Name SOLUTIONS UID

Physics 1CH Midterm #1

April 21, 2022

On all problems, you need to show your work to get full credit.

Below are a set of numerical constants. If you have any questions, please raise your hand to ask for help.

Acceleration of gravity (Earth)	g	$10.0\ m/s^2$
Boltzmann constant	k	1.38×10^{-23} J/K
Electron charge	e	1.60×10^{-19} C
Electron mass	m_e	9.11×10^{-31} kg
		$0.511\ MeV/c^2$
Electron-volt	eV	1.60×10^{-19} J
Permeability of free space	μ_o	$4\pi \times 10^{-7}\ N/A^2$
Permittivity of free space	ε_o	$8.85 \times 10^{-12}\ C^2/N\text{-}m^2$
Planck constant	h	6.63×10^{-34} J-s
		4.14×10^{-15} eV-s
Proton mass	m_p	1.67×10^{-27} kg
		$938\ MeV/c^2$
Speed of light in vacuum	c	3.00×10^{8} m/s
Speed of sound in air (20° C)	v_s	340 m/s
Temperature conversion		0° C = 273 K

Small angle approximation: $\tan(\theta) = \sin(\theta) = \theta$ (for θ in radians)

$\cos(\theta) = \sin(90° - \theta)$ $\sin^2\theta + \cos^2\theta = 1$

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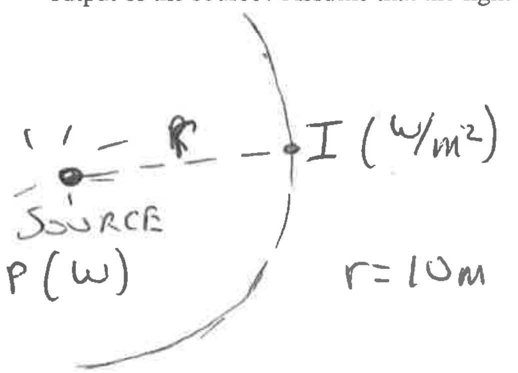
Problem 1: Short Answer (40 points total):

a) The maximum electric field at a distance of 10 m from a point source of light is 5.0 V/m. What is the power output of the source? Assume that the light output of the source is isotropic.



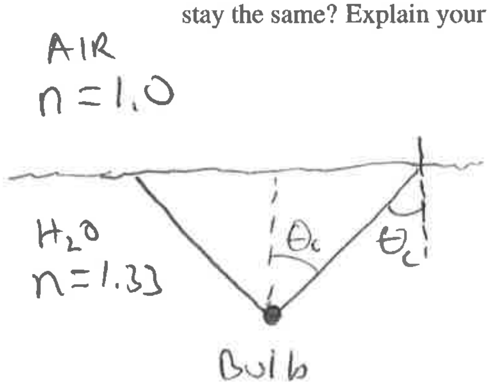
$I = \frac{1}{2}\epsilon_0 c E_0^2 \qquad E_0 = 5.0\ V/m$

$I = 0.033\ W/m^2$

$P = I(4\pi r^2) = \boxed{41.7\ W}$

IRRADIANCE $I = \frac{P}{4\pi r^2}$ (POWER)

b) A light bulb is immersed in a pool of water. Light travels out isotropically from the light bulb but only some of the light escapes the pool (i.e. crosses the water's surface and exits into the air above). What happens to the fraction of light that escapes the pool as the bulb is moved deeper into the water ... does it increase, decrease or stay the same? Explain your answer and providing a diagram would be useful.



$\sin\theta_c = \frac{1.0}{1.33}$

$\theta_c = 48.8°$

LIGHT SHINES OUT IN ALL DIRECTIONS, BUT THE ONLY LIGHT THAT ESCAPES THE POOL IS THAT HITTING THE SURFACE AT $\theta_1 < \theta_c$. THUS LIGHT WITHIN A CONE OF ANGLE θ_c WILL ESCAPE.

FRACTION OF LIGHT ESCAPING $f = \frac{\Omega_{cone}}{4\pi}$

WHERE Ω_{cone} IS SOLID ANGLE OF CONE

WHEN BULB IS MOVED DEEPER, THE CIRCULAR AREA ON SURFACE INCREASES BUT THE SOLID ANGLE OF THE CONE IS UNCHANGED. HENCE $\boxed{\text{f STAYS THE SAME}}$

Problem 1 (continued):

c) A harmonic wave is moving on a string in the negative y-direction with an amplitude of 2.0 m, a speed of 8 m/s and a wavelength of 4π m. The wave's displacement is in the z-direction. At time $t = 0$, the displacement at the origin is 2.0 m. Write the equation for the wave in our standard form (i.e. using wave number and angular frequency) using the complex representation. Substitute numerical values for all quantities.

WAVE IS OF FORM

$z(y,t) = A\cos(ky+\omega t)$
$= e^{i(ky+\omega t)}$

SINCE { WAVE MOVES ALONG $-y$; DISPLACEMENT IS IN z; $z(0,0)=A$ }

HERE, $\lambda = 4\pi\,m$

$A = 2.0\,m$, $V = 8\,m/s$

$k = 2\pi/\lambda = 0.5\,m^{-1}$

$\omega = vk = 4$ rads/s

$$\boxed{z(y,t) = (2.0\,m)\,e^{i(0.5y+4t)}}$$

WHERE $Re[z(y,t)]$ IS IMPLIED

d) No matter where you stand in front of a certain mirror, your image appears upright. What type of mirror (convex, concave, planar) could this be? Explain your answer.

$\boxed{\text{EITHER CONVEX OR PLANAR}}$

THESE BOTH ALWAYS GIVE UPRIGHT (VIRTUAL) IMAGES.

$i < 0$, SO $m = -i/o > 0$ UPRIGHT

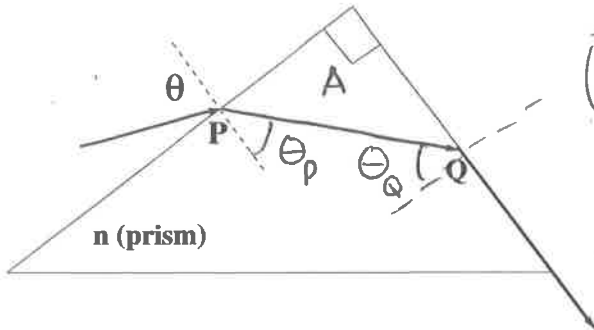
A CONCAVE MIRROR WILL GIVE EITHER AN UPRIGHT OR INVERTED IMAGE, DEPENDING ON THE OBJECT DISTANCE.

Problem 2: (25 Points Total):

As shown in the figure below, a ray of light, initially in air, strikes a 90° prism at point P. It refracts there and travels through the prism to refract again at point Q, whereupon it travels along the right-side prism surface. Assume that the index of refraction of air is 1.0.

$n_{air} = 1.0$

$n = n_{PRISM}$



TRIANGLE A

$(90° - \theta_P) + (90° - \theta_Q) + 90° = 180°$

$\theta_P + \theta_Q = 90°$ (I)

a) Determine an expression for the index of refraction of the prism, n_{prism}, in terms of the angle of incidence θ. Your expression should not depend on angles other than the angle of incidence. For an angle of incidence of 60°, what must the index of refraction be for the light ray to take this path?

POINT P, SNELL'S LAW! $n_{air} \sin\theta = n_{prism} \sin\theta_P$

$\sin\theta_P = \sin\theta / n$ (II)

POINT Q, θ_Q IS AT CRITICAL ANGLE

$\sin\theta_Q = \frac{1}{n}$, $n \sin\theta_Q = 1$

USE (I) $n \sin(90° - \theta_P) = 1$

$n \cos(\theta_P) = 1 \rightarrow n\sqrt{1 - \sin^2\theta_P} = 1$

USE (II) $n\sqrt{1 - \left(\frac{\sin\theta}{n}\right)^2} = 1$

$n^2 - \sin^2\theta = 1$

$\boxed{n = n_{PRISM} = \sqrt{1 + \sin^2\theta}}$

For $\theta = 60°$

$n_{PRISM} = \sqrt{1 + (\sqrt{3}/2)^2} = \sqrt{1.75}$

$= \boxed{1.32}$

Problem 2 (continued)

b) What is the upper bound on the value of the index of refraction, for light to have such a path through the prism?

MAXIMUM VALUE OF $\sin^2\theta$ IS 1.0

WHEN $\theta = 90^\circ$ ($\theta = -90^\circ$ DOESN'T make sense)

[JUST LESS THAN 90° ACTUALLY]

$$(n_{prism})_{MAX} = \sqrt{1+1} = \sqrt{2} \quad \boxed{= 1.41}$$

$\theta_P = 45^\circ$

$\theta_Q = 45^\circ$

WITH n_{prism} Larger THAN THIS

THIS RAY GEOMETRY IS NOT POSSIBLE

AND RAY WILL TOTALLY INTERNAL Reflect AT Q.

e.g. $n_{prism} = 1.5$

TAKE $\theta = 90^\circ$, $\theta_P = 41.8^\circ$

$\theta_Q = 48.2^\circ$ BUT $\theta_C = 41.8^\circ$

So TIR @ Q

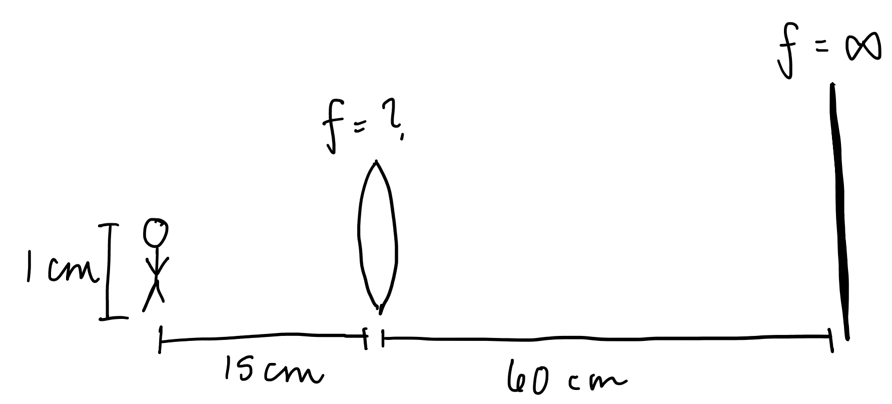
Midterm Q3

Tuesday, April 19, 2022 8:19 PM

Problem 3: (35 points total)

A lens of unknown focal length and a plane mirror are located on an optical bench. The mirror is located 60 cm to the right of the lens. An upright object, O_1, is 1.0 cm high and located 15 cm to the left of the lens. The lens creates a real image (I_1) of O_1 that is twice the size of O_1 in magnitude.

a) What is the focal length of the lens and is it converging or diverging?



real image: $i > 0$

twice the size: $M_T = \left|-\frac{i}{o}\right| = 2$

$o = 15$ cm, so $i = 30$ cm

then the Gaussian lens formula is:

$$\frac{1}{30} + \frac{1}{15} = \frac{1}{f} \rightarrow \frac{3}{30} = \frac{1}{f}, \text{ or } f = 10\text{ cm}$$

which is positive, so the lens is converging

b) Determine the location, size, and orientation of the image of the mirror, I_2. Is this image real or virtual?

O_2 is I_1 from part a, which is an inverted, 2 cm tall real image 30 cm to the right of the lens, and 30 cm to the left of the mirror.

$$\frac{1}{30} + \frac{1}{i} = \frac{1}{\infty} \rightarrow i = -30\text{ cm}, \text{ so}$$

I_2 is located 30 cm to the right of the planar mirror

$M_T = -\left(\frac{-30}{30}\right) = +1$, so it is an erect image of O_2 of the same size, 2 cm tall

O_2/I_1 is inverted relative to the original object, so I_2 is inverted

this is a virtual image

Problem 3 (continued)

c) Consider the light that returns back through the lens to form a final image, I_3. Determine the location, size and orientation of I_3. Is this image real or virtual?

O_3 is I_2 from part b, located 90 cm to the right of the lens. so $o = 90$ cm, $f = 10$ cm, and we have:

$$\frac{1}{90} + \frac{1}{i} = \frac{1}{10} \qquad \frac{1}{i} = \frac{9}{90} - \frac{1}{90} = \frac{8}{90} \rightarrow i = 90/8 = 11.25$$

so I_3 is located 11.25 cm left of the lens

$M_T = \frac{-11.25}{90} = \frac{-90/8}{90} = -\frac{1}{8} \rightarrow$ so the image is 1/8 of the size of O_3 (2 cm), which is 0.25 cm high

it is also inverted w.r.t. O_3 (inverted), so I_3 is upright with respect to the original object

the image is real

d) On the axis below, draw the lens, the mirror, and the original object O_1. Then provide a ray-trace for at least two rays from O_1 to I_1, from O_2 to I_2, and from O_3 to I_3. Indicate the positions of O_1, O_2, O_3, I_1, I_2, I_3, and F_1.