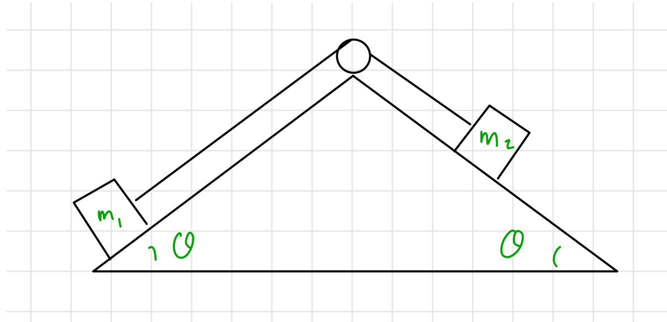
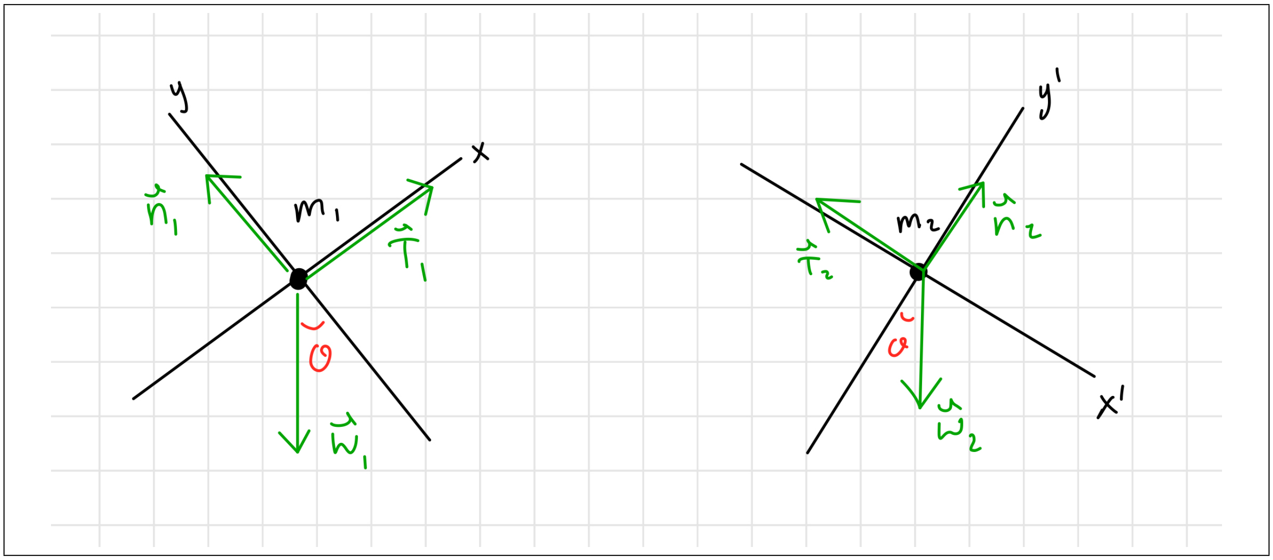
Problem 1

Two boxes of masses m_1, m_2 are connected by a massless rope and strung over a massless pulley, as shown. The boxes are on a wedge whose faces each make an angle of θ with the floor.



(a): – points

Assuming the ramp is frictionless, draw a free body diagram for each block. Make sure to clearly label all forces and angles. For simplicity, use a tilted coordinate system for each diagram, and make sure to clearly label your axes.



(a): – points

(b): − points

Calculate the magnitude of the normal forces acting on each block. Give your answer in terms of m_1, m_2, g, and/or θ.

In the "vertical" direction for each block, Newton's second law gives us

$$0 = n_1 - (w_1)_y$$
$$0 = n_2 - (w_2)_{y'}.$$

Thus

$$n_1 = m_1 g \cos\theta$$
$$n_2 = m_2 g \cos\theta$$

(b): − points

(c): − points

Calculate the magnitude of the tension in the rope. Give your answer in terms of m_1, m_2, g, and/or θ.

In the "horizontal" direction for each block, Newton's second law gives us

$$m_1 a = T - (w_1)_x$$
$$m_2 a = (w_2)_{x'} - T,$$

or

$$a = T/m_1 - g\sin\theta$$
$$a = g\sin\theta - T/m_2,$$

Here we used the fact that the tension will have the same magnitude acting on either box, and the acceleration along the surface of the wedge will be the same for both blocks. These are two equations and two unknowns (T and a). Eliminating a, we find

$$2g\sin\theta = T\left(\frac{1}{m_1} + \frac{1}{m_2}\right),$$

or

$$T = \frac{2g\sin\theta}{\frac{1}{m_1} + \frac{1}{m_2}} = \frac{m_1 m_2}{m_1 + m_2} 2g\sin\theta.$$

(c): − points

(d): − points

Calculate the acceleration of the blocks. Give your answer in terms of m_1, m_2, g, and/or θ.

Using the same two equations and eliminating T, we find

$$(m_1 + m_2)a = (m_2 - m_1)g\sin\theta,$$

or

$$a = \frac{m_2 - m_1}{m_2 + m_1} g\sin\theta.$$

(d): − points

Problem 2

You push a book across a table. It leaves your hand with a speed of 0.50 m/s, and comes to a stop after traveling a distance of 0.80 m. What is the coefficient of kinetic friction between the book and the table? Answer to two significant figures.

The acceleration of the book can be found using the kinematic equations of constant acceleration:

$$v_f^2 = v_i^2 + 2a\Delta x,$$

which gives

$$a = \frac{v_i^2}{2\Delta x}.$$

Newton's second law tells us that the net force parallel to the table must have been

$$F = ma = \frac{mv_i^2}{2\Delta x}.$$

This was supplied entirely by the force of kinetic friction, $f_k = \mu_k n = \mu_k mg$. Equating these, we find

$$\frac{mv_i^2}{2\Delta x} = \mu_k mg.$$

Solving for μ_k, we find

$$\mu_k = \frac{v_i^2}{2g\Delta x} = 0.016$$