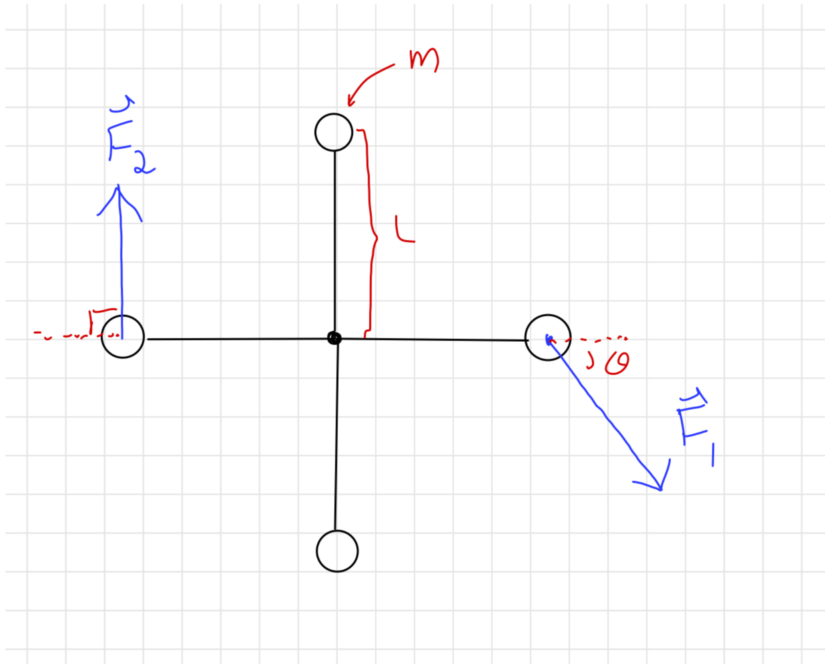
Problem 1

– points

Four small identical balls each of mass m are attached at the ends of two crossed very light rigid rods, and allowed to rotate around the point of intersection. The distance from each ball to the axle is L. Two forces, $\vec{F}_1$ and $\vec{F}_2$, are applied to the device as shown. In this problem, give your answers in terms of any combination of the known quantities F_1, F_2, θ, m, and L.



(a): – points

Calculate the net torque on this device. Be sure to state your sign convention.

With CCW+, we have

$$\tau = L(F_1 \sin\theta + F_2).$$

(a): – points

(b): – points

Calculate the angular acceleration due to the torque in part (a).

The moment of inertia is $4mL^2$, so

$$\alpha = \tau/I = \frac{F_1 \sin\theta + F_2}{4mL}$$

(b): – points

(c): – points

If the system started at rest, and the forces were such that the angular acceleration you calculated in part (b) remained constant throughout the motion, how long would it take for the system to make two full rotations?

By kinematics, we know $\Delta\theta = \frac{1}{2}\alpha t^2 = 4\pi$ rad, so

$$t = \sqrt{\frac{2(4\pi)}{\alpha}} = \sqrt{\frac{32\pi mL}{F_1 \sin\theta + F_2}}.$$

(c): − points

(d): − points

After two full rotations, the forces disappear. From this point on, what is the tension in the rods?

The angular velocity at this point is

$$\omega = \alpha t = \sqrt{8\pi\alpha} = \sqrt{2\pi \frac{F_1 \sin\theta + F_2}{mL}}$$

(d): − points

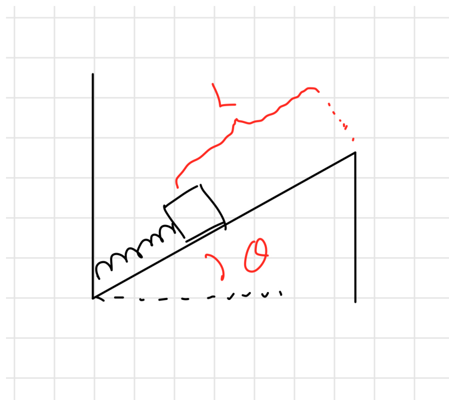
The rods are supplying a centripetal force of $F = m\omega^2 L$, or

$$F = 2\pi(F_1 \sin\theta + F_2)$$

Problem 2

– points

A block of mass m and a spring with spring constant k sit on a frictionless ramp that makes an angle θ with the horizontal. The block compresses the spring down the ramp a displacement L from its equilibrium point. The block is then released from rest; the spring decompresses, and the block shoots off the ramp.



(a): – points

What is the speed of the block as it leaves the ramp? Give your answer in terms of any combination of the given quantities m, k, L, θ, and g

The system converts spring potential energy into kinetic energy and gravitational potential energy:

$$\frac{1}{2}mv^2 + mgL\sin\theta = \frac{1}{2}kL^2.$$

Solving for v,

$$v = \sqrt{kL^2/m - 2gL\sin\theta}$$

(a): – points

(b): – points

After the block leaves the ramp, how high above the top of the ramp does the block reach?

The initial vertical velocity of the block is $v_y = v\sin\theta$. Then kinematics tells us that $v_y^2 = -2a\Delta y$, or

$$\Delta y = v_y^2/2g = (kL^2/2mg - L\sin\theta)\sin^2\theta.$$

(b): – points

(c): – points

When the block is at its highest point, what is its speed?

The easiest way to solve this is to note that the horizontal component of velocity is unchanged from when it leaves the ramp, and at the apex the vertical velocity is zero. Thus

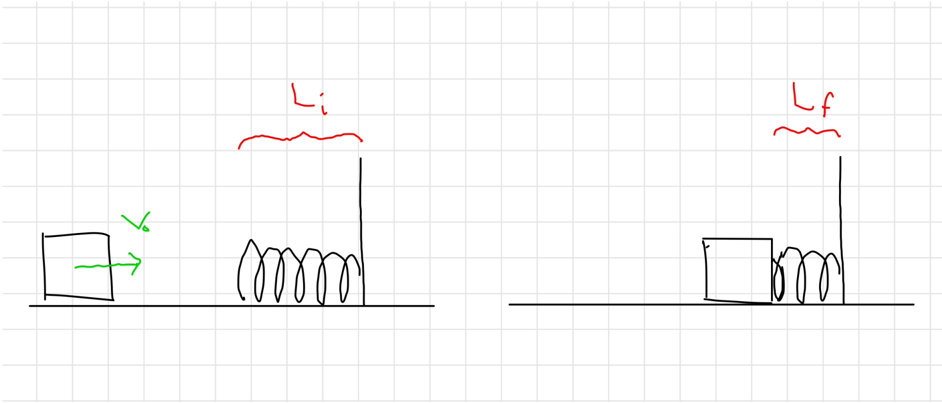
$$v_{\text{apex}} = v\cos\theta = \sqrt{kL^2/m - 2gL\sin\theta}\cos\theta$$

(c): – points

Problem 3

– points

A block of mass m is given a push across a horizontal table and released with initial speed v_0. The coefficients of kinetic and static friction between the block and the table are μ_k and μ_s, respectively. The block slides into a spring of equilibrium length L_i with spring constant k, and compresses the spring to a length of L_f, where it comes to rest. Shown are the initial and final positions of the block.



(a): – points

How much energy was converted into thermal energy throughout this process? Give your answers in terms of any combination of the given quantities $m, k, v_0, L_i, L_f, \mu_k, \mu_s$, and/or g.

The initial energy is $\frac{1}{2}mv_0^2$ and the final energy is $\frac{1}{2}k(L_i - L_f)^2 + E_{\text{th}}$, and so

$$E_{\text{th}} = \frac{1}{2}mv_0^2 - \frac{1}{2}k(L_i - L_f)^2$$

(a): – points

(b): – points

What can you say about the coefficient of static friction μ_s? Give your answer as in inequality, e.g. "$\mu_s > \ldots$". Give your answers in terms of any combination of the given quantities m, k, v_0, L_i, L_f, and/or g.

When the block is at rest, it feels a horizontal force $F = k(L_f - L_i)$, so the static friction force must be

$$f_s = k(L_f - L_i).$$

We know that this must be less than $\mu_s n = \mu_s mg$, so

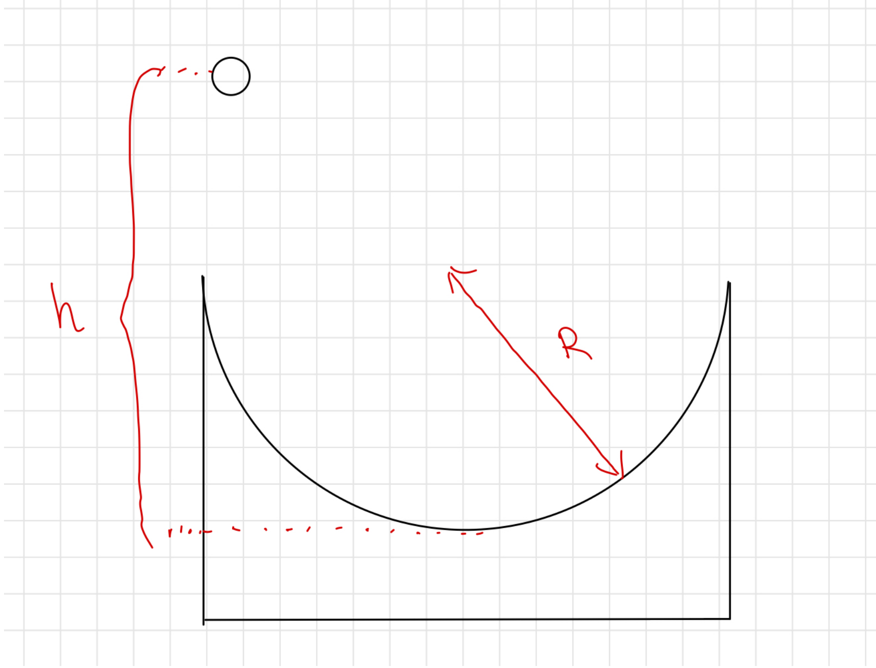
$$\mu_s > \frac{k(L_f - L_i)}{mg}$$

(b): – points

Problem 4

– points

A ball of mass m is dropped from rest into a frictionless semicircular bowl of radius R, from a height h above the bottom of the bowl. Assume that the ball can be modeled as a point particle. Give all answers in this question in terms of any combination of the given quantities m, h, R, and g.



(a): – points

What is the speed of the ball when it reaches the bottom of the bowl?

By conservation of energy, $\frac{1}{2}mv^2 = mgh$, so

$$v = \sqrt{2gh}$$

(a): – points

(b): – points

What is the magnitude of the normal force acting on the ball when it reaches the bottom of the bowl?

The sum of forces must have magnitude mv^2/R, so

$$n - mg = mv^2/R,$$

or

$$\begin{aligned} n &= mg + mv^2/R \\ &= mg + 2mgh/R \\ &= mg(1 + 2h/R) \end{aligned}$$

(b): – points

Problem 5

– points

For each of the following statements, indicate whether the statement is true or false, and explain your reasoning.

(a): – points

An object with nonzero acceleration can have a constant speed.

True. See: any object in uniform circular motion.	(a): – points

(b): – points

At any given time, the work being done by the force of gravity in keeping the Moon in its circular orbit around the Earth is nonzero.

False. The force is always perpendicular to the velocity, so no work is done.	(b): – points

(c): – points

In order for an object to move foward at a constant velocity, there must be some force pointing in the direction of motion.

False. See Newton's first law.	(c): – points

(d): – points

Two identical balls are thrown from the same point with the same speed; one is thrown vertically and one is thrown horizontally. They will reach the ground at the same time.

False. They will reach the ground with the same speed, but at different times, according to $\Delta y = (v_y)_i - \frac{1}{2}gt^2$.	(d): – points