Final Exam

Physics 5A Lecture 1 – David Schriver

December 8, 2021

Name: ______________________________ Student I.D.# ___________________________

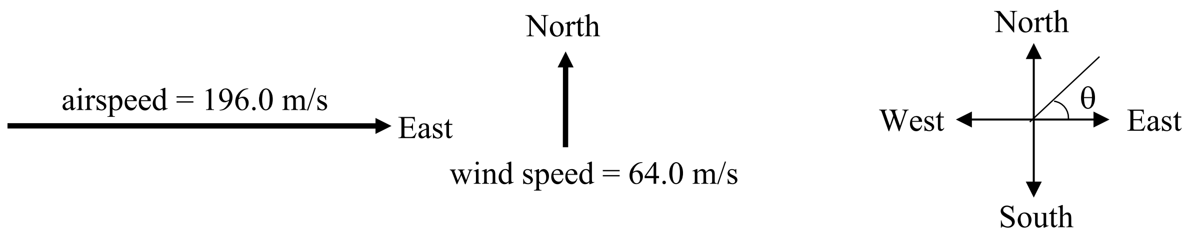
Signature: ______________________________

Please do the following 6 problems. Show all work and reasoning. Use the back of the page if necessary and circle or box your final answer.

Problem #	Score
1	
2	
3	
4	
5	
6	
Total	

Problem 1

An airplane is headed due East with an airspeed (speed relative to the surrounding air) of 196 m/s when a wind blowing due North with a speed of 64.0 m/s starts. Arrow lengths below are not to scale.



[8 pts] (a) When the airplane is flying through the wind, what is the airplane's velocity (magnitude and direction) with respect to the ground? Specify the airplane's direction in degrees with zero degrees due East and angle θ increasing counterclockwise as shown in the diagram at the far right.

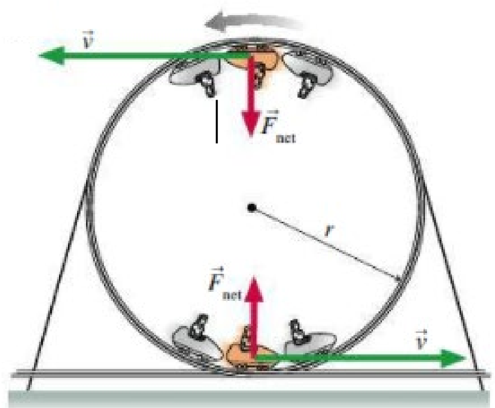
[5 pts] (b) If the airplane travels for 1 hour (3600 s) in the wind, how far (total distance) will the plane be from its initial position when the wind started blowing North?

[7 pts] (c) If the airplane pilot wants to maintain a heading due East with respect to the ground with the wind blowing to the North at 64.0 m/s (maintaining the airspeed of 196.0 m/s), what direction must the airplane head? Use the same convention for the angle as in part (a).

[5 pts] (d) For the situation in part (c) with the plane flying due East, what is the plane's speed with respect to the ground?

Problem 2

A roller coaster car travels on a track with a circular loop with radius $r = 25.0$ m at a constant speed of $v = 28.3$ m/s as shown in the diagram.



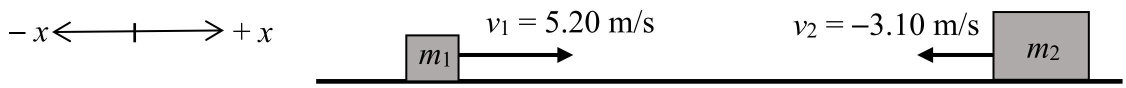
[7 pts] (a) What is the centripetal acceleration of the roller coaster car as it travels in uniform circular motion around the loop?

[9 pts] (b) What is the apparent weight (normal force in Newtons) of a 70.0 kg passenger in the car when the car is at the very top of the loop?

[9 pts] (c) What is the apparent weight (normal force in Newtons) of the 70.0 kg passenger in the car when the car is at the very bottom of the loop?

Problem 3

A block of mass $m_1 = 1.70$ kg slides left to right at $v_1 = 5.20$ m/s towards a second block with mass $m_2 = 3.95$ kg sliding right to left at $v_2 = -3.10$ m/s on a frictionless surface.



[4 pts] (a) What is the momentum (magnitude and direction) of each mass before they collide? Note there are two quantities asked for in this part.

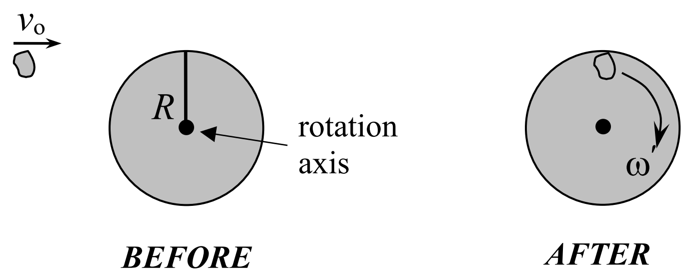
[8 pts] (b) When the two masses collide, they stick together in a perfectly inelastic collision. What is the velocity (magnitude and direction) of the combined masses?

[6 pts] (c) What is the impulse block m_1 exerts on block m_2 during the collision?

[7 pts] (d) If the time of the collision is 0.100 s, what is the average force block m_1 exerts on block m_2 during the collision?

Problem 4

A cylindrical disk of mass M = 4.60 kg and radius R = 0.250 m is sitting motionless on a fixed axis. A piece of putty of mass m = 1.25 kg moving with horizontal velocity v_o = 3.44 m/s strikes the edge of the disk and sticks to it as shown. After this, the cylinder and putty rotate together clockwise as shown. The cylindrical disk has a moment of inertia $I = \frac{1}{2}MR^2$. Ignore friction on the rotation axis.



[8 pts] (a) What is the angular momentum of the disk/putty system just before the collision?

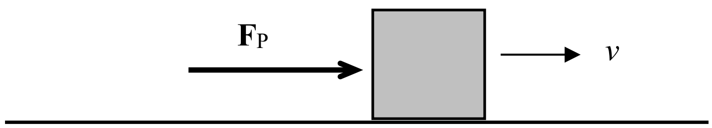
[4 pts] (b) What is the angular momentum of the disk/putty system just after the collision?

[9 pts] (c) What is the angular velocity (ω') of the disk/putty system just after the collision?

[4 pts] (d) For the angular velocity found in part (c) for the disk/putty system, how long will it take to make one complete revolution, i.e., what is the period T of rotation? Assume ω' to be constant for the complete revolution.

Problem 5

A worker is pushing a crate of mass m = 10.2 kg on a frictional horizontal surface with a constant pushing force F_P = 65.5 N pointing in the horizontal direction with respect to the horizontal. The worker pushes the crate a distance d = 30.0 m at a constant velocity v = 3.20 m/s.



[5 pts] (a) What is the coefficient of kinetic friction μ_k between the crate and the floor? (assume μ_k is constant)

[5 pts] (b) How much work is done by friction on the crate?

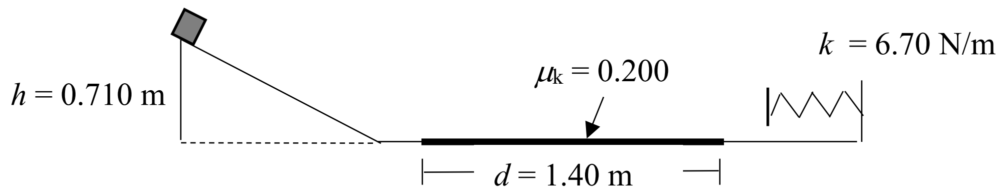
[5 pts] (c) How much work is done by gravity on the crate?

[5 pts] (d) How much work is done by the worker exerting the pushing force (F_P)?

[5 pts] (e) How much power is delivered by the worker exerting the pushing force?

Problem 6

A block with mass $m = 0.250$ kg starts at rest at the top of a frictionless incline with vertical height $h = 0.710$ m. At the bottom of the incline, the block encounters a frictional patch with $\mu_k = 0.200$ that has a length $d = 1.40$ m. The block then hits an uncompressed/unstretched spring with a spring constant $k = 6.70$ N/m. Other than the frictional patch, all surfaces, including the incline and the portion under the spring are frictionless.



[8 pts] (a) What is the speed of the block when it reaches the bottom of the incline on the horizontal part of the track before it hits the frictional patch?

[9 pts] (b) What is the speed of the block after passing over the entire frictional patch, but before hitting the spring?

[8 pts] (c) When the mass hits the spring, how far will the spring be compressed (i.e., when the mass compresses the spring and comes to a momentary stop)?

Final Exam, Physics 5A Lecture 1, Fall 2021 Name ____________________________
David Schriver

Constants (in SI units) and Mathematical Relations:

$g = 9.81$ m/s^2	acceleration of gravity at Earth
$\sin\theta$ = opposite/hypotenuse	sine of angle theta
$\cos\theta$ = adjacent/hypotenuse	cosine of angle theta
$\tan\theta$ = opposite/adjacent	tangent of angle theta

Motion Relationships

d	distance traveled
$s = d/\Delta t$	speed
$\Delta x = x_f - x_i$	displacement vector (1D)
$v = \Delta x/\Delta t$	velocity vector (1D)
$a = \Delta v/\Delta t$	acceleration vector (1D)

Equations of Motion in 1D with constant acceleration

$x_f = x_i + v_i\Delta t + (½)a\Delta t^2$	position as a function of time
$v_f = v_i + a\Delta t$	velocity as a function of time
$v_f^2 = v_i^2 + 2a(x_f - x_i)$	velocity-position relationship

Equations of Motion in 2D for projectile motion ($a_x = 0$, $a_y = -g$)

$x_f = x_i + v_{xi}\Delta t$	x position as a function of time ($v_{xi} = v_i \cos\theta_i$)
$v_{xf} = v_{xi}$ = constant	x velocity is constant
$y_f = y_i + v_{yi}\Delta t - (½)g\Delta t^2$	y position as a function of time ($v_{yi} = v_i \sin\theta_i$)
$v_{yf} = v_{yi} - g\Delta t$	y velocity as a function of time
$v_{yf}^2 = v_{yi}^2 - 2g(y_f - y_i)$	y velocity-position relationship
$y = x\tan\theta_i - x^2\left(\frac{g}{2v_i^2\cos^2\theta_i}\right)$	y-x relationship (for $x_i = 0$ and $y_i = 0$)
$R = \left(\frac{v_i^2}{g}\right)\sin 2\theta_i$	horizontal range equation (only for $y_f = y_i$)

Relative Motion

$\mathbf{v}_{OS} = \mathbf{v}_{OM} + \mathbf{v}_{MS}$ — relative velocity equation (vector addition)
O = object
S = stationary reference frame (non-moving)
M = moving reference frame (constant velocity)

Uniform Circular Motion

$a_c = v^2/r$	centripetal acceleration (points towards center of circle)
$v = 2\pi r/T$	velocity as a function of radius and period
$f = 1/T$	frequency-period relationship

Newton's Laws of Motion and Forces

$\mathbf{v}$ = constant, if $\Sigma\mathbf{F} = 0$	Newton's 1st Law
$\Sigma\mathbf{F} = m\mathbf{a}$	Newton's 2nd Law
$\mathbf{F}_{21} = -\mathbf{F}_{12}$ ($F_{12} = F_{21}$)	Newton's 3rd Law
$\mathbf{F} = m\mathbf{g}$	Gravitational force

Forces

$\Sigma\mathbf{F} = 0$	equilibrium for a point object
$w_{app} = mg + ma$	apparent weight when accelerating up or down
$F \le \mu_s n$	static friction force (n = normal force)
$F = \mu_k n$	kinetic friction force (n = normal force)
$F = \mu_r n$	rolling friction force (n = normal force)

Angular Quantities

$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T} = 2\pi f$	angular velocity
$v = \frac{\Delta s}{\Delta t} = \omega r$	tangential (linear) velocity-angular velocity relationship
$a_c = v^2/r = \omega^2 r$	centripetal acceleration for uniform circular motion
$\Sigma F = m(v^2/r) = m\omega^2 r$	uniform circular motion dynamics

Gravity and Circular Orbits

$F = \frac{Gm_1 m_2}{r^2}$	Universal Law of Gravitation ($G = 6.67 \times 10^{-11}$ N·m²/kg²)
$v = \sqrt{\frac{GM}{r}}$	orbital velocity ($G = 6.67 \times 10^{-11}$ N·m²/kg²)
$T^2 = \frac{4\pi^2 r^3}{GM}$	orbital period ($G = 6.67 \times 10^{-11}$ N·m²/kg²)

Rotational Motion (sign convention: + counterclockwise, – clockwise)

$\alpha = \frac{\Delta\omega}{\Delta t}$	angular acceleration
$a_t = \frac{\Delta v}{\Delta t} = r\alpha$	tangential acceleration-angular acceleration relationship
$\theta_f = \theta_i + \omega_i \Delta t + \frac{1}{2}\alpha\Delta t^2$	angle position (angular acceleration α = constant)
$\omega_f = \omega_i + \alpha\Delta t$	angular velocity (α = constant)
$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$	angular velocity-angle relationship (α = constant)
$\tau = rF_\perp = I\alpha$	torque on extended (rigid) object (I = moment of inertia)
$x_{CG} = \frac{\Sigma m_i x_i}{M}$	position of center of gravity, extended object (1 dimension)
$\Sigma\mathbf{F}_{ext} = M\mathbf{a}_{CG}$	translational motion of center of gravity, extended object
$\Sigma\mathbf{F} = 0$ and $\Sigma\tau = 0$	static equilibrium for an extended rigid object

Elasticity

$F_{spring} = -k\Delta x$	Hooke's law for springs (k = spring constant)
$F = \left(\frac{YA}{L}\right)\Delta L$	restoring force for rigid objects (Y = Young's modulus)

Momentum

$\mathbf{p} = m\mathbf{v}$	momentum (single object)
$\mathbf{J} = \Delta\mathbf{p} = \mathbf{F}_{\text{avg}}\Delta t$	impulse
$\mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2 + \ldots + \mathbf{p}_\text{n}$	total momentum (n objects)
$\Delta\mathbf{P} = 0 \rightarrow \mathbf{P}_\text{f} = \mathbf{P}_\text{i}$	conservation of momentum ($\Sigma\mathbf{F}_{\text{ext}} = 0$)
$m_\text{A}\mathbf{v}_\text{A} + m_\text{B}\mathbf{v}_\text{B} = (m_\text{A} + m_\text{B})\mathbf{v}'$	perfectly inelastic collision
$L = rp_\perp = mr^2\omega$	angular momentum (single point object)
$L = I\omega$	angular momentum (solid object) (+L counterclockwise, −L clockwise)
$\Delta L = 0 \rightarrow L_\text{f} = L_\text{i}$	conservation of angular momentum ($\Sigma\tau_{\text{ext}} = 0$)

Work and Energy

$W = \mathbf{F}\cdot\mathbf{d} = F_\parallel d = Fd\cos\theta$	work
$K = \frac{1}{2}mv^2$	kinetic energy
$K = \frac{1}{2}I\omega^2$	rotational kinetic energy
$W_{net} = \Delta K = \frac{1}{2}mv_\text{f}^2 - \frac{1}{2}mv_\text{i}^2$	work-energy principle (linear)
$W_{net} = \Delta K = \frac{1}{2}I\omega_\text{f}^2 - \frac{1}{2}I\omega_\text{i}^2$	work-energy principle (rotational)
$\Delta U_\text{g} = mg\Delta y$	gravitational potential energy
$\Delta U_\text{s} = \frac{1}{2}k(\Delta x)^2$	spring potential energy
$\Delta K + \Delta U = 0$	conservation of mechanical energy ($K + U = \text{constant}$)
$\Delta E_{\text{th}} = f_k\Delta x = \mu_k n\Delta x$	thermal energy
$\Delta K + \Delta U = -\Delta E_{th}$	conservation of mechanical energy with dissipation
$P = \frac{W}{\Delta t} = Fv$	power