

UNIVERSITY OF CALIFORNIA, LOS ANGELES
Civil and Environmental Engineering Department

CEE 110 Introduction to Probability and Statistics for Engineers

Midterm Exam
(April 30, 2020 at 4:00 PM PDT)

- Students are allowed to bring one page (one side, approximately 8.5·11 inches) of summary notes with them for the exam. No other material such as textbooks, class notes, or homework is permitted.
- Useful tables are included at the end of the exam.
- Students may use any calculator during the exam, but the entire answer process must be clear on the written answer sheet. All problems must be worked as if only a basic calculator was available.
- The exam has **five** problems with multiple parts. The credit for each part is indicated.
- Full credit will be given for answers that are worked out correctly. You may leave your answer in terms of a correct symbolic equation if you like (partial credits). To obtain credit, be sure to show as much work as possible! You should draw a box around your final answer to each problem.
- You cannot leave the room until the end of the exam. Please use your restroom before the exam.
- You are not allowed to talk during the exam.
- Before submit your solution, make sure all pages of your solution are legible
- Make sure your name is on all exam sheets
- You must turn in your cheat sheet with the exam with your name is on it
- Honor Code: Section 102.01 of UCLA's UCLA Student Conduct Code prohibits all forms of academic misconduct or research misconduct, including, but not limited to, cheating, fabrication or falsification, plagiarism, multiple submissions, facilitating academic dishonesty, coercion regarding grading or evaluation of coursework, or unauthorized collaboration. By submitting your assignment and exam for grading you acknowledge these terms / you declare that your work is solely your own and that you have not communicated with anyone other than the instructor and proctors in any way during the exam."
- You should write this in the beginning of your solution: **"On my honor, I have neither received nor given any unauthorized assistance on this examination. Agreed by Your Name and Signature"**

Good Luck!

Name: _____ Student ID: _____

1. Suppose the following table shows January rainfall in Los Angeles.

Year	JAN
2011	3.00
2012	1.62
2013	1.12
2014	2.05
2015	1.25
2016	2.84
2017	5.92
2018	4.82
2019	3.06
2020	4.20

- a) Calculate the mean and the standard deviation of the rainfall. Round the values to the nearest hundredth if needed. (5 pts)
- b) What is the median of rainfall? What are lower and upper quartiles? Round the values to the nearest hundredth if needed. (5 pts)
- c) Draw the box plot for the rainfall. How many outliers are there? (5 pts)
- d) Which measure, the mean or the median, do you think better represents the January rainfall? Why? (5 pts)

2. Use the rainfall data in Problem 1.

- a) Suppose a rainfall of 5 inches and above issues flood warnings (2), a rainfall of 3.5 inches and above and less than 5 inches issues flood advisory (1), and a rainfall less than 3.5 inches does not issue flood (0). Draw the Venn diagram in terms of no flood (0), flood advisory (1) and flood warnings (2) with the outcomes. Are they mutually exclusive? Are they collectively exhaustive? (5 pts)
- b) Construct the pmf table for flooding. (5 pts)

X	0 (No Flood)	1 (Flood Advisory)	2 (Flood Warning)
P(X)			

- c) Calculate the probability that issues either flood advisory or warning (Hint: use $X=0, 1,$ or 2) (5 pts)
- d) What are the expected levels of flood using the pmf in part b? (Hint: use $X=0, 1,$ or 2) Your final expected flood levels should be either No Flood, Flood Advisory or Flood Warning. (5 pts)

Name: _____ Student ID: _____

3. A valve is given a quality score of either A, B or C. 20% of the valves were given a quality score A, 30% were given a quality score B, and 50% were given a quality score C. In addition, 3% of the valves given a quality score A eventually failed, and the failure rate was 4% for valves given quality score B, and 5% for valves given a quality score C.
 - a) Draw a tree diagram. (5 pts)
 - b) What is the probability that the valve will fail? (5 pts)
 - c) If the valve fails, what is the probability that its quality score is either B or C? (5 pts)
 - d) If the valve does not fail, what is the probability that its quality score is A?
Round the values to the nearest hundredth if needed. (5 pts)

4. The maximum contaminant levels from water treatment plants are specified to have nitrogen concentration of 10 mg/L. From 10 days of daily monitoring, the probability of violation of the maximum contaminant level is 0.1.
 - a) What is the probability that at most 8 days of monitoring will comply (=not violate) with the maximum contaminant level? (5 pts)
 - b) What is the probability that at least 8 days of monitoring will comply (=not violate) with the maximum contaminant level? (5 pts)
 - c) What is the probability that exactly 2 days of monitoring will violate the maximum contaminant level?(5 pts)
 - d) Calculate expected value and standard deviation of the days with compliance (= no violation). (5 pts)

5. Suppose vehicles arrive at a COVID-19 drive-through test station with the rate of 24 per hour on average. Round the values to the nearest hundredth if needed.
 - a) What is the probability that exactly one vehicle arrives during next five minutes? (5 pts)
 - b) What is the probability that there will be at least one vehicle during the next five minutes? (5 pts)
 - c) Suppose that an arriving vehicle will have test positive with probability .01. What is the probability that at least 1 vehicle arrives during the next 5 minutes and has test positive? (10 pts)

Table: Cumulative Binomial probabilities

$$P[X \leq c] = \sum_{x=0}^c \binom{n}{x} p^x (1-p)^{n-x}$$

		p										
c		0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.95
n = 1	0	0.950	0.900	0.800	0.700	0.600	0.500	0.400	0.300	0.200	0.100	0.050
	1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
n = 2	0	0.903	0.810	0.640	0.490	0.360	0.250	0.160	0.090	0.040	0.010	0.003
	1	0.998	0.990	0.960	0.910	0.840	0.750	0.640	0.510	0.360	0.190	0.098
	2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
n = 3	0	0.857	0.729	0.512	0.343	0.216	0.125	0.064	0.027	0.008	0.001	0.000
	1	0.993	0.972	0.896	0.784	0.648	0.500	0.352	0.216	0.104	0.028	0.007
	2	1.000	0.999	0.992	0.973	0.936	0.875	0.784	0.657	0.488	0.271	0.143
	3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
n = 4	0	0.815	0.656	0.410	0.240	0.130	0.063	0.026	0.008	0.002	0.000	0.000
	1	0.986	0.948	0.819	0.652	0.475	0.313	0.179	0.084	0.027	0.004	0.000
	2	1.000	0.996	0.973	0.916	0.821	0.688	0.525	0.348	0.181	0.052	0.014
	3	1.000	1.000	0.998	0.992	0.974	0.938	0.870	0.760	0.590	0.344	0.185
	4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
n = 5	0	0.774	0.590	0.328	0.168	0.078	0.031	0.010	0.002	0.000	0.000	0.000
	1	0.977	0.919	0.737	0.528	0.337	0.188	0.087	0.031	0.007	0.000	0.000
	2	0.999	0.991	0.942	0.837	0.683	0.500	0.317	0.163	0.058	0.009	0.001
	3	1.000	1.000	0.993	0.969	0.913	0.813	0.663	0.472	0.263	0.081	0.023
	4	1.000	1.000	1.000	0.998	0.990	0.969	0.922	0.832	0.672	0.410	0.226
	5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
n = 6	0	0.735	0.531	0.262	0.118	0.047	0.016	0.004	0.001	0.000	0.000	0.000
	1	0.967	0.886	0.655	0.420	0.233	0.109	0.041	0.011	0.002	0.000	0.000
	2	0.998	0.984	0.901	0.744	0.544	0.344	0.179	0.070	0.017	0.001	0.000
	3	1.000	0.999	0.983	0.930	0.821	0.656	0.456	0.256	0.099	0.016	0.002
	4	1.000	1.000	0.998	0.989	0.959	0.891	0.767	0.580	0.345	0.114	0.033
	5	1.000	1.000	1.000	0.999	0.996	0.984	0.953	0.882	0.738	0.469	0.265
	6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
n = 7	0	0.698	0.478	0.210	0.082	0.028	0.008	0.002	0.000	0.000	0.000	0.000
	1	0.956	0.850	0.577	0.329	0.159	0.063	0.019	0.004	0.000	0.000	0.000
	2	0.996	0.974	0.852	0.647	0.420	0.227	0.096	0.029	0.005	0.000	0.000
	3	1.000	0.997	0.967	0.874	0.710	0.500	0.290	0.126	0.033	0.003	0.000
	4	1.000	1.000	0.995	0.971	0.904	0.773	0.580	0.353	0.148	0.026	0.004
	5	1.000	1.000	1.000	0.996	0.981	0.938	0.841	0.671	0.423	0.150	0.044
	6	1.000	1.000	1.000	1.000	0.998	0.992	0.972	0.918	0.790	0.522	0.302
	7	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table: Cumulative Binomial probabilities (*continued*)

		P										
c		0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.95
n = 8	0	0.663	0.430	0.168	0.058	0.017	0.004	0.001	0.000	0.000	0.000	0.000
	1	0.943	0.813	0.503	0.255	0.106	0.035	0.009	0.001	0.000	0.000	0.000
	2	0.994	0.962	0.797	0.552	0.315	0.145	0.050	0.011	0.001	0.000	0.000
	3	1.000	0.995	0.944	0.806	0.594	0.363	0.174	0.058	0.010	0.000	0.000
	4	1.000	1.000	0.990	0.942	0.826	0.637	0.406	0.194	0.056	0.005	0.000
	5	1.000	1.000	0.999	0.989	0.950	0.855	0.685	0.448	0.203	0.038	0.006
	6	1.000	1.000	1.000	0.999	0.991	0.965	0.894	0.745	0.497	0.187	0.057
	7	1.000	1.000	1.000	1.000	0.999	0.996	0.983	0.942	0.832	0.570	0.337
	8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
n = 9	0	0.630	0.387	0.134	0.040	0.010	0.002	0.000	0.000	0.000	0.000	0.000
	1	0.929	0.775	0.436	0.196	0.071	0.020	0.004	0.000	0.000	0.000	0.000
	2	0.992	0.947	0.738	0.463	0.232	0.090	0.025	0.004	0.000	0.000	0.000
	3	0.999	0.992	0.914	0.730	0.483	0.254	0.099	0.025	0.003	0.000	0.000
	4	1.000	0.999	0.980	0.901	0.733	0.500	0.267	0.099	0.020	0.001	0.000
	5	1.000	1.000	0.997	0.975	0.901	0.746	0.517	0.270	0.086	0.008	0.001
	6	1.000	1.000	1.000	0.996	0.975	0.910	0.768	0.537	0.262	0.053	0.008
	7	1.000	1.000	1.000	1.000	0.996	0.980	0.929	0.804	0.564	0.225	0.071
	8	1.000	1.000	1.000	1.000	1.000	0.998	0.990	0.960	0.866	0.613	0.370
	9	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
n = 10	0	0.599	0.349	0.107	0.028	0.006	0.001	0.000	0.000	0.000	0.000	0.000
	1	0.914	0.736	0.376	0.149	0.046	0.011	0.002	0.000	0.000	0.000	0.000
	2	0.988	0.930	0.678	0.383	0.167	0.055	0.012	0.002	0.000	0.000	0.000
	3	0.999	0.987	0.879	0.650	0.382	0.172	0.055	0.011	0.001	0.000	0.000
	4	1.000	0.998	0.967	0.850	0.633	0.377	0.166	0.047	0.006	0.000	0.000
	5	1.000	1.000	0.994	0.953	0.834	0.623	0.367	0.150	0.033	0.002	0.000
	6	1.000	1.000	0.999	0.989	0.945	0.828	0.618	0.350	0.121	0.013	0.001
	7	1.000	1.000	1.000	0.998	0.988	0.945	0.833	0.617	0.322	0.070	0.012
	8	1.000	1.000	1.000	1.000	0.998	0.989	0.954	0.851	0.624	0.264	0.086
	9	1.000	1.000	1.000	1.000	1.000	0.999	0.994	0.972	0.893	0.651	0.401
	10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
n = 11	0	0.569	0.314	0.086	0.020	0.004	0.000	0.000	0.000	0.000	0.000	0.000
	1	0.898	0.697	0.322	0.113	0.030	0.006	0.001	0.000	0.000	0.000	0.000
	2	0.985	0.910	0.617	0.313	0.119	0.033	0.006	0.001	0.000	0.000	0.000
	3	0.998	0.981	0.839	0.570	0.296	0.113	0.029	0.004	0.000	0.000	0.000
	4	1.000	0.997	0.950	0.790	0.533	0.274	0.099	0.022	0.002	0.000	0.000
	5	1.000	1.000	0.988	0.922	0.753	0.500	0.247	0.078	0.012	0.000	0.000
	6	1.000	1.000	0.998	0.978	0.901	0.726	0.467	0.210	0.050	0.003	0.000
	7	1.000	1.000	1.000	0.996	0.971	0.887	0.704	0.430	0.161	0.019	0.002
	8	1.000	1.000	1.000	0.999	0.994	0.967	0.881	0.687	0.383	0.090	0.015
	9	1.000	1.000	1.000	1.000	0.999	0.994	0.970	0.887	0.678	0.303	0.102
	10	1.000	1.000	1.000	1.000	1.000	1.000	0.996	0.980	0.914	0.686	0.431
	11	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Cumulative Poisson Distribution Table

Table shows cumulative probability functions of Poisson Distribution with various λ . Example: to find the probability $P(X \leq 3)$ where X has a Poisson Distribution with $\lambda = 2$, look in row 4 and column 4 to find $P(X \leq 3) = 0.8571$ where X is Poisson(2).

x	λ									
	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
0	0.6065	0.3679	0.2231	0.1353	0.0821	0.0498	0.0302	0.0183	0.0111	0.0067
1	0.9098	0.7358	0.5578	0.4060	0.2873	0.1991	0.1359	0.0916	0.0611	0.0404
2	0.9856	0.9197	0.8088	0.6767	0.5438	0.4232	0.3208	0.2381	0.1736	0.1247
3	0.9982	0.9810	0.9344	0.8571	0.7576	0.6472	0.5366	0.4335	0.3423	0.2650
4	0.9998	0.9963	0.9814	0.9473	0.8912	0.8153	0.7254	0.6288	0.5321	0.4405
5	1.0000	0.9994	0.9955	0.9834	0.9580	0.9161	0.8576	0.7851	0.7029	0.6160
6	1.0000	0.9999	0.9991	0.9955	0.9858	0.9665	0.9347	0.8893	0.8311	0.7622
7	1.0000	1.0000	0.9998	0.9989	0.9958	0.9881	0.9733	0.9489	0.9134	0.8666
8	1.0000	1.0000	1.0000	0.9998	0.9989	0.9962	0.9901	0.9786	0.9597	0.9319
9	1.0000	1.0000	1.0000	1.0000	0.9997	0.9989	0.9967	0.9919	0.9829	0.9682
10	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990	0.9972	0.9933	0.9863
11	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9991	0.9976	0.9945
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9992	0.9980
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9993
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

x	λ									
	5.5	6	6.5	7	7.5	8	8.5	9	9.5	10
0	0.0041	0.0025	0.0015	0.0009	0.0006	0.0003	0.0002	0.0001	0.0001	0.0000
1	0.0266	0.0174	0.0113	0.0073	0.0047	0.0030	0.0019	0.0012	0.0008	0.0005
23	0.0884	0.0620	0.0430	0.0296	0.0203	0.0138	0.0093	0.0062	0.0042	0.0028
3	0.2017	0.1512	0.1118	0.0818	0.0591	0.0424	0.0301	0.0212	0.0149	0.0103
4	0.3575	0.2851	0.2237	0.1730	0.1321	0.0996	0.0744	0.0550	0.0403	0.0293
5	0.5289	0.4457	0.3690	0.3007	0.2414	0.1912	0.1496	0.1157	0.0885	0.0671
6	0.6860	0.6063	0.5265	0.4497	0.3782	0.3134	0.2562	0.2068	0.1649	0.1301
7	0.8095	0.7440	0.6728	0.5987	0.5246	0.4530	0.3856	0.3239	0.2687	0.2202
8	0.8944	0.8472	0.7916	0.7291	0.6620	0.5925	0.5231	0.4557	0.3918	0.3328
9	0.9462	0.9161	0.8774	0.8305	0.7764	0.7166	0.6530	0.5874	0.5218	0.4579
10	0.9747	0.9574	0.9332	0.9015	0.8622	0.8159	0.7634	0.7060	0.6453	0.5830
11	0.9890	0.9799	0.9661	0.9467	0.9208	0.8881	0.8487	0.8030	0.7520	0.6968
12	0.9955	0.9912	0.9840	0.9730	0.9573	0.9362	0.9091	0.8758	0.8364	0.7916
13	0.9983	0.9964	0.9929	0.9872	0.9784	0.9658	0.9486	0.9261	0.8981	0.8645
14	0.9994	0.9986	0.9970	0.9943	0.9897	0.9827	0.9726	0.9585	0.9400	0.9165
15	0.9998	0.9995	0.9988	0.9976	0.9954	0.9918	0.9862	0.9780	0.9665	0.9513
16	0.9999	0.9998	0.9996	0.9990	0.9980	0.9963	0.9934	0.9889	0.9823	0.9730
17	1.0000	0.9999	0.9998	0.9996	0.9992	0.9984	0.9970	0.9947	0.9911	0.9857
18	1.0000	1.0000	0.9999	0.9999	0.9997	0.9993	0.9987	0.9976	0.9957	0.9928
19	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9995	0.9989	0.9980	0.9965
20	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9991	0.9984
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9993
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000