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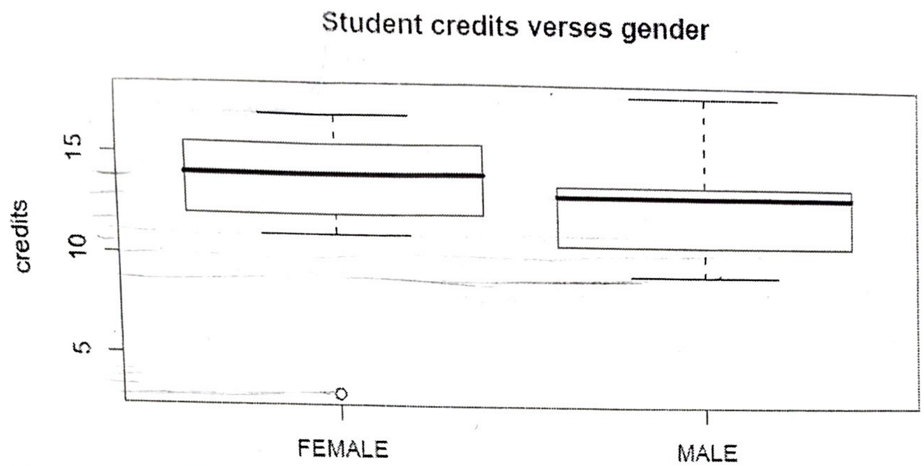
1. The following box and whisker plot shows the number of credits that the College of Engineering students earned based on their gender.



a. What is the median number of credits for each gender? Which gender has a higher median number of credits? (5 pt)

the median number of credits for females is ≈ 14 credits and the median for males is ≈ 13 credits

Females have a higher median number of credits

b. What is the approximate IQR (=fourth spread) of each gender? What percent of the data falls within the IQR? (5 pt)

$IQR_{female} = Q_3 - Q_1 \approx 16 - 12 = 4$ credits

$IQR_{male} = Q_3 - Q_1 \approx 13.5 - 10.5 = 3$ credits

50% of the data falls within the IQR

c. Which gender has a larger range of credits? (5 pt)

range(female) = max−min ≈ 17.5 − 1 = 16.5 credits

range(male) = max − min ≈ 19 − 9 = 10 credits

Females have a larger range

d. If someone reported the mean as the typical number of credits, would it be valid in this case? Justify your answer. (5 pt)

The mean is not the typical number of credits because the mean is the average value. The mean, unlike the median, is not resistant to outliers that skew the average.

2. The sample space of an experiment consists of a total of eight outcomes: S={1,2,3,4,5,6,7,8}. The events of X, Y, Z are defined as: X={even numbers}, Y={>5}, Z={3, 7}

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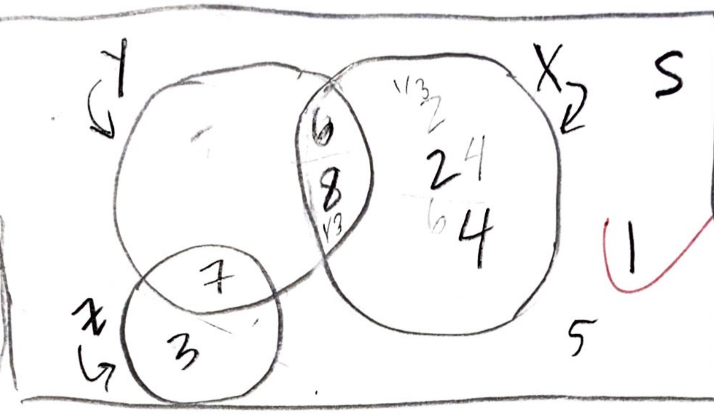
a. Draw the Venn diagram of the sample space and the events. Are X, Y, Z are mutually exclusive and collectively exhaustive? Justify your answers. (5 pt)

Since $X \cap Y = \{6,8\} \neq \emptyset$, $Y \cap Z = \{7\} \neq \emptyset$,

$\boxed{\text{X, Y, and Z are not all mutually exclusive.}}$



Since $X \cup Y \cup Z = \{2,3,4,6,7,8\} \neq S$,

$\boxed{\text{X, Y, Z are not collectively exhaustive}}$

Suppose we know that P(X∩Y)=1/3; X and Y are independent events; and probabilities of all even numbered outcomes are equal, in other words, p(2)=p(4)=p(6)=p(8).

b. Find P(X). (5 pt).

$P(X \cap Y) = 1/3 \rightarrow P(6) + P(8) = 1/3 \rightarrow P(\overset{\text{any one}}{\text{even}}) = 1/6$

$P(X) = P(\overset{\text{any one}}{\text{even}}) \cdot 4 = 4/6 = \boxed{P(X) = 2/3}$

c. Find P(Y). (5 pt)

$P(Y) = \dfrac{P(X \cap Y)}{P(X|Y)} = \dfrac{1/3}{P(X)} = \dfrac{1/3}{2/3} = \boxed{P(Y) = 1/2}$

$P(X|Y) = P(X)$

d. Find P(X∪Y). (5 pt)

$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y) =$

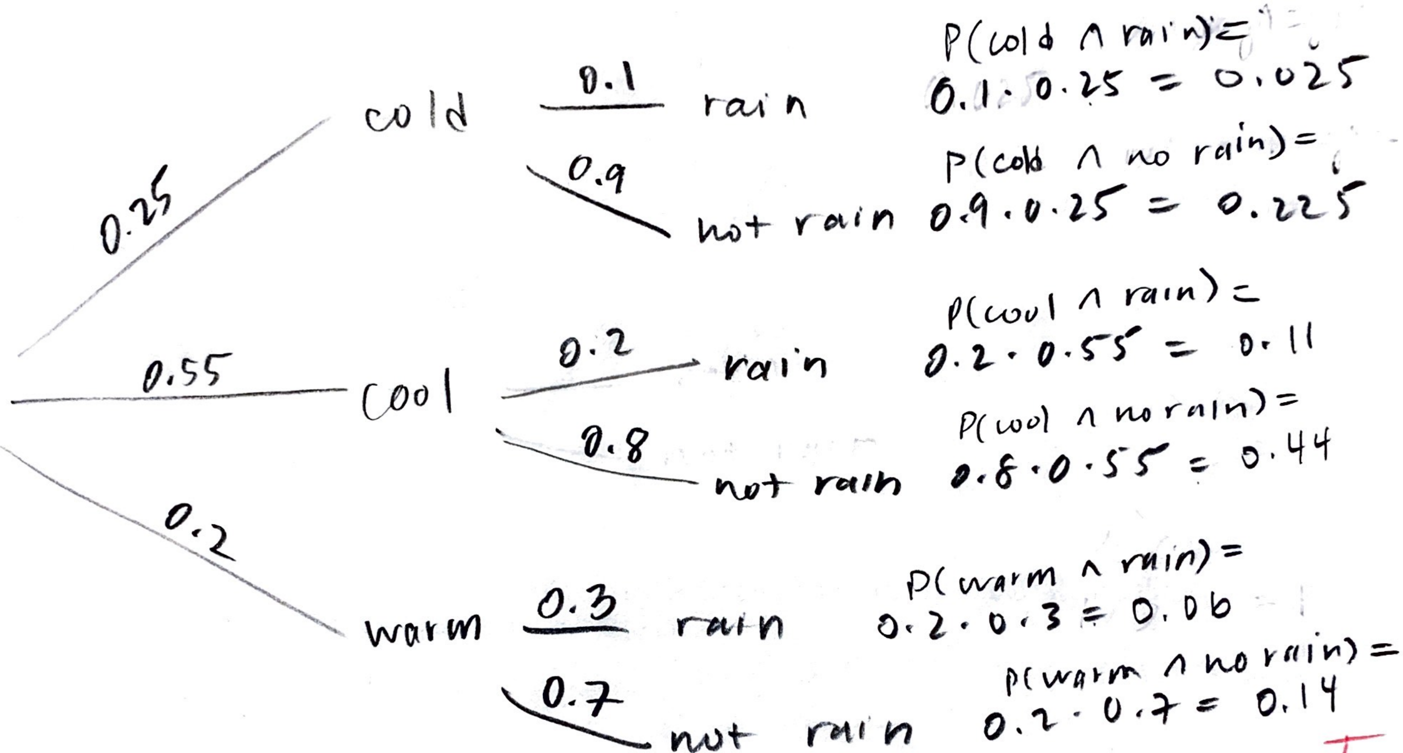
$2/3 + 1/2 - 1/3$

$\boxed{P(X \cup Y) = 5/6}$

3. In Los Angeles, the weather on a spring day is classified as either cold, cool, or warm. The probability that it is cold is 0.25, the probability that it is cool is 0.55 and the probability that it is warm is 0.2. In addition, on cold days the probability that it will rain is 0.1 and on cool days the probability that it will rain is 0.2 and on warm days the probability that it will rain is 0.3.

+5

a. Draw a tree diagram and show the probability with the associated outcomes. (5 pt)



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b. Find the probability that it will rain. (5 pt)

$$P(\text{rain}) = P(\text{rain} \cap \text{cold}) + P(\text{rain} \cap \text{cool}) + P(\text{rain} \cap \text{warm})$$

$$= 0.1 \cdot 0.25 + 0.2 \cdot 0.55 + 0.3 \cdot 0.2$$

$$= 0.025 + 0.11 + 0.06$$

$$\boxed{P(\text{rain}) = 0.195}$$

c. If it rains, what is the probability that it is either cold or cool? Round the values to two decimal places if needed. (5 pt)

+5

$$P(\text{cold or cool} \mid \text{rain}) = \frac{P(\text{rain} \cap \text{cold}) + P(\text{rain} \cap \text{cool})}{P(\text{rain})} =$$

$$\frac{0.025 + 0.11}{0.195} = \boxed{0.69}$$

d. If it is not raining on a particular day, what is the probability that it is cold? Round the values to two decimal places if needed. (5 pt)

+5

$$P(\text{cold} \mid \text{not raining}) = \frac{P(\text{cold} \cap \text{not raining})}{P(\text{not raining})}$$

$$= \frac{0.225}{P(\text{cold} \cap \text{no rain}) + P(\text{cool} \cap \text{no rain}) + P(\text{warm} \cap \text{no rain})}$$

$$= \frac{0.225}{0.225 + 0.44 + 0.14} = \frac{0.225}{0.805} =$$

$$\boxed{P(\text{cold} \mid \text{not raining}) = 0.28}$$

4. Suppose the sample space of discrete random variable is {0, 1, 2, 3, 4}. The pmf of the random variable is p(x)=cx, in other words p(0)=c×0, p(1)=c×1, p(2)=c×2, p(3)=c×3, p(4)=c×4.

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a. What is the value of c? (5 pt)

$0 + c + 2c + 3c + 4c = 1$

$10c = 1$

$\boxed{c = 0.1}$

+5

b. What is the cdf? (5 pt)

$F(0) = P(X \le 0) = 0$

$F(1) = P(X \le 1) = F(0) + P(1) = 0 + 0.1 = 0.1$

$F(2) = P(X \le 2) = F(1) + P(2) = 0.1 + 0.2 = 0.3$

$F(3) = P(X \le 3) = F(2) + P(3) = 0.3 + 0.3 = 0.6$

$F(4) = P(X \le 4) = F(3) + P(4) = 0.6 + 0.4 = 1$

$$\boxed{F(x) = \begin{cases} 0.1 & x=1 \\ 0.3 & x=2 \\ 0.6 & x=3 \\ 1 & x=4 \\ 0 & \text{otherwis} \end{cases}}$$

+5

c. Find E(X) and V(X). (5 pt)

$E(X) = \sum_{x \in D} x_i \cdot p(x_i) = 0 \cdot 0 + 1 \cdot 0.1 + 2 \cdot 0.2 + 3 \cdot 0.3 + 4 \cdot 0.4 = \boxed{E(X) = 3}$

$E(X^2) = \sum_{x \in D} x_i^2 \cdot P(x_i) = 0^2 \cdot 0 + 1^2 \cdot 0.1 + 2^2 \cdot 0.2 + 3^2 \cdot 0.3 + 4^2 \cdot 0.4 = E(X^2) = 10$

$V(X) = E(X^2) - E(X)^2 = 10 - 3^2 = 10 - 9 = \boxed{V(X) = 1}$

+5

d. Suppose Y = 2X-5, find E(Y) and V(Y). (5 pt)

$E(Y) = E(2X-5) = 2 \cdot E(X) - 5 = 2 \cdot 3 - 5 = \boxed{E(Y) = 1}$

$V(Y) = V(2X-5) = 2^2 \cdot V(X) = 4 \cdot 1 = \boxed{V(Y) = 4}$

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5. A certain large shipment comes with a guarantee that it contains no more than 15% defective items. If the proportion of defective items in the shipment is greater than 15%, the shipment may be returned. You draw a random sample of 10 items. Let x be the number of defective items in the sample.
   a. If in fact 15% of the items in the shipment are defective (so that the shipment is good, but just barely), what is the probability of at least seven defective items in the sample? (5 pt)

$P(\text{defective} \geq 7)$

$= P(\text{defective} = 7) + P(\text{defective} = 8) + P(\text{defective} = 9) + P(\text{defective} = 10)$

$= bpdf(7; 10, 0.15) + bpdf(8; 10; 0.15) + bpdf(9; 10; 0.15) + bpdf(10; 10; .15)$

where $bpdf(x; n; p) = \binom{n}{x} p^x (1-p)^{n-x}$

$= \boxed{P(\text{defective} \geq 7) = 0,000135}$

   b. Based on the answer to part a, if 15% of the items in the shipment are defective, would 7 defectives in a sample of size 10 be an unusually large number? If you found that 7 of the 10 sample items were defective, would this be convincing evidence that the shipment should be returned? Explain. (5 pt)

If we found 7 defectives, this would be evidence that the shipment should be returned because the $p(\text{defective} \geq 7)$ in a sample size of 10 where 15% are defective is only $0.000135 < 0.01$ which is statistically significant so we reject the hypothesis only $\leq 15\%$ of the shipment is defective.

c. If in fact 15% of the items in the shipment are defective, what is the probability of at least two defectives in the sample? (5 pt)

$P(\text{defective} \geq 2) = 1 - P(\text{not defective} = 1)$

$= 1 - \text{bpdf}(1; 10; 0.85) - \text{bpdf}(0; 10; 0.85)$

$= 1 - \binom{10}{1} 0.85^1 \cdot 0.15^9 - \binom{10}{0} 0.85^0 \cdot 0.15^{10} =$

$\boxed{P(\text{defective} \geq 2) = 0.4557}$

d. Based on the answer to part c, if 15% of the items in the shipment are defective, would 2 defectives in a sample of size 10 be an unusually large number? If you found that 2 of the 10 sample items were defective, would this be convincing evidence that the shipment should be returned? Explain. (5 pt)

2 is not an unusually large number because the probability that we find at least 2 defectives in a sample size of 10 where 15% are defective is $0.4557 > 0.01$ so this is not statistically significant and we can accept the hypothesis that $\leq 15\%$ of the items are defective and so we do not have evidence that the items should be returned.