Name SOLUTIONS

UID

Physics 1CH Midterm #2

May 12, 2022

On all problems, you need to show your work to get full credit.

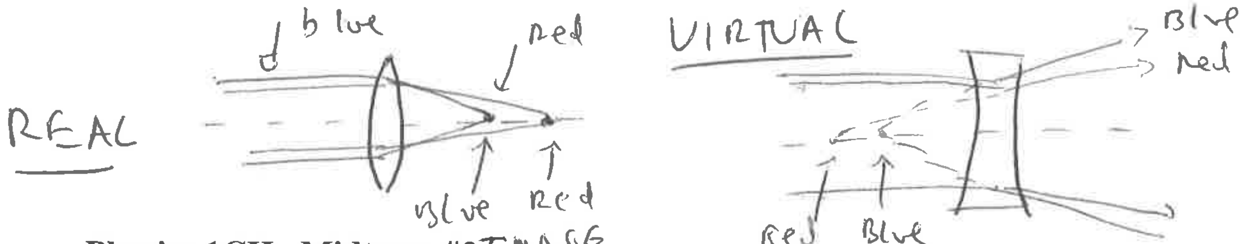
Below are a set of numerical constants. If you have any questions, please raise your hand to ask for help.

Acceleration of gravity (Earth)	g	10.0 m/s^2
Boltzmann constant	k	1.38×10^{-23} J/K
Electron charge	e	1.60×10^{-19} C
Electron mass	m_e	9.11×10^{-31} kg
		0.511 MeV/c^2
Electron-volt	eV	1.60×10^{-19} J
Permeability of free space	μ_o	$4\pi \times 10^{-7}$ N/A^2
Permittivity of free space	ε_o	8.85×10^{-12} $C^2/N\text{-}m^2$
Planck constant	h	6.63×10^{-34} J-s
Proton mass	m_p	1.67×10^{-27} kg
		938 MeV/c^2
Speed of light in vacuum	c	3.00×10^{8} m/s
Speed of sound in air (20° C)	v_s	340 m/s
Temperature conversion		0° C = 273 K

Small angle approximation (θ in radians): $\sin(\theta) \approx \tan(\theta) \approx \theta$

Series expansion for x << 1: $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots$

Index of refraction for air (STP): 1.000293

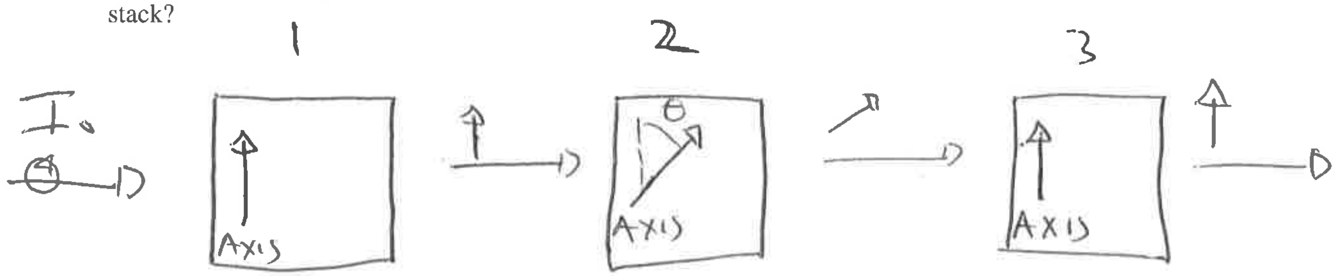


Problem 1: Short Answer (40 points total):

a) True or False? Aberrations only occur for real images. Explain your answer and provide a figure if you like.

FALSE. THE SAME ISSUES THAT LEAD TO ABERRATIONS WITH REAL IMAGES PRODUCE ABERRATIONS WITH VIRTUAL IMAGES. AS AN EXAMPLE, TAKE CHROMATIC ABERRATION – CAUSED BY DISPERSION AND HENCE DIFFERENT FOCAL LENGTH FOR DIFFERENT λ. THE f WILL CHANGE WITH λ FOR BOTH TYPES OF IMAGES. SEE ABOVE.

b) Circularly polarized light of irradiance I_0 is normally incident on a stack of three linear polarizers. The polarization axes of the first and third polarizers are oriented in the same direction. The polarization axes of the first and second polarizer have a relative angle of 30^o. What is the irradiance of the light transmitted by the stack?



I_0 $\qquad \frac{I_0}{2}$ $\qquad \frac{I_0}{2}\cos^2\theta$ $\qquad \frac{I_0}{2}\cos^4\theta$

$$I_{TOT} = \frac{I_0}{2}\cos^4\theta \qquad \theta = 30^\circ$$

$$I_{TOT} = \frac{I_0}{2}\left(\frac{\sqrt{3}}{2}\right)^4 = \frac{I_0}{2}\left(\frac{3}{4}\right)^2 = \boxed{\frac{9}{32} I_0}$$

V = Speed of Sound

Problem 1 (continued):

LET $V' = V_G = V_m$ EITHER G. OR M. MOVING

c) Suppose that George blows a whistle and Martha hears it. She will hear an increased frequency whether she is running towards George or George is running towards her. Are the increases in frequency the same in each case? If not, which apparent frequency would be higher? Assume the same speed of running by either George or Martha. Explain your answer.

INCREASES IN f ARE NOT THE SAME. HIGHER f WHEN SOURCE (GEORGE) IS MOVING

① MOVING SOURCE (GEORGE)

$$f_G = f_0\left(\frac{V}{V-V'}\right) = f_0\left(\frac{V}{V(1-V'/V)}\right)$$

$$f_G = f_0\left(\frac{1}{1-V'/V}\right) = f\left[1 + \frac{V'}{V} + \frac{V'^2}{V^2} + \cdots\right]$$

CLEARLY $f_G > f_M$

② MOVING Detector (Martha)

$$f_m = f_0\left(\frac{V+V'}{V}\right)$$

$$= f_0\left[1 + \frac{V'}{V}\right]$$

Some NUMBERS $V' = 5\ m/s$, $V = 340\ m/s$

$f_G = (1.0149) f_0$

$f_m = (1.0147) f_0$

d) You are in a store examining sunglasses displayed in a glass case. The salesperson claims that the sunglasses have Polaroid filters. You suspect that the sunglasses are just tinted plastic. You ask to see a couple of the sunglasses. Describe two ways in which you could determine the truth of the salesperson's claims (in the store). Explain your answer.

1) TRANSMISSION

TAKE TWO SUNGLASSES AND LOOK AT LIGHT GOING THRU BOTH OF THEM. ROTATE ONE - CHECK IF TRANSMISSION → 0 (MALUS' LAW)

2) REFLECTION

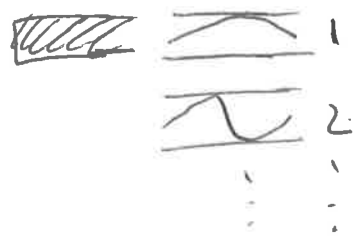
USE ONE PAIR TO OBSERVE LIGHT REFLECTED OFF GLASS [$n_1 = 1.0$, $n_2 \simeq 1.5$ $\theta_B \simeq 56°$ BREWSTER ANGLE] GLASS AT $\theta \simeq 56°$ WILL REFLECT LIGHT INCIDENT ON GLASS AT $\theta \simeq 56°$ WITH LINEAR POLARIZATION. CHECK THIS WITH GLASSES.

Problem 2: (30 Points Total): $V_s = 340\ m/s$

A road tunnel leading straight through a mountain greatly amplifies sounds at frequencies of 135 Hz and 138 Hz.
a) Find the shortest length the tunnel can be. Explain why you know that this is the shortest length. (Note that the speed of sound is given on Page 1).

TUNNEL = OPEN-OPEN PIPE ⟹ STANDING WAVES

$\lambda_n = \frac{2L}{n}, \quad n = 1, 2, 3 \ldots$ (LIKE STRING)



LET $f_a = 135\ Hz$ a, b INTEGERS
$f_b = 138\ Hz$

i) ASSUME $b = a + 1$ (MODES DIFFER BY 1)

$V_s = f_a \lambda_a = f_b \lambda_b \Rightarrow f_a \frac{2L}{a} = f_b \left(\frac{2L}{a+1}\right)$

$a = \left(\frac{f_b}{f_a} - 1\right)^{-1} = 45$

$b = 46$

$\lambda_a = \frac{V_s}{f_a} = 2.52\,m = \frac{2L}{a} \Rightarrow \boxed{L = 56.7\,m}$

ii) NOW TRY $b = a + 2$, YOU GET $a = 90$, $b = 92$ $L = 113\,m$

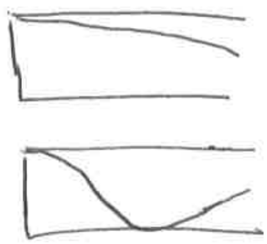
HENCE SOLUTION WE FOUND FOR $b = a + 1$ IS FOR THE SHORTEST LENGTH OF TUNNEL.

Problem 2 (continued)

b) Suppose one end of the tunnel is completely closed off due to a rock slide. What would be the lowest frequency sound the tunnel would greatly amplify now? Would either 135 Hz of 138 Hz be amplified in this case?

NOW TUNNEL = OPEN-CLOSED PIPE

n=1
n=3



$$\lambda_n = \frac{4L}{n}, \quad n = 1, 3, 5 \ldots$$

LOWEST FREQUENCY corresponds to LARGEST λ

$\lambda_1 = 4L = 227\,m$

$f_1 = \frac{V_s}{\lambda_1} = \boxed{1.5\,Hz}$ LOWEST TONE

CHECK $f_a = 135\,Hz \rightarrow a = 90$

$f_b = 138\,Hz \rightarrow b = 92$

NOT modes of open-closed pipe

TONES OF 135 Hz, 138 Hz WOULD **NOT** BE AMPLIFIED

Physics 1CH Midterm #2 $\sin\theta \simeq \tan\theta \simeq \theta$ **May 12, 2022**

Problem 3: (30 points total) SMALL ANGLE APPROX. VALID

Plane waves of monochromatic light of wavelength 600 nm are normally incident on two identical slits, with a separation distance *a*. In the interference pattern observed on a screen 1.5 m from the slits, the bright fringes are 3 mm apart. For this problem you can ignore diffraction effects.

a) What is the slit separation *a*? $L = 1.5\,m$, $\lambda = 600\,nm$

INTERFERENCE

MAX $a \sin\theta = m\lambda$, $m = 0, \pm1, \pm2...$

$m = 0$, $\theta_0 = 0$, $Y_0 = 0.0\,cm$

$m = 1$, $\sin\theta_1 \simeq \theta_1 = \lambda/a$, $Y_1 = 3.0\,mm$

$$Y_1 = L \tan\theta_1 \simeq L\theta_1 = \frac{L\lambda}{a} \Rightarrow a = \frac{L\lambda}{Y_1}$$

$$a = 3\times10^{-4}\,m = \boxed{0.3\,mm}$$

b) Derive an expression for $\Delta\theta$, defined as the angle between the two points on a fringe where the irradiance is one-half that at the center of the fringe. What is the value of $\Delta\theta$?

IRRADIANCE $I_{TOT} = 4I_0 \cos^2(\delta/2)$, $\delta = ka\sin\theta$

Take Central fringe, $\theta = 0$. FIND ANGLE $\theta_{1/2}$ WHERE I_{TOT} IS DOWN BY FACTOR OF 2 - Clearly THIS IS FOR $\frac{\delta}{2} = \pi/4$

$$\delta = ka\sin\theta_{1/2} = \pi/2$$

$$\frac{2\pi a}{\lambda}\sin\theta_{1/2} = \pi/2$$

$$\sin\theta_{1/2} \simeq \theta_{1/2} = \frac{\lambda}{4a}$$

$$\Delta\theta = 2\theta_{1/2} = \boxed{\frac{\lambda}{2a}} = \boxed{1.0\times10^{-3} \text{ OR } 1\,mrad}$$

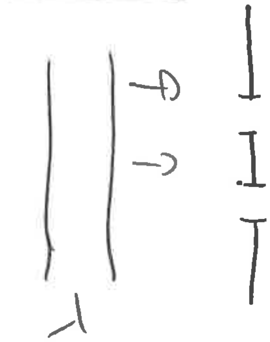
Problem 3 (continued)

Now a piece of clear glass of thickness 0.75 μm is placed on the top slit. The glass is non-dispersive and has an index of refraction of 1.4.

c) With the glass in place, what happens to the interference pattern on the screen? Discuss how the pattern changes in a quantitative way, if at all.

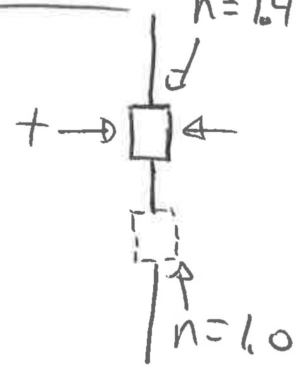
THE GLASS INTRODUCES AN OPTICAL PATH DIFF. (Δ) BETWEEN THE SLITS

BEFORE



$\Delta = 0.0$m

AFTER



$t = 0.75\mu m = 750 nm$

$\Delta = n_g t - n_{air} t$

$= (n_g - n_{air}) t$

$= 300 nm$

$\boxed{\Delta = \lambda/2}$

THE UPPER SLIT HAS AN ADDITIONAL $\lambda/2$ PATH, SO THE INTERFERENCE PATTERN WILL SHIFT UP BY HALF A FRINGE, OR BY $\boxed{1.5 mm}$.

ALL INTERFERENCE MAX → MIN ON SCREEN
MIN → MAX

CENTER $\theta = 0, y = 0$ IS NOW A MINIMUM POINT