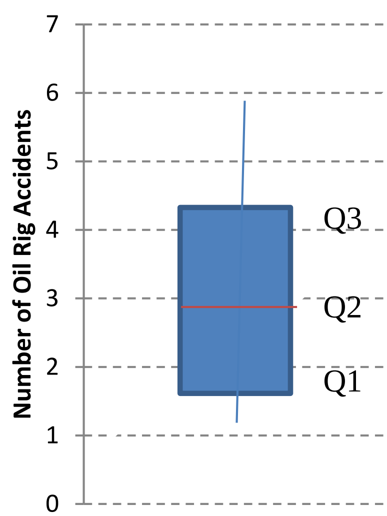
JAN
1.12
1.25
1.62
2.05
2.84
3.00
3.06
4.20
4.82
5.92

$Q2_{jan} = \tilde{x}_{jan} = \frac{2.84+3.00}{2} = 2.92$

$Q1_{jan} = 1.62$

$Q3_{jan} = 4.20$

c) Draw the box plot for each month. How many outliers are there? (5 pts)



IQR = 4.2-1.62 = 2.58
1.5* IQR = 1.5*2.58 = 3.87
$Q3_{Jan}$+1.5* IQR = 4.2+3.87 = 8.07 ∴ Max = 5.92
$Q1_{Jan}$-1.5* IQR = 1.62-3.87 = -2.25 ∴ Min = 1.12

No outliers

d) Which measure, the mean or the median, do you think better represents the January rainfall? Why? (5 pts)

Both looks similar because there is no outlier.
(Median is generally better because it is not affected by the outliers)

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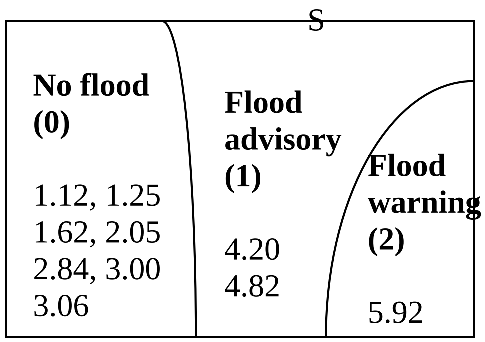
2. Use the rainfall data in Problem 1.

a) Suppose a rainfall of 5 inches and above issues flood warnings (L2), a rainfall of 3.5 inches and above and less than 5 inches issues flood advisory (L1), and a rainfall less than 3.5 inches does not issue flood (L0). Draw the Venn diagram in terms of L0, L1 and L2 with the outcomes. Are they mutually exclusive? Are they collectively exhaustive? (5 pts)



L0, L1 and L2 are mutually exclusive and collectively exhaustive.

b) Construct the pmf table for flooding (0: no flood, 1: flood advisory, 2: flood warning). (5 pts)

X	0 (No Flood)	1 (Flood Advisory)	2 (Flood Warning)
P(X)	7/10 = 0.7	2/10 = 0.2	1/10 = 0.1

c) Calculate the probability that issues either flood advisory or warning (5 pts)

$$
\begin{aligned}
P(L1 \cup L2) &= P(L1)+P(L2)-P(L1 \cap L2) \\
&= 0.2 + 0.1 - 0 \\
&= 0.3
\end{aligned}
$$

d) What are the expected levels of flood using the pmf in part b? (5 pts)

$E(X)=0\times0.7 + 1\times0.2 + 2\times0.1 = 0 + 0.2 + 0.2 = 0.4 \cong$ No Flood

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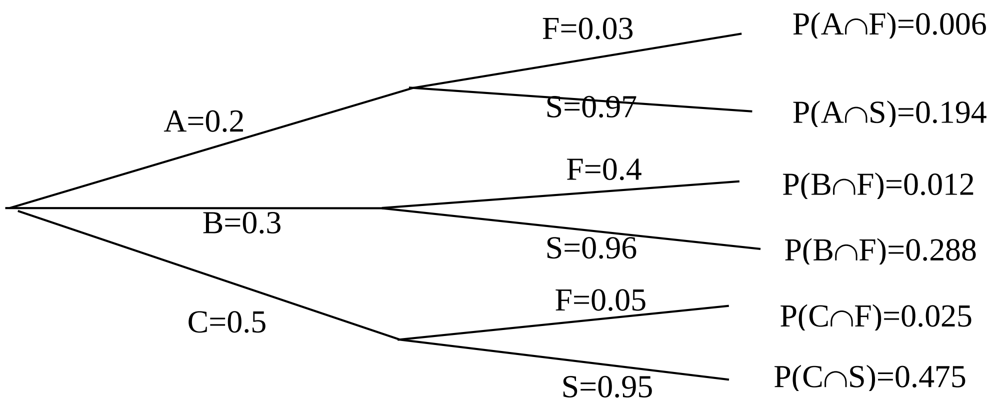
3. A valve is given a quality score of either A, B or C. 20% of the valves were given a quality score A, 30% were given a quality score B, and 50% were given a quality score C.
In addition, 3% of the valves given a quality score A eventually failed, and the failure rate was 4% for valves given quality score B, and 5% for valves given a quality score C.

a) Draw a tree diagram. (5 pts)



b) What is the probability that the valve will fail? (5 pts)

P(A) = 0.2 P(B) = 0.3 P(C) = 0.5
P(F|A) = 0.03 P(F|B) = 0.04 P(F|C) = 0.05

$$
\begin{aligned}
P(F) &= P(F|A)\times P(A) + P(F|B)\times P(B) + P(F|C)\times P(C)\\
&= 0.03\times0.2+0.04\times0.3+0.05\times0.5\\
&= 0.006+0.012+0.025\\
&= 0.043
\end{aligned}
$$

c) If the valve fails, what is the probability that its quality score is either B or C? (5 pts)

$$P(B\,|\,F) = \frac{P(F\,|\,B)\times P(B)}{P(F)} = \frac{0.04\times0.3}{0.043} = 0.28$$

$$P(C\,|\,F) = \frac{P(F\,|\,C)\times P(C)}{P(F)} = \frac{0.05\times0.5}{0.043} = 0.58$$

and therefore P(B|F) + P(C|F) = 0.28 + 0.58 = 0.86

d) If the valve does not fail, what is the probability that the quality score of the valve is A? Round the values to the nearest hundredth if needed. (5 pts)

$$P(A\,|\,S) = \frac{P(A)\times P(S\,|\,A)}{P(S)} = \frac{0.2\times(1-0.03)}{(1-0.043)} = 0.2$$

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4. The maximum contaminant levels from water treatment plants are specified to have nitrogen concentration of 10 mg/L. From 10 days of daily monitoring, the probability of violation of the maximum contaminant level is 0.1.

   a) What is the probability that at most 8 days of monitoring will comply with the maximum contaminant level? (5 pts)

   X: number of days for compliance
   P=1-0.1
   X ~ Bin (10,0.9)
   From the cumulative Binomial Table, $P(X\leq 8) = 0.264$ (or 0.26)

   b) What is the probability that at least 8 days of monitoring will comply with the maximum contaminant level? (5 pts)

   $P(X\geq 8) = P(X=8)+ P(X=9)+ P(X=10)$

$$= \binom{10}{8}\cdot 0.9^8(0.1)^2 + \binom{10}{9}\cdot 0.9^9(0.1)^1 + \binom{10}{10}\cdot 0.9^{10}(0.1)^0$$
$$= \frac{10!}{8!(2)!}0.9^8(0.1)^2 + \frac{10!}{9!(1)!}0.9^9(0.1)^1 + \frac{10!}{10!(0)!}0.9^{10}(0.1)^0$$
$$= 0.194 + 0.387 + 0.349 = 0.93$$

   Alternatively, $P(X\geq 8) = 1 - P(X<8) = 1 - P(X\leq 7)$
   From the cumulative Binomial Table, $P(X\leq 7) = 0.07$
   Therefore, 1 - 0.07 = 0.93

   c) What is the probability that exactly 2 days will violate the maximum contaminant level? (5 pts)

   Either p(y=2)~Bin(10,0,1) for 2 days with violation or p(x=8)~Bin(10,0.9) for 8 days with compliance

$$= \binom{10}{2}\cdot 0.1^2(1-0.1)^8 = \binom{10}{8}\cdot 0.9^8(1-0.9)^2 = 0.194(0.19)$$

   Alternatively, $P(X=8) = P(X\leq 8) - P(X\leq 7) = 0.264-0.07=0.194$ (or 0.19)

   d) Calculate expected value and standard deviation of the days with compliance. (5 pts)

   $E(X) = np = 10\times 0.9 = 9$
   $V(X)= np(1-p) = 10\times 0.9\times(1-0.9) = 0.9$
   $STD = \sqrt{0.9} = 0.95$

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5. Suppose vehicles arrive at a COVID-19 drive-through test station with the rate of 24 per hour on average.

a) What is the probability that exactly one vehicle arrives during next five minutes? (5 pts)

a = 24/hour, t = 5/60 hour
$\lambda = 24\times1/12 = 2$, Poisson Distribution

$$p(x:\lambda) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$p(X_t = 1) = e^{-2}\frac{2^1}{1!} = 2\times e^{-2} = .2707(= 0.27)$$

Alternatively, from Poisson table = p(x=1, λ=2) - p(x=0, λ=2)
= 0.4060 - 0.1353 = 0.2707

b) What is the probability that there will be at least one vehicle during the next five minutes? (5 pts)

$$p(X \geq 1) = 1 - p(X = 0) = 1 - \frac{e^{-2}(2)^0}{0!} = 0.8647(0.86)$$

Alternatively, from Poisson table = 1 - p(x=0, λ=2)
= 1- 0.1353 = 0.8647

c) Suppose that an arriving vehicle will have test positive with probability .01. What is the probability that at least 1 vehicle arrives during the next 5 minutes and has test positive? (10 pts)

$p(X\geq1) \times p(x=\text{test positive})$
$= 0.8647\times0.01$
$= 0.008647$ (0.0086)