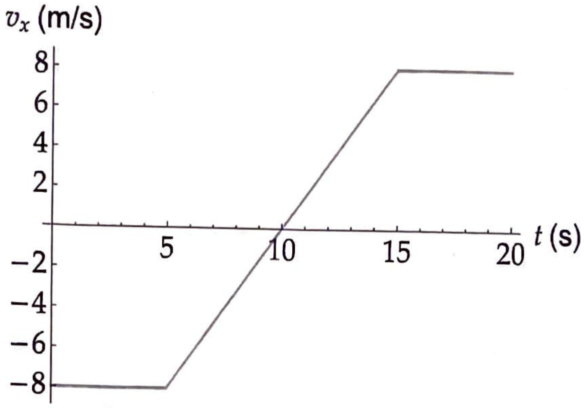
Problem 1 (20 points). Consider the following velocity vs. time graph for a drone moving along the x-axis. The drone starts at $x = 0$ at $t = 0$.

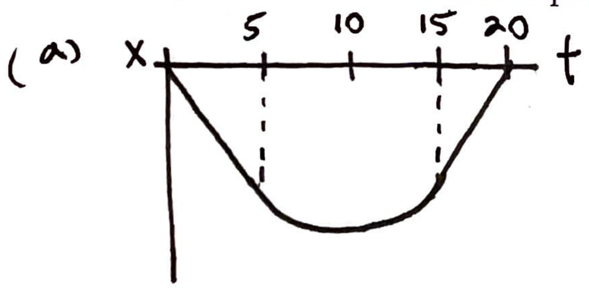


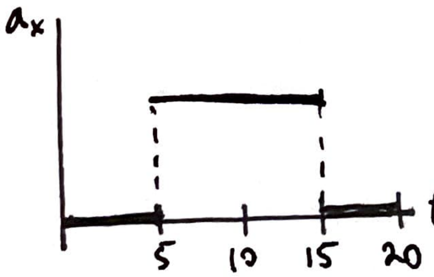
(a) (10 points) Sketch x vs. t and a_x vs. t graphs for the drone from $t = 0$ to $t = 20$ s. (Only the t axes need numerical labels.)

(b) (4 points) What is the x-component of the drone's acceleration at $t = 10$ s?

(c) (6 points) What is the drone's position at $t = 10$ s?

(a)





(b) $a_x = \frac{\Delta v_x}{\Delta t} = \frac{8\,m/s - (-8\,m/s)}{10\,s} = \boxed{1.6\,m/s^2}$

(c) $x = x_0 + \Delta x$

$= 0 + (-8\,m/s)(5\,s) + \frac{1}{2}(-8\,m/s)(5\,s)$

$= \boxed{-60\,m}$

Problem 2 (16 points). A passenger train moving at 25 m/s sees a freight train 200 m away. The freight train is moving at 15 m/s in the same direction as the passenger train. The passenger train brakes, causing a constant acceleration of 0.1 m/s^2 opposite the train's velocity.

(a) (8 points) Will the two trains collide? If so, what is their position when they collide?

(b) (6 points) In a single plot, sketch position vs. time graphs for the two trains.

(c) (2 points) What is the minimum magnitude acceleration the passenger train needs in order for the two trains not to collide?

(a) $x_P(t) = v_P t + \frac{1}{2} a_P t^2$ $\qquad v_P = 25\,\text{m/s}, \quad a_P = -0.1\,\text{m/s}^2$

$x_F(t) = x_{0F} + v_F t$ $\qquad x_{0F} = 200\,\text{m}, \quad v_F = 15\,\text{m/s}$

$$x_P = x_F \rightarrow v_P t + \frac{1}{2} a_P t^2 = x_{0F} + v_F t$$

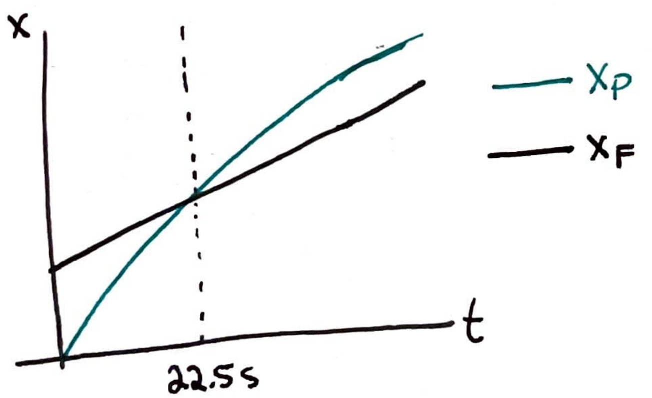
$$\rightarrow \frac{1}{2} a_P t^2 + (v_P - v_F) t - x_{0F} = 0$$

$$\rightarrow t = \frac{-(v_P - v_F) \pm \sqrt{(v_P - v_F)^2 + 2 a_P x_{0F}}}{a_P} = \begin{cases} 22.5\,\text{s} \\ 177\,\text{s} \end{cases}$$

since there are solutions, **the trains do collide!**

Where? $x_F(22.5\,\text{s}) = \boxed{540\,\text{m}}$

(b)



(c) No solutions if

$$(v_P - v_F)^2 - 2|a_P| x_{0F} < 0$$

$$\rightarrow |a_P| > \frac{(v_P - v_F)^2}{2 x_{0F}} = \boxed{0.25\,\text{m/s}^2}$$

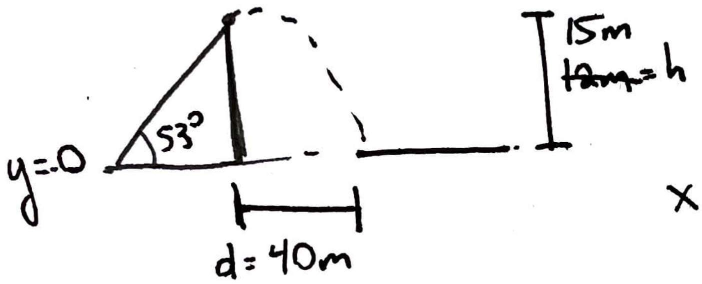
Problem 3 (14 points). Alex attempts to jump across a river on a motorcycle. She takes off from a ramp inclined at 53° above the horizontal. The river is 40 m wide, and the river bank on the far side of the river is 15 m below the top of the ramp.

(a) (8 points) At what speed does Alex need to leave the ramp to just reach the far bank of the river?

(b) (6 points) If she takes off with the initial speed you found in part (a), what will Alex's velocity be just before she lands?

(a)

(Not to scale...)



$$x(t) = v_0 \cos\theta \, t$$

$$y(t) = h + v_0 \sin\theta \, t - \frac{1}{2} g t^2$$

$$x = d \rightarrow v_0 \cos\theta \, t = d \rightarrow t = \frac{d}{v_0 \cos\theta}$$

$$y = 0 \rightarrow h + v_0 \sin\theta \left(\frac{d}{v_0 \cos\theta}\right) - \frac{1}{2} g \left(\frac{d}{v_0 \cos\theta}\right)^2 = 0$$

$$\rightarrow h + d\tan\theta - \frac{g d^2}{2 v_0^2 \cos^2\theta} = 0$$

$$\rightarrow h + d\tan\theta = \frac{g d^2}{2 v_0^2 \cos^2\theta}$$

$$\rightarrow v_0 = \sqrt{\frac{g d^2}{2(h + d\tan\theta)\cos^2\theta}} = \boxed{~~18.2~~ \text{ m/s}} \quad 17.8$$

(b)

$$t = \frac{d}{v_0 \cos\theta} = ~~3.64 \text{ s}~~ \quad t = 3.73 \text{ s}$$

$$v_x = v_0 \cos\theta = \boxed{11 \text{ m/s}} \quad \text{constant.}$$

$$v_y = v_0 \sin\theta - g t = \boxed{~~-21~~ \text{ m/s}} \quad -22.3 \text{ m/s}$$