CEE 110 Probability and Statistics for Engineers and Scientists
Midterm
Good luck!! Please show ALL work.

100

Problem 1: Short answer:

A)
More and more fuses are selected until four defective fuses are found. The number of fuses tested up to and including the fourth defective fuse is recorded.

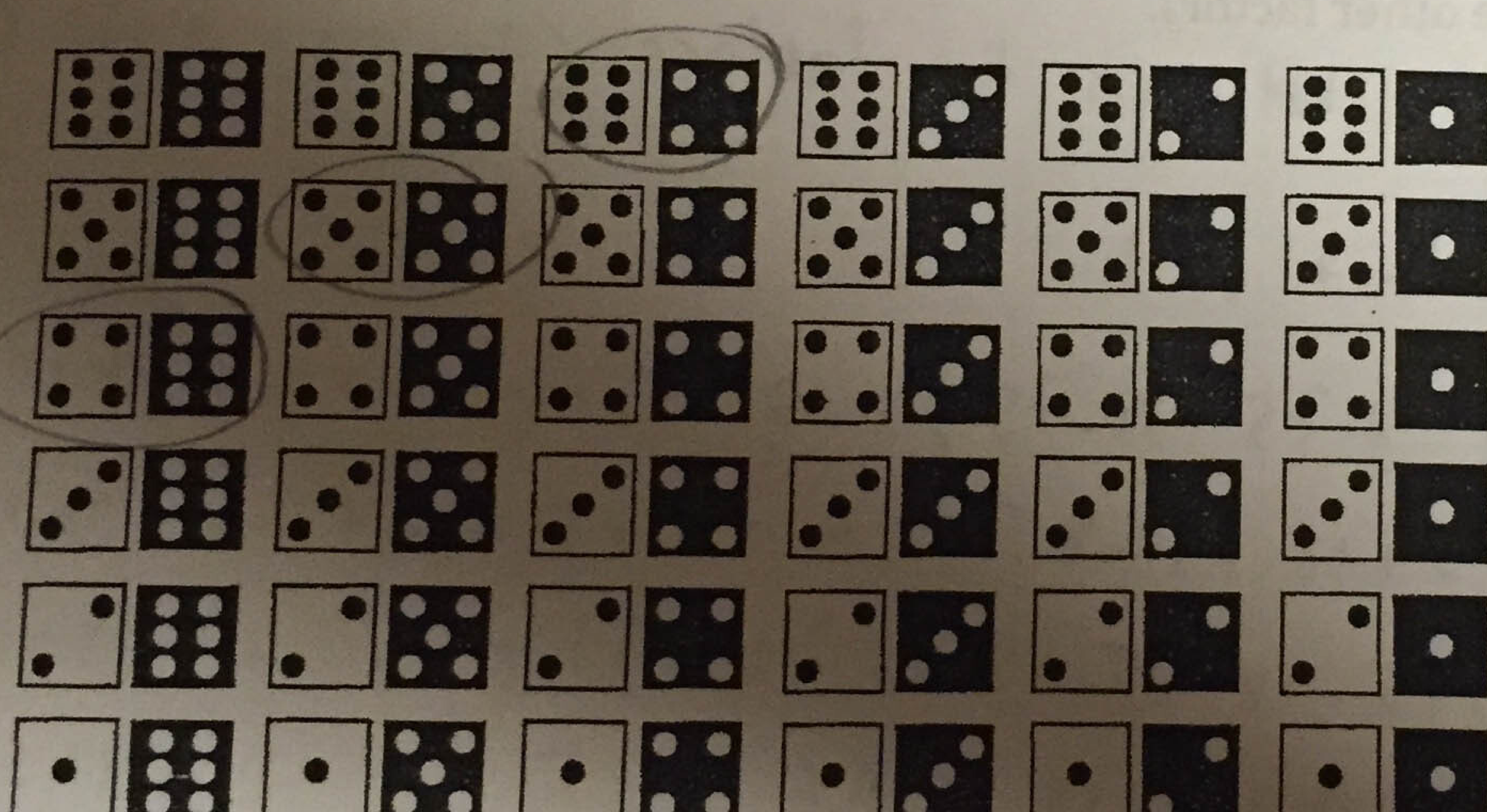
$S_x = (4, 5, \ldots, \infty)$

B)
Three students from this class are selected to each rate a new movie on a scale of 1-4 stars. The sample space contains how many possible outcomes?

$4 \cdot 4 \cdot 4 = \boxed{64}$

C)
What are the odds of two dice summing to ten?

4+6, 5+5, 6+4



$3/36 = \boxed{1/12}$

D) What is the probability of rolling at least one double "1's" in eighteen rolls of two dice?

$1 - \left(\frac{35}{36}\right)^{18} = \boxed{.40}$

Problem 2:

A pound is trying desperately to tame many dogs for adoption. Both giving treats and playing with the dogs are factors contributing to how friendly a dog will be after a week-long training program. They do an experiment to find out how best to efficiently tame the dogs. Considering the following data on average dog friendliness in the four different groups at the end of the program, please calculate main effects for each factor, identify whether the factors are additive or interaction, and please quantify any interaction, if any. (Solve for the mean you'd expect for dogs treated with both treats and playtime if the factors were additive.)

	No treats	With treats	Row averages	Main row effects
No playtime	40	70	55	$\alpha_1 = -8.75$
With playtime	50	95	72.5	$\alpha_2 = 8.75$
Column averages	45	82.5	63.75	
Main column effects	$\beta_1 = -18.75$	$\beta_2 = 18.75$		

Please draw the graph used to visualize interaction (one factor along x axis and lines drawn for each case for the other factor).

There is interaction between the factors. $(70-40 = 30) \neq (95$
$=$

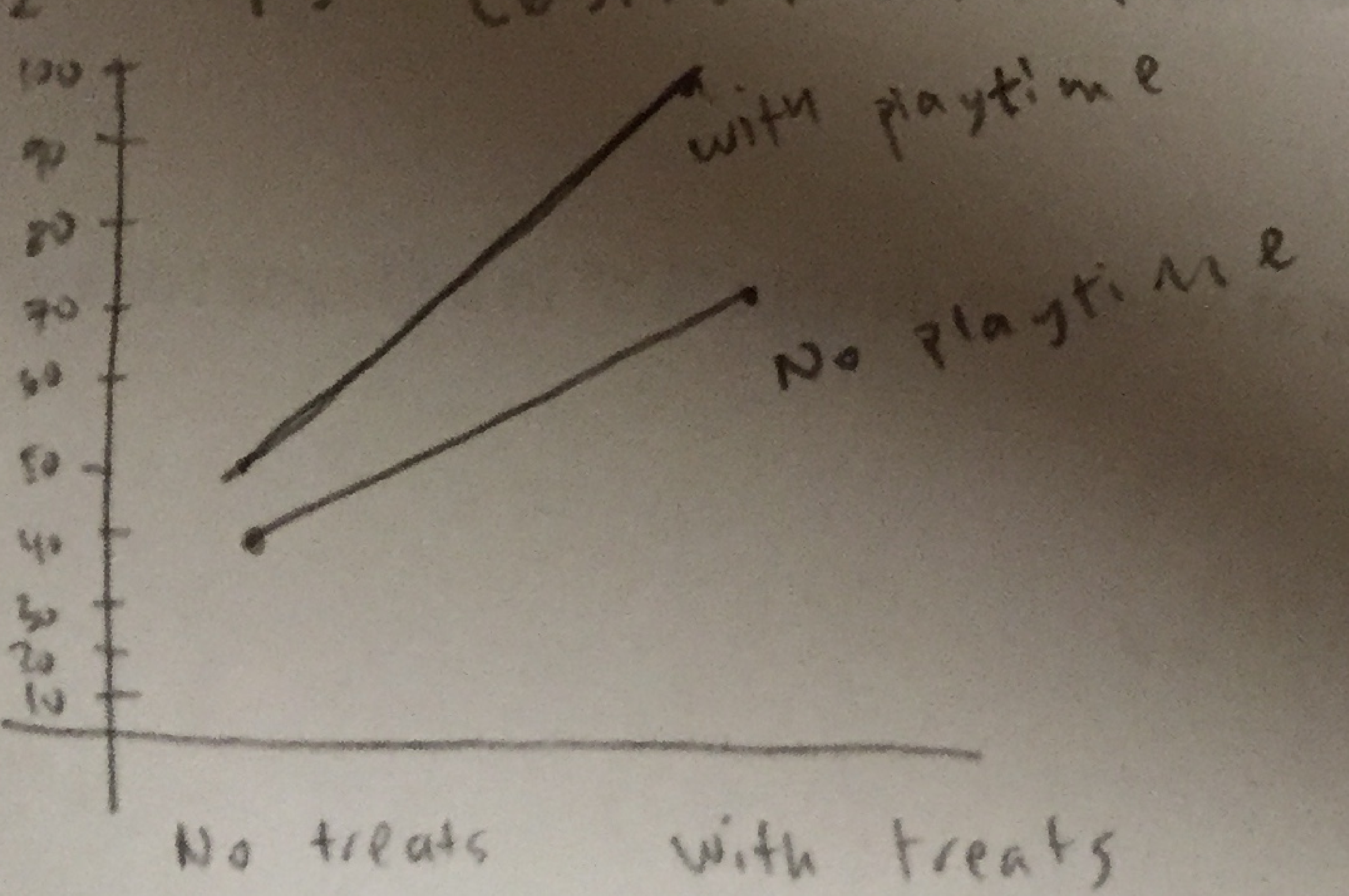
$\gamma_{ij} = \mu_{ij} - (\mu_{..} + \alpha_i + \beta_j)$

$\gamma_{11} = 40 - (63.75 - 8.75 - 18.75) = 3.75$, expected mean for additive: 36.25

$\gamma_{12} = 70 - (63.75 + 18.75 - 8.75) = -3.75$, expected mean: 73.75

$\gamma_{21} = \overset{50}{\cancel{45}} - (63.75 - 18.75 + 8.75) = -3.75$, " " : 53.75

$\gamma_{22} = 95 - (63.75 + 8.75 + 18.75) = 3.75$, " - " : 91.25



Note that the two lines a
not parallel - this further
demonstrates interaction b
the factors.

Problem 3:

A leadership committee of four students is to be formed from ten students.

a) How many different committees are possible?
b) The ten students are five biology majors, three chemists, and two physicists. How many committees would have two biology majors, one chem major, and one physics major?
c) If all committees were equally likely to form, what is the likelihood of a committee as outlined in part b.

a.) $\binom{10}{4} = 210$

b.) $\binom{5}{2}\binom{3}{1}\binom{2}{1} = 60$

c.) $\frac{60}{210} = .29$

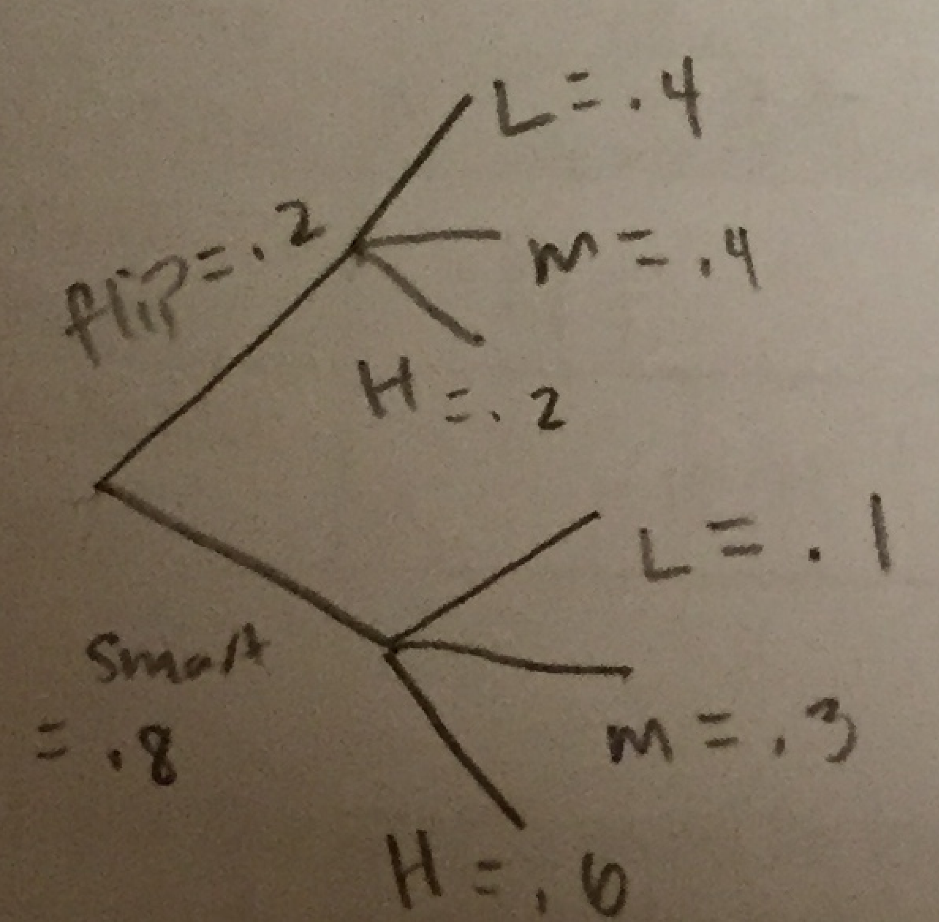
Problem 4:

Eighty percent of all students have smart phones while 20% have flip phones. Of those with smart phones, 60% check email with high frequency (every hour or two), 30% check email with middle frequency (three or four times a day) and 10% check it with low frequency (once or twice per day). Of those using flip phones, 20%, 40%, and 40% ~~have incomes~~ check emails in the high, middle and low categories, respectively.

Please draw tree diagram

a) What is the probability that a randomly chosen student only checks email once or twice a day?

b) Given a low frequency of email checking, what is the probability that s/he has a smart phone?



a.) $P(L) = .2 \cdot .4 + .8 \cdot .1 = \boxed{.16}$

b.) $\dfrac{.8 \cdot .1}{.8 \cdot .1 + .2 \cdot .4} = \boxed{.5}$

Problem 5:

On average, 2.6 spotted owls land in a certain grove of Redwoods per hour.

a) What is the probability that four or less birds will fly by in an hour?
b) What is the probability that two, three, or four birds will fly by in an hour?
c) What is the probability that at least two birds will fly by in the hour?

a.) $X \sim \text{Poisson}(2.6)$, $\lambda = 2.6$

$P(X \leq 4) = \boxed{.877}$

b.) $P(X \leq 4) - P(X \leq 1) = .877 - .267 = \boxed{.61}$

c.) $P(X \geq 2) = 1 - P(X \leq 1) = 1 - .267 = \boxed{.733}$

probability of 0 or 1 birds

Problem 6:

A habitat contains 20 snow leopards, of which five are fitted with a radio collar. five snow leopards are captured at a later time, and let X be the number of collare snow leopards in the group (assuming the collar does not affect the probability o being captured).

a) Is X:
   a. Binomial
   b. Hypergeometric
   c. Negative binomial
   d. Poisson

b) What is the probability that $X = 2$?

c) What is the expected value of X?

$$P(X=x) = \frac{\binom{m_1}{x}\binom{m_2}{n-x}}{\binom{m_1+m_2}{n}}$$

$n = 5$

$x = 2$

$m_1 = 5$

$m_2 = 15$

$N = 20$

$$P(X=2) = \frac{\binom{5}{2}\binom{15}{3}}{\binom{20}{5}} = \boxed{.293}$$

c.) $\mu_x = n\frac{m_1}{N} = \frac{5 \cdot 5}{20} = \boxed{1.25}$

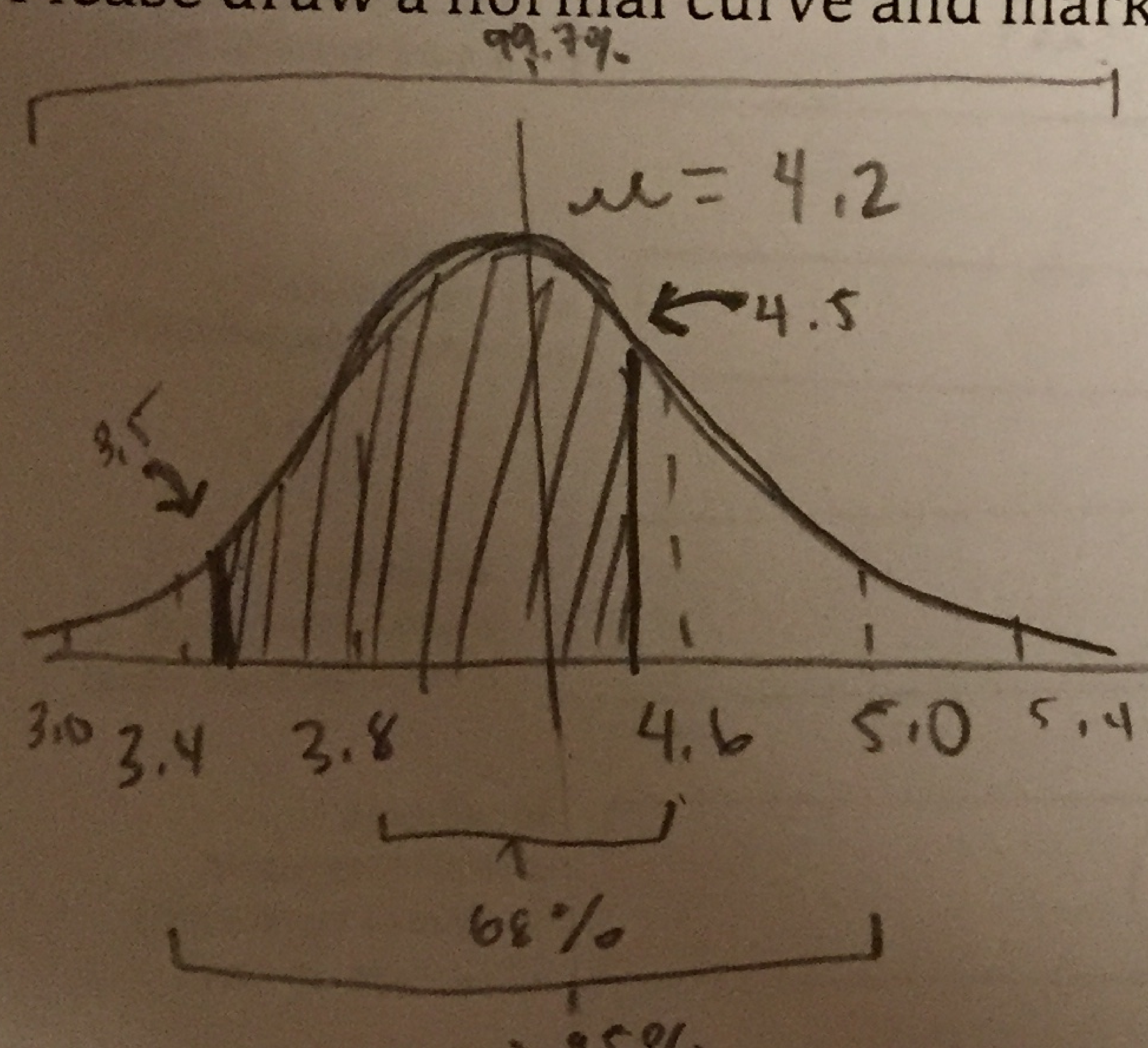
Problem 7:

Adult frogs of a certain species are being studied for effects due to climate change. Their length is normally distributed with mean length of 4.2 cm and standard deviation 0.4 cm. An adult frog is considered in a health range if its length is between 3.5 and 4.5 cm.

a) What is the probability that a randomly captured frog will be in the healthy range?

b) What is the probability that of four randomly and independently captured frogs, two are acceptable?

Please draw a normal curve and mark off the area you are solving for.



a.) $z_1 = \frac{3.5 - 4.2}{.4} = -1.75$

$z_2 = \frac{4.5 - 4.2}{.4} = .75$

normalCDF(−1.75, .75) = $\boxed{.733}$

b.) $p = .733$, Let X = acceptable frog

$P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$

$P(X=2) = \binom{4}{2} p^2 (1-p)^2 = \boxed{.23}$

Equation Sheet

Main Row and Column Effects	$\alpha_i = \overline{\mu}_{i.} - \overline{\mu}_{..}, \quad \beta_j = \overline{\mu}_{.j} - \overline{\mu}_{..}$	(1.8.4)
Cell Means under Additivity	$\mu_{ij} = \overline{\mu}_{..} + \alpha_i + \beta_j$	(1.8.5)
Interaction Effects	$\gamma_{ij} = \mu_{ij} - (\overline{\mu}_{..} + \alpha_i + \beta_j)$	(1.8.6)
Number of Permutations of k Units Selected from n	$P_{k,n} = n(n-1)\cdots(n-k+1) = \dfrac{n!}{(n-k)!}$	(2.3.2)
Number of Permutations of n Units Among Themselves	$P_{n,n} = n!$	(2.3.3)
Number of Combinations of k Units Selected from n	$\dbinom{n}{k} = \dfrac{P_{k,n}}{P_{k,k}} = \dfrac{n!}{k!(n-k)!}$	(2.3.4)
Number of Arrangements of n Units into r Groups of Sizes $n_1, n_2, \ldots, n_r$	$\dbinom{n}{n_1, n_2, \ldots, n_r} = \dfrac{n!}{n_1! n_2! \cdots n_r!}$	(2.3.6)
Multiplication Rule for Two Events	$P(A \cap B) = P(A)P(B\|A)$	(2.5.3)
Multiplication Rule for Three Events	$P(A \cap B \cap C) = P(A)P(B\|A)P(C\|A \cap B)$	(2.5.4)
Law of Total Probability	$P(B) = P(A_1)P(B\|A_1) + \cdots + P(A_k)P(B\|A_k)$	(2.5.7)
Bayes' Theorem	$P(A_j\|B) = \dfrac{P(A_j)P(B\|A_j)}{\sum_{i=1}^{k} P(A_i)P(B\|A_i)}$	(2.5.8)
Total Area Under the Curve of a PDF Must Equal 1	$\int_{-\infty}^{\infty} f(x)\,dx = 1$	(3.2.13)
Definition of Expected Value for Continuous X	$E(X) = \mu_X = \int_{-\infty}^{\infty} x f(x)\,dx$	(3.3.3)

General Definition of Variance of a Random Variable X

$$\sigma_X^2 = E\left[(X-\mu_X)^2\right] \tag{3.3.5}$$

Mean Value of a Function of a Discrete Random Variable X

$$E(h(X)) = \sum_{x \text{ in } S_X} h(x)p_X(x).$$

2. If X is continuous and $h(x)$ is a function, the expected value of $Y = h(X)$ can be computed using the PDF $f_X(x)$ of X as

Mean Value of a Function of a Continuous Random Variable X

$$E(h(X)) = \int_{-\infty}^{\infty} h(x)f(x)dx.$$

3. If the function $h(x)$ is linear, that is, $h(x) = ax + b$, so $Y = aX + b$, then

Mean Value of a Linear Function of a General Random Variable X

$$E(h(X)) = aE(X) + b.$$

Short-cut Formula for Variance of a Random Variable X

$$\sigma_X^2 = E(X^2) - [E(X)]^2 \tag{3.3.6}$$

PMF of the Binomial Distribution

$$P(Y = y) = \binom{n}{y} p^y(1-p)^{n-y}, \quad y = 0, 1, \ldots, n \tag{3.4.3}$$

Probability Mass Function of the Hypergeometric Distribution

$$P(X = x) = \frac{\binom{M_1}{x}\binom{M_2}{n-x}}{\binom{M_1+M_2}{n}} \tag{3.4.8}$$

Mean and Variance of the Hypergeometric Distribution

$$\mu_X = n\frac{M_1}{N}, \quad \sigma_X^2 = n\frac{M_1}{N}\left(1-\frac{M_1}{N}\right)\frac{N-n}{N-1} \tag{3.4.9}$$

PMF of the Geometric Distribution

$$P(X = x) = (1-p)^{x-1}p, \quad x = 1, 2, 3, \ldots \tag{3.4.11}$$

CDF of the Geometric Distribution

$$F(x) = 1 - (1-p)^x, \quad x = 1, 2, 3, \ldots \tag{3.4.12}$$

The mean value and variance of a geometric random variable X are derived in Examples 3.3-3 and 3.3-12, respectively, and summarized below.

Mean and Variance of the Geometric Distribution

$$E(X) = \frac{1}{p}, \quad \sigma^2 = \frac{1-p}{p^2} \tag{3.4.13}$$

PMF of the Negative Binomial Distribution

$$P(Y=y)=\binom{y-1}{r-1}p^r(1-p)^{y-r} \qquad (3.4.15)$$

Mean and Variance of the Negative Binomial Distribution

$$E(Y)=\frac{r}{p}, \quad \sigma_Y^2=r\frac{1-p}{p^2} \qquad (3.4.16)$$

PMF of the Poisson Distribution

$$P(X=x)=e^{-\lambda}\frac{\lambda^x}{x!}, \quad x=0,1,2,\ldots \qquad (3.4.17)$$

Mean and Variance of the Poisson Distribution

$$\mu_X=\lambda, \quad \sigma_X^2=\lambda \qquad (3.4.18)$$

Mean, Variance, and Percentiles of the Exponential Distribution

$$\mu=\frac{1}{\lambda}, \quad \sigma^2=\frac{1}{\lambda^2}, \quad x_\alpha=-\frac{\log(\alpha)}{\lambda} \qquad (3.5.2)$$

$$\phi(z)=\frac{1}{\sqrt{2\pi}}e^{-z^2/2} \quad \text{and} \quad \Phi(z)=\int_{-\infty}^{z}\phi(x)\,dx$$

Corollary 3.5-1 If $X \sim N(\mu,\sigma^2)$, then

1. $\dfrac{X-\mu}{\sigma} \sim N(0,1)$, and
2. $P(a \le X \le b)=\Phi\left(\frac{b-\mu}{\sigma}\right)-\Phi\left(\frac{a-\mu}{\sigma}\right)$.

Corollary 3.5-2 Let $X \sim N(\mu,\sigma^2)$, and let x_α denote the $(1-\alpha)$-100th percentile of X. Then,

$$x_\alpha=\mu+\sigma z_\alpha. \qquad (3.5.5)$$

x	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0
0	0.301	0.247	0.202	0.165	0.135	0.111	0.091	0.074	0.061	0.050
1	0.663	0.592	0.525	0.463	0.406	0.355	0.308	0.267	0.231	0.199
2	0.879	0.833	0.783	0.731	0.677	0.623	0.570	0.518	0.469	0.423
3	0.966	0.946	0.921	0.891	0.857	0.819	0.779	0.736	0.692	0.647
4	0.992	0.986	0.976	0.964	0.947	0.928	0.904	0.877	0.848	0.815
5	0.998	0.997	0.994	0.990	0.983	0.975	0.964	0.951	0.935	0.961
6	1.000	0.999	0.999	0.997	0.995	0.993	0.988	0.983	0.976	0.966
7		1.000	1.000	0.999	0.999	0.998	0.997	0.995	0.992	0.988
8				1.000	1.000	1.000	0.999	0.999	0.998	0.996
9							1.000	1.000	0.999	0.999

Table A.3 The Cumulative Distribution Function for the Standard Normal Distribution: Values of $\Phi(z)$ for Nonnegative z

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990